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INSTITUTES
OF
NATURAL PHILOSOPHY,
THEORETICAL AND PRACTICAL.

By WILLIAM ENFIELD, LL. D.

FIRST AMERICAN FROM THE SECOND LONDON EDITION,

WITH SOME CORRECTIONS;
CHANGE IN THE ORDER OF THE BRANCHES;
AND THE ADDITION OF
AN APPENDIX TO THE ASTRONOMICAL PART,
SELECTED FROM
MR. EWING'S PRACTICAL ASTRONOMY.

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THE INTRODUCTION TO THE
FIRST PRINCIPLES OF CHEMISTRY,
BEING MADE AN APPENDIX.

OMNIS PHILOSOPHIÆ DIFFICULTAS IN EO VERSARI VIDETUR, UT A PHÆNOMENIS MOTUUM INVESTIGEMUS VIRES NATURÆ, DEINDE AB HIS VIRIBUS DEMONSTREMUS PHÆNOMENA RELIQUA.—*Newton.*

The Improvements ſecured according to Act of Congreſs.

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JAN. 1802.

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REV. JOSEPH PRIESTLEY,

DOCTOR OF LAWS, FELLOW OF THE ROYAL SOCIETY, &c.

IN TESTIMONY OF RESPECT FOR

A CHARACTER EMINENTLY DISTINGUISHED

BY

COMPREHENSIVE AND ENLARGED VIEWS OF SCIENCE,

ASSIDUOUS AND SUCCESSFUL RESEARCHES INTO NATURE,

AN ARDENT LOVE OF TRUTH,

INDEFATIGABLE ZEAL IN THE SERVICE OF RELIGION,

SIMPLICITY OF MANNERS,

AND

AN ACTIVE SPIRIT OF PHILANTHROPY,

THIS WORK

IS INSCRIBED,

BY HIS AFFECTIONATE FRIEND,

AND OBEDIENT SERVANT,

WILLIAM ENFIELD.

PREFACE.

NOTHING can be an adequate apology for obtruding upon the world a new Elementary Work, in a branch of Science already well underſtood, except the plea of utility. It is wholly upon this ground, that I venture to ſubmit the following Treatiſe to the public inſpection.

The difficulty which I met with, in providing my Claſſes[*] with a Text-book in Natural Philoſophy, neither, on the one hand, materially deficient in Mathematical Demonſtration, nor, on the other, too copious, or too abſtruſe, for the purpoſe of elementary inſtruction, firſt ſuggeſted the idea of this work. And the apprehenſion, that others may have met with the ſame difficulty, induces me to make it public, in hopes that it may be of ſome uſe to thoſe who wiſh to ſtudy, or to teach, this Science ſyſtematically.

To that claſs of readers who are ſatisfied with general views, this work will be of little ſervice. Sketches of philoſophy, ſufficiently comprehenſive to anſwer their purpoſe, will eaſily be found. But the knowledge which is gathered up in this curſory manner, muſt unavoidably be ſuperficial, and will, in many particulars, be confuſed and inaccurate. What Cicero ſays of Philoſophy in general, is particularly true of Natural Philoſophy: *Difficile eſt enim in philoſophia pauca eſſe ei nota, cui non ſint aut pleraque,*

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[*] In the Warrington Academy.

pleraque, aut omnia.[*] It may be laid down as an univerſal maxim, that there is no eaſy method of attaining excellence. The ſmall portion of learning, or ſcience, which is to be acquired by the help of facilitating expedients, has been juſtly compared [†] to a temporary edifice built for a day. It is as unreaſonable, to hope to acquire knowledge without undergoing the labour by which it is uſually gained, as it would be to expect, that an acorn will become an oak without paſſing through the ordinary proceſs of vegetation.

All the knowledge of Natural Philoſophy which can be acquired by curſory reading, without the aſſiſtance of mathematical learning, muſt conſiſt in an acquaintance with leading facts and general concluſions. To underſtand the manner in which the laws of nature have been inferred from theſe facts, and to be able with certainty and preciſion to apply theſe laws to the explanation of particular phenomena, neceſſarily requires a previous knowledge of the elements of Geometry, Trigonometry, the Conic Sections, and Algebra. A mechanic who ſhould ſet about making a machine without the requiſite tools, would not act more abſurdly, than a ſtudent who ſhould attempt to underſtand the ſcience of Natural Philoſophy without theſe helps. A Preceptor who profeſſes to teach this ſcience in the eaſy and amuſing method of experiment alone, is an architect without his rule, plumb-line, and compaſſes.

Facts are, it is true, the materials of ſcience ; and much praiſe is unqueſtionably due to thoſe who have increaſed the public ſtore, by new experiments accurately made, and faithfully related. But it is not in the mere knowledge, nor even in the diſcovery of facts, that philoſophy conſiſts. One who proceeds thus far, is an experimentaliſt ; but he alone, who, by examining the nature, and obſerving the relation of facts, arrives at general truths, is a philoſopher. A moderate ſhare of induſtry may ſuffice for the former : patient attention, deep reflection, and acute penetration, are neceſſary in the latter. It is therefore no wonder, that amongſt many experimentaliſts there ſhould be few philoſophers.

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[*] Tuſcul. Quæſt. II. 1.

[†] Knox on Liberal Education, §. 9.

The hardy perſeverance, and the vigorous exertions, which are neceſſary to form this character, are ſo contrary to that effeminacy and frivolity which diſtinguiſh the preſent age, that, if it were not for the proviſion which is made in our Univerſities, and other Seminaries, for the propagation of ſound learning of every kind, there would be ſome reaſon to apprehend, that all the more abſtruſe and difficult branches of ſcience would be excluded from the modern ſyſtem of education, and conſequently would fall into diſeſteem and neglect.

It is by no means the intention of this Treatiſe to encourage the indolent ſpirit of the times, by opening a bye-path to the Temple of Philoſophy. The known and beaten road is the ſafeſt and the beſt. It has been with the view of aſſiſting the ſtudent in his progreſs, that I have attempted to arrange the leading truths of Natural Philoſophy in a perſpicuous method, and to demonſtrate them with conciſeneſs; adding a brief deſcription of Experiments, adapted to illuſtrate and confirm the Propoſitions to which they are reſpectively ſubjoined.

Being more deſirous to be uſeful than to appear original, I have freely ſelected from a variety of Authors ſuch materials as ſuited my deſign. Thoſe who are converſant with this claſs of writers will perceive that, amongſt many others, I have made uſe of the works of NEWTON, *Keil*, *Whiſton*, *Graveſande*, *Cotes*, *Smith*, *Helſham*, *Rowning*, and laſtly, *Rutherforth*, whoſe arrangement I have in part adopted.

With reſpect to any inaccuracies or miſtakes which may have eſcaped my attention, I muſt rely on that candour, which thoſe who are beſt acquainted with the extent and difficulty of this undertaking will be moſt inclined to exerciſe.

ADVERTISEMENT

To the SECOND *LONDON* EDITION,

By the EDITOR.

IN laying before the public a new edition of "*The Inſtitutes of Natural Philoſophy, by the late Dr. Enfield*," the Editor feels it incumbent on him to aſſure the Reader, that he has endeavoured, as far as was conſiſtent with an elementary book, to avail himſelf of thoſe advantages which the publication of new diſcoveries, and new works in ſcience, has afforded him; and although the limits of an advertiſement will not allow him to particularize all the additions that will be found interwoven with the various parts of the volume, yet it may be expected that, in this place, ſome notice ſhould be taken of the moſt material of them; and it is preſumed that the following account will be deemed ſufficient for the purpoſe.

In the firſt book, the propoſitions on the diviſibility of matter, and the attraction of coheſion, are more fully diſcuſſed, and a very uſeful corollary is drawn from that on the attraction of capillary tubes. To the firſt and third propoſitions of the ſecond book, conſiderable additions are ſubjoined; and in the ſecond chapter is inſerted a new propoſition, from which, in conjunction with others, are deduced many corollaries and ſcholiums, connected with the remaining parts of the book.[*] Several examples are alſo given in the two firſt ſections of the fifth chapter, which will be found uſeful to the young ſtudent as illuſtrative of the theory of falling bodies.

In the third book is given, independently of the additions noticed in the margin,[†] an important propoſition on the ſpecific gravities of bodies, with which are connected examples, and a table of the comparative weights of many of the moſt uſeful ſubſtances in nature. Deſcriptions, accompanied with figures, are likewiſe given of the Pyrometer, Air-pump, Barometer, with its application to the meaſuring of altitudes, &c. Farenheit's Thermometer, with a Table of Heat; different kinds of Hygrometers, the Steam-engine, and the Hydrometer.

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[*] See Prop. A, (p. 14.) and Cors. Schols. &c. to Prop. 14, 17, 24, 26, 28, 30, 31, 36, 44, 46, 49, 52, 53, 54, 57, and 58.

[†] See additions to Prop. 3, 6, 12, 13, 18, A, (p. 103) 50, and 55.

The principal additions to the book of Optics, will be found connected with the propoſitions mentioned below;* in the courſe of which are introduced, Mr. Delaval's Theory of Colours; brief accounts of Dr. Blair's achromatic Lenſes, and Dr. Herſchel's grand Teleſcope.

On the ſubject of Aſtronomy, are arranged under the different articles, ſeveral uſeful Tables, and the important diſcoveries of the illuſtrious Dr. Herſchel, which have been carefully ſelected from the laſt twenty volumes of the Philoſophical Tranſactions. The reference in the margin,† will direct the reader to thoſe propoſitions to which the moſt material additions are ſubjoined.

Some valuable treatiſes, on Magnetiſm and Electricity, particularly thoſe of Mr. Cavallo, having appeared ſince the original publication of this volume, it was thought neceſſary very conſiderably to enlarge this part of the work; and it is hoped that the principal diſcoveries in theſe branches of ſcience will now be found under their reſpective heads.

By the ſuggeſtion of a friend, on whoſe judgment the public has long placed great confidence, it has been deemed proper that the firſt principles of Chemiſtry ſhould form a part of the preſent volume; and although we have chiefly confined ourſelves to the intereſting diſcoveries of the philoſophers Black, Prieſtley, and Lavoiſier, on Heat and the Factitious

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* See Prop. 5, 13, 22, A, (p. 185) 42, B, (192) 61, 62, 66, 68, 69, 76, D, (p. 212) 94, 122, 128, 144, and 145.

† See Def. 1, 12. Prop. 8, 16, 20, 32, 35, 39, 51, 57, 72, 78, 79, 83, 109, 116, 117, 118, 119, 120, 123, 136, 167, 168, 177, 179, A, B, (p. 351) 182, 183.

Airs, it is nevertheleſs preſumed, that enough has been ſaid on theſe ſubjects to render the doctrines and introductory practice to modern chemiſtry perfectly intelligible to any perſon who may be deſirous of farther proſecuting the ſtudy of this amuſing and uſeful ſcience.

The Reader ought to be apprized, that beſides additions to the old plates, two new ones are now given :—one, as already noticed, accompanying deſcriptions of ſeveral pneumatic and hydraulic machines, and the other containing figures relating to ſubjects in magnetiſm, electricity, and chemiſtry.

It is hoped that the augmentations to the volume, although they compoſe about one-third of the whole work, will be found ſuch as ought at this period, to be comprehended in an elementary book of ſcience ; and that the ſpeculations of Dr. Herſchel, towards the end of the aſtronomical part, will not be conſidered as an exception : they are at leaſt the ſpeculations of a great mind, and capable of exciting, in every well-diſpoſed heart, emotions of intereſt and exquiſite pleaſure, inaſmuch as they lead to the grandeſt and moſt ſublime notions of the great Author of the univerſe.

The Editor will only add, that in the additions to this work, he has uniformly aimed at conciſeneſs ; and he will conſider his exertions well rewarded, if it be found, by a candid and diſcerning public, that he has not ſacrificed perſpicuity to brevity, and that he has not omitted, within his preſcribed limits, any material article that might ſerve to render the original work, in its preſent enlarged ſtate, generally acceptable and uſeful.

May 14, 1799.

ADVERTISEMENT

To this FIRST AMERICAN EDITION.

THE principal object in undertaking this American Edition of Dr. Enfield's Inſtitutes of Natural Philoſophy was to ſupply our Colleges with a book, which is held in ſo high eſtimation for the uſe of Students in recitations to their Inſtructors. In ſome of theſe ſeminaries of learning Ferguſon's Aſtronomy is recited, and in one or more of them after the exerciſes in Enfield's Philoſophy, excluſive of the Aſtronomical part. It was the opinion of the Rev. Dr. Willard, Preſident of Harvard College, and Profeſſor Webber, that it would be a valuable improvement in this part of Collegial Inſtruction to ſubſtitute Enfield's Aſtronomy for Ferguſon's, provided ſome additions were made, which ſhould be equally or more important than certain Articles in the latter, particularly thoſe relating to the Calculation and Projection of Solar and Lunar Eclipſes, including the neceſſary Tables. On account of ſome particular circumſtances, meaſures were not taken to make the propoſed additions till the printing was far advanced, and aſſurances given, that it would be finiſhed within a ſhort period. Hence it was extremely difficult to execute the plan. Profeſſor Webber, however, conſented to attempt what then ſeemed to be practicable, and what, it was hoped, might in ſome good degree anſwer the purpoſe. And there appears to be a propriety in giving a particular account of the alterations, that have been made.

The errors, which occurred, are corrected. And it was thought expedient to retain the Introduction to the Firſt Principles of Chemiſtry, but to annex it as an Appendix.

A change is made in the order of the branches by inſerting Magnetiſm and Electricity between Pneumatics and Optics, inſtead of placing them after Aſtronomy; as it was thought a more natural and uſeful arrangement for a regular courſe of Inſtruction, the propriety of cloſing with Aſtronomy being particularly obvious.

In the Aſtronomical part the alterations are compriſed in a few particulars, the ſubſtance of which is almoſt entirely taken from Ferguſon's Aſtronomy, as well as the figures, to which they refer.*

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* The particular additions are the explanation of the circles of perpetual apparition and occultation under Prop. XX.—Scholium 2 to Prop. LX.—Scholium to Prop. LXXXII.—Scholium to Prop. CXIV.—Explanation of the figures of orbits of Satellites under Scholium to Prop. CXVI.—Scholium to Prop. CXLIII.—Cor. to Prop. CLXXIII.—Cor. to Prop. CLXXV. A ſmall alteration is made in the demonſtration of Prop. CXIX. The Scholium to Prop. CXIX. and Scholium 3 to Prop. CXX. are omitted. The figures, together with four projections of eclipſes, which are alſo added with the Appendix to the Aſtronomical part, fill two plates.

An Appendix of about 80 pages is ſubjoined to the Aſtronomical part, and conſtitutes the moſt diſtinguiſhing and important peculiarity of this Edition. It contains the moſt uſeful Solar and Lunar Tables, and a Table of Logiſtical Logarithms, together with their explanation, and twenty-two Problems, illuſtrating their uſe and application, and exemplifying the Projection of Eclipſes. Theſe are ſelected from a work of Mr. Alexander Ewing, Teacher of Mathematics at Edinburg, entitled "Practical Aſtronomy," and publiſhed in 1797. With reſpect to the Projection of Eclipſes, there is a ſmall alteration in making two diſtinct Problems for the Lunar and Solar Projections, inſtead of placing them under the reſpective Problems for calculating thoſe Eclipſes. And two Notes are added to the Problem for projecting a Solar Eclipſe. The former contains the neceſſary directions for a different mode of projection; and the latter the Rev. Preſident Willard's Method of finding the point on the ſun's limb, where a Solar Eclipſe begins; the knowledge of which point is of great importance to obſervers.

In the whole execution of the work it has been the unfeigned endeavour of the Editors to merit the approbation of the Public.

BOSTON, *January*, 1802.

ERRATA.

Page 40, *line* 46, for Ggn, read gGn.

Page 79, *line* 17 and 18, inſtead of "(for El. I. 13) RPI, IPZ, are equal to HPR, HPZ; and taking away the common part HPI," read for (Simſon's Conic Sect. II. 11 Cor.)

Page 85, *line* 15, for C, read D.

Page 110. Experiments 9, 10, 11, 12 and 13 do not ſhew the truth of the Propoſition, under which they are placed. In Ex. 12 the effect is produced by the *elaſticity* of air; and in Exps. 9, 10, 11 and 13 by the *increaſed elaſticity* of *condenſed* air.

In Ex. 6 the aſcent of the water may be attributed to the *weight* or *elaſticity* of the air, according as there is a *free* communication or *not* with the external air.

Page 171, *line* 5, after "incidence" inſert, and refraction.

Page 282, *line* 6 from bottom, and in the margin oppoſite, for Plate 1. Fig. 14. read Plate 14. Fig. 1.

Page 291, *line* 4, for Fig. 14. Plate 2. read Plate 14 Fig. 2. as in the margin oppoſite.

Page 305, *line* 1, and in the margin oppoſite, for Plate 15. read Plate 14.

Page 318, *line* 9 from bottom, and in the margin oppoſite, for Plate 14. read Plate 15.

Page 366. Add 1″ to the difference of longitude in time between Greenwich and Cambridge, that is, call it 4h. 44m. 28″, and make the corrections, that this alteration requires.

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BOOK I.

OF MATTER.

Definition I. MATTER is an extended, ſolid, inactive, and moveable ſubſtance.

Scholium. Extenſion and ſolidity are diſcovered to be properties of matter by the ſenſes. Both by the ſight and touch we perceive material ſubſtances to have length, breadth and thickneſs, that is, to be extended: and from the reſiſtance which they make to the touch, we acquire the idea, and infer the property, of ſolidity. It is unneceſſary here to inquire whether ſolidity neceſſarily ſuppoſes impenetrability. Natural Philoſophy, being employed in inveſtigating the laws of nature by experiment and obſervation, and in explaining the phenomena of nature by theſe laws, has no concern with metaphyſical ſpeculations, which are generally little more than unſucceſsful efforts, to extend the boundaries of human knowledge beyond the reach of the human faculties.

Def. II. A body is any portion of matter.

Corollary. All bodies have ſome figure; for, being portions of matter, they are finite, and therefore bounded by lines either ſtraight or curved.

PROPOSITION I.

Matter is infinitely diviſible, or is capable of being divided beyond any ſuppoſed diviſion.

1. Any particle of matter, placed upon a plane ſurface, has an upper and a lower ſide, or a part which touches, and another which does not touch the plane, and is therefore diviſible.

Plate I. Fig. I. 2. Let CO, MD, be two parallel right lines, to which let AB be drawn perpendicular. In the line MD, on one ſide of the perpendicular AB, take, at equal diſtances, the points E, F, G, H. On the other ſide of AB, in the line CO, take any point C, and join CE, CF, &c. Each of the lines CE, CF, &c. will cut off a portion from AB: But whatever number of lines be drawn in the ſame manner from C to MD produced, there will ſtill remain a portion of AB not cut off, becauſe no line can be drawn from the point C to the line MD, which ſhall coincide with CO: The line AB is therefore infinitely diviſible.

Plate I. Fig. 2. 3. Let the right lines AC and GH be drawn perpendicular to the right line BF. In AC, produced at pleaſure, take any points C, C, &c. from which as centres, with the diſtances CA, CA, &c. deſcribe arcs of circles KAL, NAO, &c. touching BF in the point A, and cutting HG. The farther the central point is taken from A, the greater will be the circles, and the nearer will the arcs approach to the line BF; but (El. III. Pr. 16. Cor.) the arcs, touching BF in A cannot touch it in any other point. The line HG is therefore infinitely diviſible.

From this propoſition the following theorems are derived by Dr. Keill, in his fifth lecture.

Theorem I. Any quantity of matter, however ſmall, and any finite ſpace, however large, being given, (as for example, a cube circumſcribed about the orb of Saturn) it is poſſible for the ſmall quantity of matter to be diffuſed throughout the whole ſpace, and to fill it ſo that there ſhall be no pore or interſtice in it, whoſe diameter ſhall exceed a given finite line.

Cor. Hence there may be given a body, whoſe matter, if it be reduced into a ſpace abſolutely full; that ſpace may be any given part of its former magnitude.

Theorem II. There may be two bodies of equal bulk, whoſe quantities of matter being unequal, in any proportion; yet the ſum of their pores, or quantity of void ſpace in each of the two bodies, ſhall be to each other nearly in a ratio of equality.—Example. Suppoſe 1000 cubic inches of gold, when reduced into a ſpace abſolutely full to be equal to one cubic inch: then 1000 cubic inches of water, which is 19 times lighter than gold, will, when reduced, contain $\frac{1}{19}$th part of an inch of matter. Conſequently the void ſpaces in the gold, will be to thoſe in water as 999 to 999 $\frac{18}{19}$, or nearly in the ratio of equality.

In the preſent ſtate of knowledge, it is impoſſible to determine how far the diviſion of matter can be actually carried, or whether there be any indiviſible atoms by the arrangement and combination of which all ſenſible bodies are formed. We are, however, furniſhed both by art and nature with many aſtoniſhing inſtances of minute diviſion.—If a pound of ſilver and a grain of gold be melted together, the gold will be equally diffuſed through the whole ſilver; and if a grain of the maſs, containing only the 5761th part of a grain of gold, be diſſolved in aqua fortis, the gold will ſubſide.

A grain of gold may be ſpread by the gold-beater into a leaf containing 50 ſquare inches. and this leaf may be divided into 500.000 parts: and by a microſcope, magnifying the diameter of an object 10 times, and its area 100 times, the 100th part of each of theſe, that is, the 50 millionth part of a grain of gold will be viſible.

The *natural* diviſions of matter are ſtill more wonderful. In odoriferous bodies a ſurpriſing ſubtilty of parts is perceived: ſeveral bodies ſcarcely loſe any ſenſible part of their weight in a great length of time, and yet continually fill a very large ſpace with odoriferous particles. Dr. Keill has computed the magnitude of a particle of aſſafœtida to be only $\frac{38}{1.000.000.000\,000.000.000}$th part of a cubic inch. Again, Mr. Leewenhoeck informs us that there are more animals in the milt of a cod-fiſh than there are men on the whole earth, and a ſingle grain of ſand is larger than four millions of theſe animals. Moreover, a particle of the blood of one of theſe animalcula has been found, by calculation, to be as much ſmaller than a globe of the $\frac{1}{10}$th of an inch in diameter; as that globe is ſmaller than the whole earth. Nevertheleſs, if theſe particles be compared with the particles of light, they will be found to exceed them in bulk as much as mountains do ſingle grains of ſand.

Theſe inſtances may ſerve to ſhew the amazing fineneſs of the parts of bodies which are nevertheleſs compounded. Gold, when reduced to the fineſt leaf, ſtill retains thoſe properties which ariſe from the modifications of its parts. Microſcopic animalcula are, without doubt, organized bodies, and the globules of their blood are poſſeſſed of ſpecific qualities. Even the rays of light are compounded of an indefinite variety of particles, which, when ſeparated, have the power of exciting the ideas of colours.

Def. III. That force by which the parts of the ſame body, or of different bodies, on their contact, or near approach, are united to or tend towards each other, is called the *attraction of coheſion.*

PROP. II. The attraction of coheſion appears in ſolid bodies.

Exp. 1. Obſerve the different degrees of coheſion in different kinds of wood, ſuſpending weights, from pieces of equal diameter, placed vertically or horizontally, till they break.

2. Meaſure the different degrees of coheſion in ſilk, thread, horſe hair, &c. by weights ſuſpended from cords of each, placed vertically or horizontally.

The reſult of ſundry experiments, made by profeſſor Muſſchenbroek, to ſhew the coheſive power of different ſolids, may be ſeen in the following table. In eſtimating the abſolute coheſion of ſolid bodies, he applied weights to ſeparate them according to their length: the pieces of wood which he uſed were parallelopipedons, each ſide of which was $\frac{27}{100}$ths of an inch, and the metal wires made uſe of were $\frac{1}{10}$th of a Rhinland inch in diameter, and they were drawn aſunder by the following weights:

	℔		℔
Fir	600	Copper	299¼
Elm	950	Braſs	360
Alder	1000	Gold	500
Oak	1150	Iron	450
Beech	1250	Silver	370
Aſh	1250	Tin	49¼
Lead	29¼		

PROP. III. The attraction of coheſion takes place between particles of the ſame fluid.

Exp. 1. A drop of water, at the end of a ſmall cylinder of wood, will hang in a ſpherical form. The drop is ſpherical, becauſe, each particle exerts an equal power in every direction, drawing other particles towards it on every ſide as far as its power extends.

2. Two globules of mercury, on meeting, unite, which union can ariſe only from their ſtrong attraction. Drops of water will do the ſame.

PROP. IV. The attraction of coheſion takes place between two ſolid bodies of the ſame kind; and the more perfect the contact, the greater is the attracting force.

Exp. 1. Two plates of glaſs laid together, though perfectly dry, will cohere.

2. Two leaden balls, having each a flat ſurface of a quarter of an inch in diameter, ſcraped ſmooth, on being forcibly put together, will cohere ſo ſtrongly as ſometimes to require a weight of nearly 100 lb. to ſeparate them.

3. Two poliſhed plates of braſs, ſmeared with oil, will cohere ſtrongly.

M. Muſſchenbroek found that the adheſion of poliſhed planes, about two inches in diameter, heated in boiling water, and ſmeared with greaſe, required the following weights to ſeparate them:

	Cold Greaſe.	Hot Greaſe.
Planes of Glaſs	130 ℔	300 ℔
Braſs	150	800
Copper	200	850
Marble	225	600
Silver	150	250
Iron	300	950

PROP. V. The attraction of coheſion takes place between ſolids and fluids.

EXP. 1. A plate of glaſs, or metal, will retain drops of water, or mercury, when inverted.

2. If a plate of glaſs be in part immerſed in a veſſel of water, the water which lies contiguous to the glaſs will riſe above the level.

3. Water riſes above its level between two parallel plates of glaſs at a ſmall diſtance from each other, and in a glaſs tube having a fine bore, called a capillary tube.

4. The fluid will riſe between parallel plates, and in capillary tubes, *in vacuo*. Hence it appears, that the aſcent of fluids in capillary tubes is not owing to the preſſure of the air.

5. Human blood will riſe to a great height in a tube having an exceedingly fine bore.

6. Water will aſcend in the cavities of ſponge, ſugar, and other porous bodies.

7. If a drop of oil be poured upon a plate of glaſs laid horizontally, and another plate of glaſs be ſo placed as to meet the firſt plate at one edge, and be at ſuch a diſtance from it at the other, as juſt to touch the drop of oil; this drop, becauſe its touching ſurface is continually enlarging, will move with increaſing velocity, towards that edge. If the planes be lifted up on the ſide where they meet, the motion will be retarded, ſtopped or reverſed, according to the degree of elevation.

8. The ſame phenomenon takes place in a tube of unequal bore.

9. Two glaſs bubbles, floating near each other on water, ruſh together.

10. A glaſs bubble floating on water in a glaſs veſſel, moves towards the ſide of the veſſel.

11. Two circular pieces of cork placed upon water, and brought near each other, will be attracted.

12. A circular piece of ice, two inches and a half in diameter, exactly balanced and brought to touch the ſurface of ſome mercury, will be ſo ſtrongly attracted by the mercury, as to require more than nine pennyweights, in the oppoſite ſcale, to reſtore the equilibrium.

13. A piece of wood having a ſmooth and plane ſurface, ſuſpended from a beam and balanced, on touching a ſurface of water, will be attracted; and it will require an additional weight in the oppoſite ſcale to ſeparate them.

SCHOLIUM 1. As it is by the attraction of coheſion that the parts of a body are kept together; ſo when a body is broken this attraction is only overcome. Hence the reaſon of ſoldering of metals; glueing of wood, &c. Hence alſo may be explained why ſome bodies are *hard*, others *ſoft*, and others *fluid*, which properties may reſult from the different figures of the particles, and the greater or leſſer degree of attraction conſequent thereupon. *Elaſticity* may ariſe from the particles of a body, when diſturbed, not being drawn out of each other's attraction; as ſoon, therefore, as the force upon it ceaſes to act, they reſtore themſelves to their former poſition.

Scholium ii. Solids are diſſolved in menſtruums from the particles of the ſolid being more attracted to the fluid than to themſelves. Precipitation ariſes from a like cauſe; for if to the ſolution of any ſolid in a fluid, ſome other ſolid or fluid be added, the particles of which are attracted by the fluid with a greater force than thoſe of the ſolid which was diſſolved, the ſolid falls to the bottom in a fine powder. Thus ſilver diſſolved in aqua fortis is precipitated by copper.

PROP. VI. The heights to which a fluid riſes between parallel plates of glaſs are inverſely as the diſtances of the plates.

The abſolute attractive force of the plates, will always remain the ſame, whatever be the diſtance of the plates. The ſame weight of fluid muſt, therefore, at different diſtances of the plates, be ſupported. But the quantity of fluid ſupported can only continue the ſame, when the height of the column ſupported is reciprocally as its baſe; that is, when as much as the height is increaſed the baſe is diminiſhed, and the reverſe. Now, the length of the baſe remaining unvaried, the baſe can only be made greater or leſs, by increaſing or diminiſhing the diſtance between the plates. Therefore, the force, and the quantity of fluid ſupported, remaining the ſame, the height will be greater as the diſtance of the plates is leſs, and the reverſe.

Let H, B, D, expreſs the height, baſe and diſtance, when the plates are at any given diſtance, and h, b, d, expreſs the ſame when they are brought nearer: from what has been ſhewn, $H : h :: b : B$; but $b : B :: d : D$; therefore $H : h :: d : D$.

Exp. Let two parallel plates of glaſs be immerſed, at different diſtances from each other, in a veſſel of coloured water.

PROP. VII. The ſuſpenſion of the fluid, in capillary tubes, is owing to the attraction of the ring of glaſs contiguous to the upper ſurface of the fluid.*

Every ring of glaſs below the ſurface attracts the water above it as much downwards, as it attracts the water below it upwards, and conſequently can contribute nothing towards the ſupport of the column: and the action of the loweſt ring upon all the fluid in the tube, within its ſurface of attraction, muſt either concur with the force of gravity to bring the fluid downwards, or, acting upon it at right angles, can have no effect in ſuſpending it within the tube. The fluid therefore can only be ſupported by the ring of glaſs contiguous to its upper ſurface, which, attracting upwards, oppoſes the action of gravitation by which the fluid endeavours to deſcend.—This reaſoning may be applied to the fluid raiſed between parallel plates of glaſs.

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* This propoſition has been diſputed. Dr. Hamilton, in his ſecond lecture, ſuppoſes that the ſuſpenſion ariſes from the attraction of the annulus lying juſt within the lower orifice of the tube. But Mr. Parkinſon rejects both ſuppoſitions, and concludes, that the fluid is ſuſtained by the immediate attraction of the glaſs. See Parkinſon's Hydroſtatics, p. 39.

Exp. Let a capillary tube be compoſed of two parts, the bore of one of which is wider than that of the other: immerſe its wider orifice in water, till it is filled to any height leſs than the length of the wider part; the fluid will only riſe to the height to which it would riſe if the tube were throughout of the ſame bore with the wider part: but immerſe the tube till the fluid enters the ſmaller part, and the whole column will be ſuſpended, provided its length do not exceed that of the column which a tube of the ſmaller bore is capable of ſupporting.

Hence it is manifeſt, that the water is ſuſtained by the attraction of the narrower part of the tube, for the wider part could not ſuſtain ſo long a column: it is alſo manifeſt, that it is ſuſtained by the ring contiguous to the upper ſurface; for if it were ſuſtained by the ring at the lower ſurface, no reaſon could be aſſigned why this ring ſhould now ſupport the greater column in both parts of the tube, when it was before only able to ſuſtain a column which filled a part of the wider tube.

Next, let the tube be inverted, and the water be raiſed into the lower extremity of the wider part; when the ſuſpended column is of greater length than that which a tube of the ſame bore with the wider part is capable of ſuſtaining, it will immediately ſink: whence it is manifeſt, that the ſuſpenſion of the column in this caſe depends upon the attraction of the wider part of the tube; for the narrower part could ſuſtain a larger column: and alſo, that it is ſuſtained by the ring contiguous to the upper ſurface; for if it were ſuſtained by the ring at the lower ſurface, it has been ſeen that this ring could ſupport a much longer column.

Schol. The reaſon why the narrower or wider ring ſuſtains a column of the ſame length in the unequal tube above deſcribed, as in a tube throughout of the ſame diameter as the upper ring, is, that the moving forces of the columns are in both caſes the ſame; as will be more fully ſhewn hereafter. Book III. Pr. IV. Cor.

PROP. VIII. In capillary tubes, the heights to which the fluid riſes are inverſely as the diameter of the bores.

The fluid being ſuſpended (Prop. VII.) by the ring of glaſs contiguous to the upper ſurface, and the diſtance to which the attracting force of glaſs reaches being unvaried; the attracting force which ſuſtains the fluid will be as the number of attracting particles, that is, as the circumference, or diameter of the ring, or of the tube. Let Q, q, then, repreſent the quantities of fluid to be raiſed in two tubes of different bores; D, d, the diameters of their bores; and H, h, the heights to which fluids riſe in the tubes; becauſe Q, q, repreſent two cylinders of the fluid, from the properties of the circle and cylinder (El. XII. 2, 11, and 14.) $Q : q :: DDH : ddh$; and from the nature of this attraction, which is as the diameters of the tubes, $Q : q :: D : d$; therefore $DDH : ddh :: D : d$; and conſequently $D : d : h : H$.

Cor. From this propoſition it appears, that in any glaſs capillary tube, the height to which it will elevate water, and keep it ſuſtained, multiplied into the diameter of the tube, is a given quantity; this is found by experiment to be .053 part of an inch; by means of this value the diameter of a capillary tube being given, the height to which it will elevate water will be known, for it will be equal to .053, divided by the diameter: thus ſuppoſe the diameter is $\frac{1}{20}$ of an inch, the height to which the water will be elevated $= .053 \times 20 = 1.06$.

Exp. Let two tubes of different bores be immerſed in a veſſel of coloured water; it will be found, that the water will riſe as much higher in a ſmaller tube, as the diameter of its bore is leſs than that of the larger tube.

PROP. IX. Between two glaſs plates, meeting on one ſide, and kept open at a ſmall diſtance on the other, water will riſe unequally; and its upper ſurface will form a curve, in which the heights of the ſeveral points above the ſurface of the fluid, will be to one another reciprocally as their perpendicular diſtances from the line in which the plates meet.

Plate I. Fig. 3. Let AE be the ſurface of the fluid; AF the line in which the plates meet; HL the curve formed by the ſurface of the raiſed fluid; GB, IC, KD, LE, perpendicular to AE, expreſſing the heights of the reſpective points G, I, K, L, in the curve, above the ſurface of the fluid, and AB, AC, AD, AE, perpendiculars to AF, expreſſing the diſtances of the ſame points from the line in which the plates meet: theſe heights and diſtances are reciprocally proportional. For, let the lines GB, IC, KD, LE, repreſent pillars of fluid of an equal but exceedingly ſmall breadth: thoſe portions of the glaſs plates, which, by their attraction, ſupport theſe pillars, being equal, will ſuſtain equal quantities of fluid; that is, the pillars will be equal. But the pillars may be conſidered as ſimilar parallelipipeds, which (El. XI. 34.) are equal when their baſes and altitudes are reciprocally proportional. And the baſes, being equal in breadth, are as their lengths, that is, as the intervals between the plates: and ſince the intervals continually increaſe as the diſtance from the line in which the plates meet increaſes, theſe intervals, at the points B, C, D, E, are as their diſtances AB, AC, AD, AE, from the line AF. Since, then, the heights of the pillars are reciprocally as the intervals, the heights GB, IC, &c. are reciprocally as the diſtances AB, AC, &c.

Exp. Let coloured water riſe between two glaſs plates (their inner ſurfaces being firſt moiſtened) meeting on one ſide according to the propoſition.

PROP. X. Some bodies appear to poſſeſs a power the reverſe of the attraction of coheſion, called repulſion.

Exp. 1. If a piece of iron be laid upon mercury, the ſurface of the mercury near the iron will be depreſſed.

2. A fine needle laid upon water will ſwim.

3. Two circular plates of tinfoil being placed upon water, and preſſed down by a ſmall additional weight upon their ſurface, repelling the water, will have a cavity round them: but when they are brought near each other, they will ruſh together; the re-action of the water on the outer-ſide of the plates being greater than the re-action on the inner-ſide, where the two cavities produced by repulſion are united.

4. Mercury poured into a recurved glaſs tube, having the bore on one ſide exceedingly fine, and on the other large, will not riſe ſo high in the narrow, as in the wide bore: water will riſe higher.

5. Melted glaſs dropped into water form globules with a ſtem (called prince Rupert's drops) which on breaking the ſtem will burſt with great violence, and fall into powder.

PROP. XI. All bodies on or near the ſurface of the earth tends towards its centre, by the attraction of gravitation.

A ſtone, or other heavy body, let fall, will move towards the earth till it meet with ſome other body to obſtruct its courſe. And bodies move in lines perpendicular to the ſurface, becauſe the point to which they ultimately tend is the centre of the earth, and the line of direction produced, coincides with the radius, and is at right angles with the ſurface which is nearly ſpherical. Some bodies aſcend, becauſe they are acted upon by a force greater than the attraction of gravitation, and in a contrary direction. Vapours, ſmoke, &c. do not deſcend, becauſe they are lighter than the air, and ſupported by it.

Exp. 1. Smoke or ſteam will deſcend in an exhauſted receiver.

2. Any boiling fluid being placed in a ſcale and balanced, the balance will be deſtroyed by evaporation.

Schol. 1. When we ſpeak of attracting powers, we do not attempt to explain their nature, or aſſign their cauſes. Having derived general principles, or laws of nature, from phenomena, we only give a name to theſe principles, in order to explain other appearances by them.

Schol. 2. The tendency of all bodies towards the earth really reſults from their tendency towards the ſeveral parts of the earth. For by an experiment made by Dr. Maſkelyne upon the ſide of the mountain Schehallien, he found the attraction of that mountain ſufficient to draw the plumb-line ſenſibly from the perpendicular. See Phil. Tranſ. Vol. lxv. or Sir J. Pringle's Diſcourſes.

BOOK II.

OF MECHANICS,

OR

THE DOCTRINE OF MOTION.

CHAP. I.

Of the General Laws of Motion.

PROPOSITION I.

EVERY body will preſerve in its ſtate of reſt, or of uniform motion in a right line, until it is compelled, by ſome force, to change its ſtate.

Any body at reſt on the ſurface of the earth will always continue ſo, if no external force be impreſſed upon it to give it motion, and if the obſtacle which hinders the attraction of gravitation from carrying it towards the centre be not removed. A body being put into motion by ſome external impulſe, if all external obſtructions were removed, and the attraction of gravitation ſuſpended, would move on for ever in a right line; for there would be no cauſe to diminiſh the motion, or to alter its direction. This cannot be fully eſtabliſhed by experiment, becauſe it is impoſſible entirely to remove all obſtructions; but, ſince the leſs obſtruction remains the longer motion continues, it may be reaſonably inferred, that if all obſtacles could be removed, motion once communicated to any body would never ceaſe.

Exp. 1. A body at reſt requires ſome degree of force to put it into motion: and when in motion, it will continue to move longer on a ſmooth ſurface than on a rough one; inſtances of which occur in the uſe of friction rollers; in the exerciſe of ſkaiting, &c.

2. If a ſtone be whirled round in a ſtring, on being ſet at liberty it will continue to move with the force which it has acquired.

3. If a veſſel containing a quantity of water be moved along upon an horizontal plane, the water reſiſting the motion of the veſſel, will at firſt riſe up in the direction contrary to that in which the moving force acts: when the motion of the veſſel is communicated to the

water it will perſevere in this ſtate; and if the veſſel be ſuddenly ſtopped, reſiſting the change from motion to reſt, it will riſe up on the oppoſite ſide. In like manner, if a horſe which was ſtanding ſtill, ſuddenly ſtarts forwards, the rider will be in danger of being thrown backwards, if the horſe ſtops ſuddenly, the rider will be thrown forwards.

SCHOL. This propoſition ſuggeſts two methods of diſtinguiſhing between *abſolute* and *apparent* motions. (1.) Abſolute motion, or change of abſolute motion, may ſometimes be diſtinguiſhed from apparent, by conſidering the *cauſes* which produce them. When two bodies are abſolutely at reſt, they are relatively ſo; and the appearance is the ſame when they are moving at the ſame rate, and in the ſame direction: a relative motion, therefore, can only ariſe from an abſolute motion in one or both of the bodies, which (by the Prop.) cannot be produced but by force impreſſed. Hence, then, if we know that ſuch a cauſe exiſts, and acts upon one of the bodies, and not upon the other, we may conclude that the relative motion ariſes from a change in the ſtate of reſt, or abſolute motion of the former; and that with reſpect to the latter, the effect is merely apparent. Thus when a perſon on board a ſhip obſerves the coaſt receding from him, he knows the appearance ariſes from the motion of the ſhip upon which the wind or tide is acting.

(2.) Abſolute motion may ſometimes be diſtinguiſhed from apparent by the *effects* produced. A body in abſolute motion endeavours to proceed in the line of its direction: if the motion be only apparent, there is no ſuch tendency. It is in conſequence of the tendency to perſevere in a rectilinear motion that a body revolving in a circle conſtantly endeavours to recede from the centre. This effort is called a *centrifugal force*; and as it ariſes from abſolute motion only, whenever it is obſerved, we are convinced that the motion is real.

EXP. Let a bucket, partly filled with water, be ſuſpended by a ſtring, and turned round till the ſtring is conſiderably twiſted; then let the ſtring untwiſt itſelf. At firſt the water remains at reſt, but as it acquires the motion of the bucket, the ſurface grows concave to the centre, and the water aſcends up the ſides, thus endeavouring to recede from the axis of motion, and this effect increaſes till the water and bucket are relatively at reſt. When this is the caſe, let the bucket be ſuddenly ſtopped, and the abſolute motion of the water will be gradually diminiſhed by the friction of the veſſel; and at length, when it is again at reſt, the ſurface becomes plane. Thus the centrifugal force does not depend upon the relative, but upon the abſolute motion, with which it begins, increaſes, decreaſes, and diſappears.

PROP. II. The change of motion produced in any body is proportional to the force impreſſed, and in the direction of that force.

Effects are proportional to their adequate cauſes. If, therefore, a given force will produce a given motion, a double force will produce the double of that motion. If a new force be impreſſed upon a body in motion, in the direction in which it moves, its motion will be increaſed proportionally to the new force impreſſed: If this force acts in a direction contrary to that in which the body moves, it will loſe a proportional part of its motion: If the direction of this force be oblique to the direction of the moving body, it will give it a new direction.

Exp. Let one clay ball, ſuſpended by ſtrings, ſtrike another clay ball ſuſpended in the ſame manner, at reſt or in motion, it will communicate a degree of motion greater or leſs in proportion to the force of the ſtriking body: In the oppoſite direction, motion will be deſtroyed in the ſame proportion.

Cor. Since the effect produced by two bodies upon each other, depends upon their *relative* velocity, it will always be the ſame whilſt this remains unaltered, whatever be their *abſolute* motions.

PROP. III. To every action of one body upon another, there is an equal and contrary re-action: Or, the mutual actions of bodies on each other are equal and in contrary directions, and are always to be eſtimated in the ſame right line.

Whatever quantity of motion any body communicates to another, or whatever degree of reſiſtance it takes away from it, the acting body receives the ſame quantity of motion, or loſes the ſame degree of reſiſtance, in the contrary direction: the reſiſtance of the body acted upon producing the ſame effect upon the acting body, as would have been produced by an active force equal to, and in the direction of, that reſiſtance.

Cor. i. Hence it appears, that one body acting upon another, loſes as much motion as it communicates; and that the ſum of the motions of any two bodies in the ſame line of direction cannot be changed by their mutual action.

Cor. ii. This propoſition will explain the manner in which a bird, by the ſtroke of its wings, is able to ſupport the weight of its body. For if the force with which it ſtrikes the air below it, is *equal* to this weight, then the re-action of the air upwards is likewiſe equal to it; and the bird being acted upon by two equal forces, in contrary directions, will reſt between them. If the force of the ſtroke is *greater* than its weight, the bird will riſe with the *difference* of theſe two forces: And if the ſtroke be *leſs* than its weight, then it will ſink with the *difference*.

Exp. Let a clay ball in motion, ſtrike another equal to it at reſt: The ſtriking body will loſe half its quantity of motion, which will be communicated to the other body.

Schol. Theſe three laws of motion may be illuſtrated by experiments, but their beſt confirmation ariſes from hence, that all the particular concluſions drawn from them agree with univerſal experience. They were aſſumed by Sir Iſaac Newton as the fundamental principles of mechanics; and the theory of all motions deduced from them, as principles, being found to agree, in all caſes, with experiments and obſervations, the laws themſelves are conſidered as mathematically true.

CHAP. II.

Of the Compariſon of uniform Motions.

PROP. IV. The quantities of matter in bodies are in the compound ratio of their magnitudes and denſities.

If the magnitudes of two bodies be given, the quantities of matter will be as the denſities: If their denſities be given, the matter will be as the magnitudes: therefore the matter is univerſally in the compound ratio of the magnitudes and denſities. For example; If A and B be two balls equal in magnitude, the quantity of matter in A will be to that in B, as the denſity of A is to that of B: if both be of the ſame denſity, their quantities of matter will be as their magnitudes.

PROP. V. The velocities with which bodies move, are directly as the ſpaces they deſcribe, and inverſely as the times in which they deſcribe theſe ſpaces.

It is manifeſt, that the degree of velocity increaſes as the ſpace a body paſſes over in a given time increaſes, and as the time in which it paſſes over a given ſpace decreaſes; and the reverſe. For example; If one ball A move over ſix feet, and another ball B over three feet in the ſame time, A has double the velocity of B: but if the ball A paſſes over ſix feet in two ſeconds of time, and the ball B paſſes over ſix feet in one ſecond, the velocity of B is double of that of A.

PROP. VI. The ſpaces which bodies deſcribe are in the compound ratio of their times and velocities.

It is evident, that the longer time any body continues to move, and the greater velocity it moves with, the greater ſpace it will paſs through; and the reverſe. If, for example, the body A moves for one ſecond, and the body B moves for two ſeconds, both moving with the ſame velocity, A will move through half as much ſpace as B: If A moves with two degrees of velocity, and B with one degree of velocity, A will, in the ſame time, paſs over twice as much ſpace as B.

PROP. VII. The times in which bodies move are directly as the ſpaces, and inverſely as the velocities.

The greater ſpace any body paſſes through, and the leſs degree of velocity it moves with, the greater muſt be the portion of time taken up in the motion; and the reverſe. For example; If the ball A moves with the ſame velocity with the ball B, but paſſes over double the ſpace, A will move twice as long as B: If A moves over the ſame ſpace with B, and with half the velocity, it muſt, in this caſe alſo, move twice as long as B.

PROP. A. If bodies be acted upon by different conſtant forces, the velocities communicated will vary in a ratio compounded of the forces and times.

Let F, V, T, repreſent force, velocity, and time, and be ſuppoſed variable; it is evident that the velocity will be increaſed and diminiſhed in the ſame ratio with both force, and time, and theſe being independent of each other, V will be as $F \times T$.

COR. If, therefore, F be compared with any other known force f capable of generating a velocity equal to v in the time t, then $V : v :: F \times T : f \times t$.

PROP. VIII. The power required to move a body at reſt is as the quantity of matter to be moved.

Each particle of matter in any body reſiſting motion, a force muſt be exerted upon each particle to overcome this reſiſtance; if, therefore, two bodies containing different quantities of matter are to be moved, the greater body will require the greater force.

DEF. I. The *momentum* of any body is its quantity of motion.

PROP. IX. In moving bodies, if the quantities of matter be equal, the momenta will be as the velocities.

It is manifeſt, that if the body A be equal to the body B, but A has twice the velocity of B, A has twice as much motion as B.

PROP. X. The velocities of two bodies being equal, their momenta will be as their quantities of matter.

The bodies A and B moving with equal velocities, ſince every portion of matter in A has as much motion as an equal portion of B, it is evident, that if A has twice the quantity of matter in B, it muſt have twice as much motion.

PROP. XI. The momenta of moving bodies are in the compound ratio of their quantities of matter and velocities.

The greater quantity of matter there is in any body, and the greater velocity it moves with, the greater will evidently be its quantity of motion; and the reverſe. If, for example, the body A be double of the body B, and moves with twice its velocity, the momentum of A will be quadruple of that of B: For it will have twice the momentum of B from its double velocity, and alſo twice the momentum of B from its double quantity of matter.

Cor. Hence, if in two bodies the product of the quantities of matter and velocities are equal, their momenta are equal.

PROP. XII. The velocities of moving bodies are as their momenta directly, and their quantities of matter inverſely.

The greater momentum any body has, and the leſs quantity of matter it contains, the greater muſt be its velocity. For example; If the body A is half of B, and their momenta are equal, A will move with twice the velocity of B; and if A and B are equal, and the momentum of A is double of that of B, its velocity will alſo be double.

PROP. XIII. The force, or power of overcoming reſiſtance, in any moving body, is as its momentum.

Since a body having a certain degree of motion is able to overcome a certain degree of reſiſtance, it is manifeſt, that with an increaſed momentum, it will be able to overcome a greater reſiſtance.

Cor. Hence the momentum of any body is meaſured by its power of overcoming reſiſtance.

Schol. Let Q, q, denote the quantities of matter in any two bodies, D, d, their denſities, and B, b, their bulk or magnitude, V, v, their velocities, T, t, the times of their motion, S, s, the ſpaces over which they paſs, P, p, the moving powers, M, m, their *momenta*, and F, f, their force: The preceding propoſitions may be thus expreſſed:

Prop. IV. $Q : q :: BD : bd$

V. $V : v :: \frac{S}{T} : \frac{s}{t}$

VI. $S : s :: TV : tv$

VII. $T : t :: \frac{S}{V} : \frac{s}{v}$

A. $V : v :: F \times T : f \times t$

VIII. $P : p :: Q : q$

IX. $M : m :: V : v$: if $Q = q$.

X. $M : m :: Q : q$: if $V = v$.

XI. $M : m :: QV : qv$

XII. $V : v :: \frac{M}{Q} : \frac{m}{q}$

XIII. $F : f :: M : m$

CHAP. III.

Of the Compoſition and Reſolution of Forces.

Def. A. Equable motion is either ſimple or compound. *Simple* motion is that which is produced by the action, or impreſſed force of one cauſe. *Compound* motion is that which is produced by two or more conſpiring powers, i. e. by powers, whoſe directions are neither oppoſite nor coincident.

PROP. XIV. A body acted upon by two forces united, will deſcribe the diagonal of a parallelogram, in the ſame time in which it would have deſcribed its ſides by the ſeparate action of theſe forces.

Plate 1, Fig. 4. If, in a given time, a body, by the ſingle force M impreſſed upon it at the point A, would be carried from A to B; and by another ſingle force N impreſſed upon it at the ſame point, would be carried from A to C; complete the parallelogram ABDC; and with both forces united, the body will be carried in the ſame time through the diagonal of the parallelogram from A to D. For ſince the force N acts in the direction of the right line AC parallel to BD, this force (by Prop. II.) has no effect upon the velocity with which the body approaches towards the line BD by the action of the force M. The body will therefore arrive at the line BD in the ſame time, whether the force N is impreſſed upon it or not; and at the end of that time will be found ſomewhere in the line BD. For the ſame reaſon, at the end of the ſame time it will be found ſomewhere in the line CD; therefore it muſt be found at the point D, the interſection of theſe two lines. And (by Prop. I.) it will move in a right line from A to D.

Exp. Two equal leaden weights ſuſpended at the end of a triangular frame of wood to give them a ſteady motion, and let fall at the ſame inſtant from equal heights, ſtriking a ball ſuſpended by a cord at the point in which their lines of direction meet, will carry it forwards in the diagonal of the parallelogram of thoſe lines produced.

Cor. 1. Hence, the velocity produced by the joint action of two forces is to that with which the body moves by the action of each force ſingly, as the diagonal of the parallelogram to either ſide; for the diagonal is deſcribed in the ſame time with either ſide.

Cor. 2. If two ſides of a triangle repreſent the directions and quantities of two forces, the third ſide will repreſent the direction and quantity of a force equivalent to both acting jointly: For the third ſide may be conſidered as the diagonal of a parallelogram.

COR. 3. A body may be moved through the ſame line by different pairs of forces. In plate 1. fig. 4. AD is the diagonal both to the parallelogram ABCD, and to the parallelogram AEDF; and conſequently expreſſes a force equal to AB, AC, and to AF, AE.

COR. 4. Hence we learn why any heavy body let fall perpendicularly from the top of a maſt, when a ſhip is under full ſail, will fall to the bottom, in the ſame manner as if it had been at reſt.

SCHOLIUM. This propoſition may be farther illuſtrated. If two men ſit upon the oppoſite ſides of a boat in full ſail, and toſs a ball to one another, they will catch the ball, in their turn, juſt as they would have done if the boat had been at reſt. The ball is here acted upon by two forces: (1.) it partakes of the motion of the boat, which is common to the ball, the boat, and the men: (2.) the other force is that with which the man throws it acroſs the boat. By theſe two forces together the ball will deſcribe the diagonal of a parallelogram, one of whoſe ſides is the line that the boat has deſcribed whilſt the ball is flying acroſs; and the other ſide is a line drawn from one man to the other.

PROP. XV. The velocity produced by two joint forces, when they act in the ſame direction, will be as the ſum of the forces, and when they act in oppoſite directions, will be as their difference; and the velocity will be the greater the nearer they approach to the ſame direction, and the reverſe.

In the parallelograms ABCD, in which AB, AC, expreſs the direction and quantity of two joint forces, the ſide AB being placed at different angles with AC, it is manifeſt that as AB approaches towards AC, the diagonal increaſes, till at length it becomes equal to AC, CD, that is, to AC, AB, and the velocity is as the ſum of the forces, ſince they act in the ſame direction. Plate 1. Fig. 5.

In the parallelograms ABCD, as AB recedes from CD, the diagonal decreaſes, till at length it vaniſhes with the angle, and the two ſides AB, AC, conſtitute one right line, the parts of which, AB, AC, repreſenting forces acting in oppoſite directions, if the forces be equal, they will deſtroy each other; if unequal, the velocity will be as their difference. Plate 1. Fig. 6.

PROP. XVI. Any ſingle force or motion may be reſolved into two forces or motions; and the directions of theſe may be infinitely varied: alſo any two forces may be compounded into ſingle forces.

A body moving in the line AD, may be conſidered as receiving its direction and velocity from two forces acting jointly in the directions AB, AC, or from two other forces expreſſed by AF, AE: For (Prop. XIV. Cor. 3.) each pair would produce the ſame effect. In like manner Plate 1. Fig. 4.

manner the direction and quantities of the forces will be diverſified with every change of the ſides of the parallelogram, the diagonal remaining the ſame.

It is alſo manifeſt, that any two joint forces may be compounded into one, being expreſſed by the ſides of a parallelogram, or its diagonal.

PROP. XVII. If a body is acted upon by three forces, which are proportional to, and in the direction of, the three ſides of a triangle, the body will be kept at reſt.

Plate I. Fig. 7. Let a body placed at D be acted upon by three forces AD, GD, FD, proportional to, and in the direction of, the three ſides of the triangle GED: complete the parallelogram GEFD; and make AD equal to, and in the direction of, the diagonal ED.

If the body at D be acted upon by the forces AD, ED, equal and in oppoſite directions, it will be kept at reſt. But the force ED (Prop. XVI.) is equivalent to the two forces DG, DF, that is, DG, GE; therefore the body acted upon by the three forces AD, DG, DF, that is, by three forces proportional to, and in the direction of, the ſides of the triangle GED, will be at reſt.

Exp. Let three weights in the proportions of 3, 4, 5, be ſuſpended from cords, which paſs over pulleys and meet in a point; if the directions of the cords be parallel to the ſides of a triangle, (drawn in a plane parallel to the plane of the cords) whoſe ſides are to each other as the weights, a ball at the point in which the cords meet will be kept at reſt.

Cor. The body will be at reſt if the three forces are proportional to the three ſides of a triangle drawn perpendicular to the direction of the forces; for ſuch a triangle is ſimilar to the former. Draw A*g*, C*d*, and B*e*, perpendicular to the ſides GE, GD, DE, forming a triangle *g e d*, which is equiangular to GED; hence, the ſides about their equal angles being proportional, the forces which are proportional to the lines GE, GD, and DE, are alſo proportional to *g e*, *g d*, and *d e*.

Scholium. A boy's kite, as it reſts in the air, is an inſtance of a body reſting whilſt three forces act upon it. For the kite is acted upon by the wind; by its own weight; and by the ſtring that holds it.

PROP. XVIII. The force of oblique percuſſion is to that of direct or perpendicular action, as the ſine of the angle of incidence to radius.

Plate I. Fig. 8. Let a body ſtrike upon the plane AD, at the point D, in the direction BD: the line BD expreſſing the force of direct impulſe may be reſolved into two others, BC, BA, the one parallel, the other perpendicular to the plane: Of theſe, the force BC, parallel to the plane, cannot affect it: The whole force upon the plane may therefore be expreſſed by BA. But BA is to BD as the ſine of the angle of incidence BDA is to radius.

SCHOL. If the ſurface to be ſtruck be a curve, let AD be made tangent to the curve at D, and the proof will be the ſame.

PROP. XIX. The force of oblique action produced by percuſſion is to that of direct action, as the coſine of the angle comprehended between the direction of the force and that in which the body is to be moved, to radius.

Let FD repreſent a force acting upon a body at D, and impelling it towards E; but let DM be the only way in which it is poſſible for the body to move. The force FD may be reſolved into two forces FG, FH, or GD; of which only the force GD impels it towards M. And, FD being radius, GD is the coſine of the angle FDG, or MDE, comprehended between the direction of the force, and that in which the body is to be moved. Plate 1. Fig. 9.

CHAP. IV.

Of Motion as communicated by Percuſſion in Non-Elaſtic, and Elaſtic Bodies.

DEF. II. Bodies are non-elaſtic, which, when one ſtrikes another, do not rebound, but move together after the ſtroke.

COR. Hence their velocities after the ſtroke are equal.

DEF. III. Bodies are elaſtic, which have a certain ſpring, by which their parts, upon being preſſed inwards by percuſſion, return to their former ſtate, throwing off the ſtriking body with ſome degree of force; when the elaſticity is perfect, the body reſtores itſelf with a force equal to that with which it is compreſſed.

EXP. The exiſtence of this property is viſible in a ball of wool, cotton, or ſpunge compreſſed.

PROP. XX. When one non-elaſtic body in motion, ſtrikes upon another at reſt, or moving with leſs velocity in the ſame direction, the ſum of their momenta remains the ſame after the ſtroke as before.

For (Prop. III. Cor. 1.) as much motion as the ſtriking body communicates, ſo much it loſes; whence, if the motions of the bodies are in the ſame direction, whatever is added to the motion of the preceding body will be ſubducted from that which follows, and the ſum will remain the ſame.

PROP. XXI. When two non-elaſtic bodies, moving in an oppoſite direction, ſtrike upon each other, the ſum of their momenta after the ſtroke, will be equal to the difference of their momenta before the ſtroke.

For (from Prop. III. Cor. 1.) that body which had the leaſt motion will deſtroy a quantity equal to its own in the other; after which, they will move together with the remainder, that is, the difference.

Exp. Let two cylinders, filled with clay, A, B, be of equal weight, and ſuſpended by cords from equal heights; let two other cylinders of the ſame kind, C, D, but in weight as 2 to 1, be ſuſpended from the ſame height. The heights from which they are let fall, in the arc formed by the motion of the cylinder (from the nature of the pendulum, afterwards to be explained) will be the meaſure of their velocity; and (by Prop. XI.) their momenta will be as their velocities multiplied into their quantities of matter; whence the caſes of the two preceding propoſitions may be eſtabliſhed by the following experiments. N. B. Quantity of matter is expreſſed by q, velocity by v, and momentum by m.

No. 1. Prop. XX. Caſe 1. Let the cylinder A fall from the height of 18 inches, upon the cylinder B at reſt. The momentum of A before the ſtroke (by Prop. XI.) is 18; for the quantity of matter is 1, and the velocity 18; whence $q\ 1 \times v\ 18 = m\ 18$. After the ſtroke, the quantity of matter being (Def. II.) 2, and the velocity of each cylinder 9, the momentum will be 18 : $q\ 2 \times v\ 9 = m\ 18$.

No. 2. Caſe 2. Let A fall from 18 inches, and B from 9, in the ſame direction; their momenta before the ſtroke are $18 + 9 = 27$; after the ſtroke, the quantity of matter will be 2, and the velocity $13\frac{1}{2}$; whence $v\ 13\frac{1}{2} \times q\ 2 = m\ 27$.

No. 3. Prop. XXI. Caſe 1. Let the equal cylinders A and B fall in oppoſite directions, from the height of 12 inches; the momenta being equal and oppoſite, the motion of both will be deſtroyed.

No. 4. Caſe 2. Let A fall from the height of 12 inches, and meet B falling in the oppoſite direction from 6 inches; their velocity after the ſtroke being 3, and quantity of matter 2, the momentum will be 6; $q\ 2 \times v\ 3 = m\ 6$.

No. 5. Prop. XX. Caſe 3. Let the cylinder C, double of the cylinder D, fall from 12 inches on D at reſt. Before the ſtroke, the quantity of matter in C is 2, and its velocity is 12; whence its momentum is 24; $q\ 2 \times v\ 12 = m\ 24$. After the ſtroke, the velocity will be 8, and quantity of matter 3; whence $q\ 3 \times v\ 8 = m\ 24$.

No. 6. Caſe 4. Let C fall from 12 inches, and D from 6 inches in the ſame direction. Before the ſtroke, the velocity of C is 12, and quantity of matter 2; whence its momentum is 24; $q\,2 \times v\,12 = m\,24$; and the velocity of D is 6, and its quantity of matter 1; whence $q\,1 \times v\,6 = m\,6$; therefore the whole momentum is 30. After the ſtroke, the velocity of the whole is 10, and the quantity of matter 3; whence $q\,3 \times v\,10 = m\,30$.

No. 7. Prop. XXI. Caſe 3. Let C Fall from 6 inches, and D from 12, in oppoſite directions, the quantity of matter in C being 2, and its velocity 6; and the quantity of matter in D being 1, and its velocity 12, their momenta will be equal, and being oppoſite, will deſtroy each other. C $q\,2 \times v\,6 = m\,12$; D $q\,1 \times v\,12 = m\,12$.

No. 8. Caſe 4. Let C fall from 3 inches, and D from 12, in oppoſite directions: Before the ſtroke, the momentum of C is 6; $q\,2 \times v\,3 = m\,6$, and the momentum of D is 12; $q\,1 \times v\,12 = m\,12$; whence the difference of their momenta is 6. After the ſtroke, the velocity is 2, and quantity of matter 3; whence the momentum is 6; $q\,3 \times v\,2 = m\,6$.

PROP. XXII. When one elaſtic body ſtrikes upon another of the ſame kind, the one loſes, and the other gains, twice as much momentum, as if the bodies had been void of elaſticity.

For, ſince (by Def. III.) perfectly elaſtic bodies, on percuſſion, reſtore themſelves with a force equal to that with which they are compreſſed, whatever momentum is gained by one body, or loſt by the other, on percuſſion, from the law of re-action, the ſame muſt be gained, or loſt, from the power of elaſticity.

COR. 1. Hence the momentum of elaſtic bodies after percuſſion may be found, by doubling the momentum which would have remained, had the bodies been non-elaſtic.

2. If one of the bodies, conſidered as non-elaſtic, would loſe more than half its momentum, as elaſtic, it loſes more than all, that is, acquires a negative momentum in a contrary direction.

EXP. The following experiments may be made with ivory balls ſuſpended from ſtrings; they correſpond with the preceding experiments on non-elaſtic bodies.

Let A and B be equal balls; and let C be a ball double of the ball D.

No. 1. A, falling from 18 inches on B at reſt, has 18 degrees of momentum before the ſtroke: therefore, after the ſtroke, ſuppoſing the balls non-elaſtic, the ſame momentum belonging to the two equal balls together, each has 9 degrees of momentum, and A has loſt and B gained 9. This being doubled, A, as elaſtic, will loſe 18, and B will gain 18 degrees of momentum: whence A will be at reſt, and B will move with 18 degrees of momentum.

No. 2. A, falling from 18 inches, and B from 9 in the ſame direction; as non-elaſtic, after the ſtroke, each has $13\frac{1}{2}$ momentum, or A has loſt $4\frac{1}{2}$, and B gained $4\frac{1}{2}$. As elaſtic, after the ſtroke, A loſes 9, B gains 9; therefore A riſes to 9 inches, B to 18.

No. 3. A and B, falling in oppoſite directions from 12 inches, as non-elaſtic, would loſe all their momentum: as elaſtic, each loſes 24 degrees of momentum; that is, gains 12 in the contrary direction.

No. 4. A, falling from 12 inches, and B in the oppoſite direction from 6, as non-elaſtic, the momentum of each, after the ſtroke, will be in the direction of A; whence A loſes 9, and B loſes 9, moving 3 degrees in the contrary direction. As elaſtic, A loſes 18, or has 6 in the contrary direction, and B loſes 18, or gains 12 in the contrary direction.

No. 5. C, double of D, falling from 12 inches on D at reſt, the momentum of C, before the ſtroke, being 24, and of D nothing; as non-elaſtic, C, after the ſtroke, having its momentum 16, and moving with the velocity 8, will have loſt 4 degrees of velocity, and 8 of momentum: and D will have gained 8 of each. As elaſtic, therefore, C will loſe 8 degrees of velocity, or (Prop. XI.) 16 of momentum, and D will gain 16 of each; that is, C will move with 4 degrees of velocity, and D with 16.

No. 6. C, falling from 12 inches, and D from 6 in the ſame direction, before the ſtroke, the velocity of C is 12, and its momentum 24; and the momentum of D 6. After the ſtroke, as non-elaſtic, the momentum of C is 20, becauſe $q\ 2 \times v\ 10 = m\ 20$; and the momentum of D is 10, becauſe $q\ 1 \times v\ 10 = m\ 10$; therefore C has loſt 4 degrees of momentum, or 2 degrees of velocity, and D gained 4 of each. If, therefore, the gain or loſs be doubled on account of the elaſticity of C and D, C will loſe 8 degrees of momentum, or 4 of velocity, and D will gain 8 of each; that is, C will move with 8 degrees of velocity, and D with 14.

No. 7. C, falling from 6 inches, and D from 12 in oppoſite directions, their momenta being equal, would deſtroy each other without elaſticity: Therefore, being elaſtic, each will acquire the momentum of 12 in oppoſite directions; that is, D will return to 12, and C to 6.

No. 8. C, falling from 3 inches, and D from 12 in oppoſite directions; ſince the momentum of C, before the ſtroke, is 6, and of D 12, as non-elaſtic bodies they would, after the ſtroke, move in the direction of D, with the velocity of 2; whence C would move in the direction contrary to its firſt motion with 4 degrees of momentum, and loſe 10; and D would loſe 10: Therefore, being elaſtic, C will loſe 20 degrees of momentum, and alſo D 20; whence C will move in the contrary direction with 14 degrees of momentum; that is, will return to 7; and D will return to 8.

Cor. 1. If the ſum of two conſpiring momenta, or the difference of two contrary momenta, be divided by the ſum of the quantities of matter in both the moving bodies, the quotient will give the common velocity after the ſtroke.

Schol. Let A and B be two ſpherical bodies, moving with their centres in the ſame line; and let their velocities be a and b. The momentum of A, before the ſtroke, is Aa, and that of B is Bb; their ſum, or their difference is $Aa+Bb$, or $Aa-Bb$. Therefore (by Prop. XX. and XXI.) the momentum, after the ſtroke, is expreſſed by $Aa \pm Bb$, and, their common velocity by $\frac{Aa \pm Bb}{A+B}$. Hence the momentum of A, after the ſtroke, is $\frac{AAa \pm ABb}{A+B}$; and that of B is $\frac{ABa \pm BBb}{A+B}$.

Next, ſuppoſe the bodies perfectly elaſtic. Subtract the momentum of A conſidered as non-elaſtic, after the ſtroke, $\frac{AAa \pm ABb}{A+B}$, from its momentum, before the ſtroke, Aa; and the remainder, $\frac{ABa \pm ABb}{A+B}$, will expreſs the momentum in that caſe loſt by A, and gained by B. Subtract this remainder, $\frac{ABa \pm ABb}{A+B}$, from the momentum of A, as non-elaſtic, after the ſtroke, $\frac{AAa \pm ABb}{A+B}$; and add the ſame remainder to the momentum of B, after the ſtroke, $\frac{ABa \pm BBb}{A+B}$: the difference, $\frac{AAa \pm 2\,ABb - ABa}{A+B}$, will expreſs the momentum of A, after the ſtroke, and the ſum $\frac{2\,ABa \pm BBb \mp ABb}{A+B}$, will expreſs the momentum of B, after the ſtroke, ſuppoſing them perfectly elaſtic. And $\frac{Aa \pm 2\,Bb - Ba}{A+B}$, and $\frac{2\,Aa \pm Bb \mp Ab}{A+B}$, will expreſs their reſpective velocities.

Cor. 2. If there be any number of elaſtic, equal, and ſpherical bodies, whoſe centres are placed in the ſame line, and the firſt body ſtrikes upon the ſecond in the direction of that line, all the bodies will be at reſt except the laſt, which will move off with the velocity of the firſt.

Exp. Several equal ivory balls being ſo ſuſpended as to have their centres in a right line, if the firſt be let fall upon the ſecond, the laſt will fly off, to the height from which the firſt fell.

Cor. 3. When the ſtriking ball is leſs than the quieſcent, there will be an increaſe of momentum.

Exp. Let the ball D fall from 12 inches upon C, double of D, at reſt. If they were non-elaſtic, they would proceed together, and, their velocity being the ſame, C, after the ſtroke, would have double the momentum of D; that is, C would have 8 degrees, and D 4;

whence D would have communicated more than half its momentum to C. The effect being doubled by the elaſticity of the bodies, D communicates to C 16 degrees of momentum, and loſes as much itſelf, or returns with 4 degrees of momentum in the contrary direction: while C moves forwards with 4 degrees more momentum than D had at the firſt. Thus the whole ſum of momentum is increaſed from 12 to 20 degrees; but as much as the momentum is increaſed in the direction in which D firſt moved, ſo much is given to D in the contrary direction. In this manner may momentum be continually increaſed by a ſeries of bodies.

Cor. 4. If a non-elaſtic body ſtrikes upon an immoveable obſtacle, it will loſe all its motion; an elaſtic body will return with a force equal to the ſtroke.

Exp. Let a leaden ball, and an ivory ball, ſtrike upon any fixed plane.

CHAP. V.

Of Motion as produced by the Attraction of Gravitation.

SECT. I.

Of the Laws of Gravitation in Bodies falling without Obſtruction.

PROP. XXIII. The motion of a body falling freely by the attraction of gravitation is uniformly accelerated, or its velocity increaſes equally in equal times.

A new impreſſion being made upon the falling body, at every inſtant, by the continued action of the attraction of gravity, and the effect of the former (by Prop. I.) ſtill remaining, the velocity muſt continually increaſe. Suppoſe a ſingle impulſe of gravitation, in one inſtant, to give it one degree of velocity; if after this, the force of gravitation were entirely ſuſpended, the body would continue to move with that degree of velocity, without being accelerated or retarded. But, becauſe the attraction of gravitation continues, it produces as great a velocity in the ſecond inſtant as in the firſt; which being added to the firſt, makes the velocity in the ſecond inſtant double of what it was in the firſt. In like manner, in the third inſtant, it will be tripled; quadrupled in the fourth: and in every inſtant, one degree of velocity will be added to that which the body had before; that is, the motion will be uniformly accelerated.*

Cor. The velocities of falling bodies, are as the times in which they are acquired.

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* All bodies deſcending in vacuo by gravity, whether great or ſmall, denſe or rare, are found to fall through 16.1 feet in one ſecond, and to acquire a velocity in falling which would carry them uniformly through 32.2 feet in the next ſecond, and an increaſe of velocity, equal to this, is found to be added to every ſucceeding ſecond of time.

PROP. XXIV. The force of the attraction of gravitation acting upon any body is as its quantity of matter.

For, each particle of matter in any body being acted upon by gravitation, the greater number of particles are contained in any body, the greater force muſt be exerted upon it; that is, the force increaſes as the quantity of matter increaſes.

EXP. Let two unequal balls, ſuſpended by threads of the ſame length, be let fall at the ſame time from points equally diſtant from the loweſt points of the arcs in which they move: The vibrations of each will be performed in equal times, and conſequently their velocities will be equal; whence the momenta (Prop. XI.) will be as the quantities of matter: But (by Prop. XIII.) the force producing motion, is as the quantity of motion, or momentum, produced: Therefore the force of gravitation is as the quantity of matter; that is, as much greater force is exerted upon the larger body than upon the leſs; as its quantity of matter is greater than that of the leſs.

COR. 1. The weight of any body is as its quantity of matter; for weight is the degree of force with which any body is acted upon by gravitation.

COR. 2. If the attraction of gravitation were increaſed in any ratio, the weight of a given body would be increaſed in the ſame ratio. Subſtituting, therefore, W, Q, F, for the weight, quantity of matter, and force of gravity, reſpectively, and ſuppoſing them to be variable; W will be as Q × F.

PROP. XXV. The velocities of bodies falling from the ſame height without reſiſtance, are equal.

If two bodies of different quantities of matter fall from the ſame height, the attracting force which acts upon the greater body, will (Prop. XXIV.) exceed that which acts upon the leſs, as much as the greater body exceeds the leſs in quantity of matter; whence they muſt move with equal velocities.

EXP. A guinea, and a feather, or other light body, in the exhauſted receiver of an air-pump, will fall through the ſame ſpace in the ſame time.

PROP. XXVI. The ſpaces deſcribed by falling bodies are as the ſquares of the times from the beginning of the fall, and alſo as the ſquares of the laſt acquired velocities; or in the ratio compounded of the times and velocities.

Plate 1. Fig. 10. In the triangle ABC, let AB expreſs the time in which a body is falling, and BC the velocity which it has acquired at the end of the fall; let AF, AD, be parts of the time AB; and through F, D, draw FG, DE, parallel to BC.

Becauſe the triangles ABC, ADE, are ſimilar, AB is to AD as BC to DE; but AB and AD expreſs times of deſcent, and BC expreſſes the velocity acquired in the time AB; therefore, ſince (Prop. XXIII. Cor.) the velocities are as the times, DE expreſſes the velocity acquired in the time AD. In like manner, GF, any other right line parallel to BC, expreſſes the velocity acquired in the time AF. Therefore the ſum of the lines which may be ſuppoſed drawn parallel to CB in the triangle ADE; that is, the whole triangle ADE, will repreſent the ſum of the ſeveral velocities with which the falling body moves in the time AD. For the ſame reaſon; the triangle ABC will repreſent the ſum of the velocities with which the falling body moves in the time AB. Since, therefore, it is manifeſt, that the ſpace which a body paſſes through in any moment of time is as the velocity with which it moves at that moment; and conſequently, that the ſpaces through which it paſſes in any times whatſoever, are as the ſums of the velocities with which it moves in the ſeveral moments of thoſe times; the ſpaces paſſed through in the times AD, AB, are to each other as the triangles ADE, ABC. But the triangle ADE (El. VI. 19.) is to the triangle ABC in the duplicate ratio of the homologous ſides AD, AB, and alſo of DE, BC: that is, the ſpaces are as the ſquares of the times, and alſo as the ſquares of the laſt acquired velocities, conſequently the ſpaces deſcribed are in the compound ratio of the times and the velocities.

Exp. Let there be two pendulums, one of which vibrates twice as faſt as the other, a ball let fall from ſuch a height above the ball of the ſhorter pendulum as to reach it in one vibration, muſt fall from four times this height, to reach the longer pendulum in one of its vibrations.

Cor. 1. Hence, if the forces are variable, the ſpaces deſcribed are as the forces and ſquares of the times;—or as the ſquares of the velocities directly, and the forces inverſely. For by the Prop. (calling S, V and T, the ſpace, velocity and time) S is as $T \times V$, and (by Prop. A. p. 14.) V is as $F \times T$ ∴ S is as $F \times T^2$, and S is as $\frac{V^2}{F}$.

Cor. 2. The times in which bodies fall from unequal heights, and their laſt acquired velocities, are as the ſquare roots, or in the ſubduplicate ratio of their heights. Since TT is as S, T will be as $\sqrt{S}$; and ſince VV is as S, V will be as $\sqrt{S}$.

Cor. 3. If the time of the fall of a body be divided into equal parts, the ſpaces through which it falls in each of theſe parts, taken ſeparately, will be as the odd numbers 1, 3, 5, &c. The ſpaces being as the ſquares of the times or velocities, if the times be as the numbers 1, 2, 3, 4, the ſpaces will be as 1, 4, 9, 16; whence, in the firſt time the ſpace will be as 1, in the ſecond time, the ſpace paſſed over will be as 3, in the third, as 5, &c.

SCHOL. Since S is as T^2, and as in the firſt ſecond of time a body freely deſcending by the force of gravity falls through 16.1 feet, we eaſily find the ſpace deſcribed in any given number of ſeconds; for $S = 16.1 \times T^2$. Thus in 5″ a body will fall through 402 feet for $16.1 \times 25 = 402$. Again, the ſpaces fallen through in the 1ſt, 2d, 3d, ſeconds, are 16.1; 16.1×3; 16.1×5 reſpectively.

PROP. XXVII. The ſpace which a body paſſes over in any given time from the beginning of the fall, is half that which it would paſs over in the ſame time, moving with the laſt acquired velocity.

For the triangle ABC (by Prop. XXVI.) expreſſes the ſpace paſſed over in the time AB, when the motion is uniformly accelerated; the laſt acquired velocity is expreſſed by BC; and the rectangle of AB, BC, rightly expreſſes the ſpace paſſed through in the time AB with the equable velocity BC: ſince therefore the triangle ABC is half of the rectangle AB, BC, the propoſition is manifeſt. Plate I. Fig. 10.

PROP. XXVIII. The motion of a body thrown upwards is uniformly retarded by gravitation: the time of its riſe will be equal to that in which a body falling freely acquires the ſame degree of velocity with which it is thrown up; and the height to which it riſes will be as the ſquare of the time, or firſt velocity.

The ſame force which accelerates a falling body, acting in an oppoſite direction upon one thrown upwards, muſt retard it: and, ſince the action of gravitation is uniform, in whatever time it generates any velocity in a falling body, it muſt in the ſame time deſtroy the ſame velocity in a riſing body: through whatever ſpace the falling body muſt paſs to acquire any velocity, the riſing body muſt paſs through the ſame, to loſe it: and whatever ratio the ſpaces bear to the velocities and times in one caſe, muſt take place in the other: the effect of gravitation in riſing bodies being in all reſpects the reverſe of its effect upon falling bodies.

SCHOL. As the force of gravity near the ſurface of the earth is conſtant, and known by experiment, and as the ſpaces deſcribed by falling bodies vary as the ſquares of the times, (T^2) or as the ſquares of the velocities (V^2): hence every thing relating to the deſcent of bodies, when accelerated by the force of gravity; and to their aſcent, when they are retarded by that force, may be deduced from the foregoing propoſitions.

(1.) When a body falls by the force of gravity, the velocity acquired in any time, as T″, is ſuch as would carry it uniformly over 2 F T in 1″; where F = 16.1 feet.

EXAM. The velocity acquired by a falling body in $6'' = 32.2 \times 6$, or ſuch as would carry it uniformly through 193 feet in 1″.

(2.) The ſpace fallen through to acquire the velocity V is $\frac{V^2}{4F}$. For $S : F :: V^2 : \overline{2F}^2$ or $S = \frac{V^2}{4F}$.

Exam. If a body fall from reſt till it acquire a velocity of 20 feet per ſecond, the ſpace fallen through is $\frac{\overline{20}^2}{4 \times 16.1} = 6.2$ feet.

From theſe three expreſſions, $V = 2\ F\ T$; $S = \frac{V^2}{4F}$ and $S = F\ T^2$ (Cor. 1. Prop. XXVI.) : any one of the quantities S, T, V, being given, the other two may be found.

Exam. 1. To find the time in which a body will fall 400 feet; and the velocity acquired.

Since $S = F\ T^2 \therefore T = \sqrt{\frac{S}{F}} = \sqrt{\frac{400}{16.1}} = 5''$ nearly, and V being equal $2\ F\ T = 32.2 \times 5'' = 161$ feet = velocity acquired.

Exam. 2. If a body be projected perpendicularly downwards, with a velocity of 20 feet per ſecond, to find the ſpace deſcribed in 4″.

The ſpace deſcribed in 4″ by the firſt velocity is 4×20, and the ſpace fallen through by the action of gravity is 16.1×4^2, therefore the whole ſpace deſcribed is 337.6 feet.

Exam. 3. To what height will a body riſe in 3″ which is projected perpendicularly upwards with a velocity of 100 feet per ſecond?

The ſpace deſcribed in 3″ by the firſt velocity is 300 feet, and the ſpace through which the body would fall by gravity in 3″ is $16.1 \times 3^2 = 144.9$ feet, therefore the height required is $300 - 144.9 = 155.1$ feet.

SECT. II.

Of the Laws of Gravitation in Bodies falling down inclined Planes.

Def. IV. An inclined plane, is a plane which makes an acute or obtuſe angle with the plane of the horizon.

PROP. XXIX. The motion of a body deſcending down an inclined plane, is uniformly accelerated.

In every part of the ſame plane, the accelerating force has the ſame ratio to the force of gravitation acting freely in a perpendicular direction, and is therefore (El. V. 9.) equally exerted in every inſtant of the deſcent; whence, (as was ſhewn concerning bodies falling freely, Prop. XXIII.) the motion muſt be uniformly accelerated.

COR. Hence, whatever has been demonſtrated concerning the perpendicular deſcent of bodies, is equally applicable to their deſcent down inclined planes, the motion in both caſes being uniformly accelerated by the ſame power of gravitation.

PROP. XXX. The force with which a body deſcends by the attraction of gravitation down an inclined plane, is to that with which it would deſcend freely, as the elevation of the plane to its length; or as the ſine of the angle of inclination to radius.

Let AB be the length of an inclined plane, and AC its elevation, or perpendicular height. If the force of gravitation with which any body deſcends perpendicularly be expreſſed by AC, and this force be reſolved into two forces, AD, DC, by drawing CD perpendicular to AB; becauſe the force CD is deſtroyed by the re-action of the plane, the body deſcends down the inclined plane only with the force AD. And (El. VI. 8. Cor.) AD is to AC, as AC is to AB; that is, the force of gravitation down the inclined plane is to the ſame force acting freely, as the elevation of the plane to its length, or as the ſine of the angle of inclination ABC is to the radius AB. Plate I. Fig. 11.

COR. 1. Hence, the force neceſſary to ſuſtain a body on an inclined plane, is to the abſolute weight of a body, as the elevation of the plane to its length: for the force requiſite to ſuſtain a body, muſt be equal to that with which it endeavours to deſcend; which has been ſhewn to be to that with which it would deſcend freely, as the elevation of the plane to its length.

COR. 2. If H be the height of an inclined plane, L its length, and the force of gravity be repreſented by unity; the accelerating force on the inclined plane is repreſented by $\frac{H}{L}$. For by the Prop. the accelerating force is to the force of gravity (1) as H is to L $\therefore$ the accelerating force $= \frac{H}{L}$.

COR. 3. Hence $\frac{H}{L}$ varies as the ſine of the angle of inclination.

COR. 4. If a body fall down an inclined plane, the velocity V generated in T″ is ſuch as would carry it uniformly over $\frac{H}{L} \times 2\,F\,T$ feet in 1″ where, as before, F is equal 16.1. For (by Prop. A. p. 14.) the velocity varies as the force and time, (i. e.) as $\frac{H}{L} \times T$, and the velocity generated by the force of gravity in one ſecond is 2 F, therefore $V = \frac{H}{L} \times 2\,F\,T$.

Ex. If L : H :: 2 : 1, a body falling down the plane will, at the end of 4″, acquire a velocity of $\frac{1}{2} \times 32.2 \times 4 = 64.4$ feet per ſecond.

Cor. 5. The ſpace fallen through in T″ from a ſtate of reſt, is $\frac{H}{L} \times F T^2$ for (Prop. XXVI.) the ſpaces deſcribed vary as the ſquares of the times.

Ex. 1. If $H = \frac{L}{2}$ the ſpace through which a body falls in 5″ is $\frac{1}{2} \times 16.1 \times 25 = 201\frac{1}{4}$ feet.

Ex. 2. To find the time in which a body will deſcend 40 feet down this plane. Since $S = \frac{H}{L} \times F T^2$, therefore $T = \sqrt{\frac{S \times L}{H \times F}} = \sqrt{\frac{40 \times 2}{1 \times 16.1}} = 2.2$ ſeconds.

Cor. 6. The ſpace through which a body muſt fall, from a ſtate of reſt to acquire a velocity V, is $\frac{L}{H} \times \frac{V^2}{4F}$. For (Cor. 1. Prop. XXVI.) S is as $\frac{V^2}{F}$, therefore the ſpace through which the body falls by the force of gravity, is to the ſpace through which it falls down the plane, as the ſquare of the velocity directly, and as the force inverſely in the former caſe, is to the ſame in the latter; and if F (16.1) be the ſpace fallen through by gravity, and 2 F is the velocity acquired in 1″; hence $F : S :: \overline{\frac{2F}{1}}\Big|^2 : \frac{L}{H} \times V^2$ and $S = \frac{L}{H} \times \frac{V^2}{4F}$.

Ex. 1. If $L = 2H$, and a body fall from a ſtate of reſt till it has acquired a velocity of 40 feet per ſecond, the ſpace deſcribed is $\frac{2}{1} \times \frac{40^2}{64.4} = 50$ feet nearly.

Ex. 2. If a body fall 40 feet from a ſtate of reſt down this plane, to find the velocity acquired. $V^2 = 4FS \times \frac{H}{L} = 64.4 \times 40 \times \frac{1}{2} = 1288$, and $V = 35.8$ feet per ſecond.

PROP. XXXI. The ſpace deſcribed in any given time by a body deſcending down an inclined plane, is to the ſpace through which it would fall perpendicularly in the ſame time, as the elevation of the plane to its length.

Plate I. Fig. 11. Let AC repreſent the force with which a body would fall perpendicularly: CD being drawn from C perpendicular to AB; AD, as was ſhewn (Prop. XXX.) will repreſent the force with which the body deſcends down the inclined plane AB. And, ſince the ſpaces through which a body falls in any given time muſt be as the forces which move them, the ſpace through which the body will fall down the inclined plane AB, is to that through which it will fall perpendicularly in the ſame time, as the force AD, to the force AC. But AD is to AC (El. VI. 8. Cor.) as AC the elevation to AB the length of the plane; therefore

the ſpace through which the body will fall in a given time down the inclined plane AB, will be to the ſpace through which it would fall perpendicularly in the ſame time, as the elevation of the plane to its length.

Cor. 1. A body would fall down the inclined plane from A to D, in the ſame time in which it would fall perpendicularly from A to C. For, the ſpaces paſſed through in any given time are as AC to AB, that is, (El. VI. 8. Cor.) as AD to AC: conſequently, if AC is the ſpace paſſed through in any given time by the body falling freely, AD will be the ſpace paſſed through in the ſame time, down the inclined plane AB.

Cor. 2. Having the ſpace, through which a body falls in a perpendicular direction, we can eaſily find the ſpace which a body will deſcribe in the ſame time, on planes differently inclined, by letting fall perpendiculars, as CD on thoſe planes reſpectively.

PROP. XXXII. The velocity acquired in any given time by a body deſcending down an inclined plane, is to the velocity acquired in the ſame time by a body falling freely, as the elevation of the plane to the length.

In an uniformly accelerated motion, the velocities produced in equal times are as the forces which produce them: but (by Prop. XXX.) the force with which a body deſcends down an inclined plane, is to that of its perpendicular deſcent, as the height of the plane to its length; therefore the velocities produced in equal times are in the ſame ratio.

PROP. XXXIII. The time in which a body moves down an inclined plane, is to that in which it would fall perpendicularly from the ſame height, as the length of the plane to its elevation.

The ſquare of the time in which AB is paſſed over, is to the ſquare of the time in which AD is paſſed over (compare Prop. XXVI. with Prop. XXIX. Cor.) as AB to AD, that is, ſince AB, AC, AD, (El. VI. 8. Cor.) are continued proportionals, as the ſquare of AB to the ſquare of AC. Therefore the times themſelves are as the lines AB, AC, that is, as the length of the plane to its elevation. Plate I. Fig. 11.

Cor. Hence if ſeveral inclined planes have equal altitudes, the times in which thoſe planes are deſcribed by bodies falling down them, are as the lengths of the planes. For the time of the deſcent down AC is to the time of the fall down AB, as AC to AB; and the time of the fall down AB is to the time of the deſcent down AG, as AB to AG; therefore (El. V. 11.) the time of the deſcent from A to C is to the time of deſcent from A to G, as AC to AG, that is, the times are as the lengths of the planes. Plate I. Fig. 12.

PROP. XXXIV. A body acquires the ſame velocity in falling down an inclined plane, which it would acquire by falling freely through the perpendicular elevation of the plane.

Plate I. Fig. 11. The ſquare of the velocity which a body acquires by falling to D, is (by Prop. XXVI. compared with Prop. XXIX. Cor.) to the ſquare of the velocity it acquires by falling to B, as the ſpace AD is to the ſpace AB, that is, (El. VI. 8. Cor.) as the ſquare of AD is to the ſquare of AC; and conſequently the velocity at D is to the velocity at B, as AD is to AC. But, becauſe AD and AC (Prop. XXXI. Cor.) are paſſed over in the ſame time, the velocity acquired at D is (by Prop. XXXII.) to that which is acquired at C, as AD to AC. Since then the velocity at D has the ſame ratio to the velocities at B, and at C, namely, the ratio of AD to AC, the velocities at B and C (El. V. 9.) are equal.

Plate I. Fig. 12. Cor. 1. Hence the velocities acquired by bodies falling down planes differently inclined are equal, where the heights of the planes are equal. The velocities acquired in falling from A to C, and from A to G, are each equal to the velocity acquired in falling from A to B, and therefore equal to one another.

Cor. 2. Hence if bodies deſcend upon inclined planes, whoſe heights are different, the velocities will be as the ſquare roots of their heights. For (Fig. 8. and 9.) the velocity in D is equal to that in A, and the velocity in D is equal to that in G. Therefore the velocity in D (Fig. 8.) is to that in D (Fig. 9.) as $\sqrt{AB}$ is to $\sqrt{FG}$ (by Cor. Prop. XXVI).

PROP. XXXV. A body falls perpendicularly through the diameter, and obliquely through any chord of a circle, in the ſame time.

Plate I. Fig. 12. In the circle ADB, let AB be a diameter, and AD any chord; draw BC a tangent to the circle at B; produce AD to C, and join DB. Becauſe ADB (El. III. 31.) is a right angle, a body (by Prop. XXXI. Cor.) will fall from A to D on the inclined plane in the ſame time in which it will fall from A to B perpendicularly. In like manner let the chord AE be produced to G; and becauſe AEB is a right angle, a body will fall from A to E in the inclined plane in the ſame time in which it would fall from A to B.

Cor. 1. Hence all the chords of a circle are deſcribed in equal times.

Cor. 2. Hence alſo the velocities, and accelerating forces, will be as the lengths of the chords.

PROP. XXXVI. If a body deſcends along ſeveral contiguous planes, the velocity which it acquires by the whole deſcent, provided it loſt no motion in going from one to another, is the ſame which it would acquire, if it fell from the ſame perpendicular height along one continued plane; and this velocity will be the ſame with that which would be acquired by the perpendicular fall from the elevation of the planes.

Plate 1. Fig. 13.

Let AB, BC, CD, be ſeveral contiguous planes; through the points A and D, draw HE, DF, parallel to the horizon, and produce the contiguous planes CB, CD, to G and E. By Prop. XXXIV. Cor. the ſame velocity is acquired at the point B, whether the body deſcends from A to B, or from G to B. Therefore, the line BC being the ſame in both caſes, the velocity acquired at C muſt be the ſame, whether the body deſcends through AB, BC, or along GC. In like manner, it will have the ſame velocity at D, whether it falls through AB, BC, CD, or along ED, that is, (by Prop. XXXIV.) its velocity will be equal to the velocity acquired by the perpendicular fall from H to D.

Cor. Hence if a body deſcends along any arc of a circle, or any other curve, the velocity acquired at the end of the deſcent is equal to the velocity acquired by falling down the perpendicular height of the arc; for ſuch a curve may be conſidered as conſiſting of indefinitely ſmall right lines, repreſenting contiguous inclined planes.

Schol. The velocity of a body paſſing from one inclined plane to another is diminiſhed in the ratio of radius, to the co-ſine of the angle between the directions of the planes. Let BC, or B m, (Fig. 20.) repreſent the velocity acquired at B, and reſolve BC into B n and C n, by letting fall the perpendicular C n : $m\,n$ will be the velocity loſt, therefore the velocity at B is to the velocity diminiſhed by paſſing from AB to BD as BC to B n, or as radius to the co-ſine of the angle between the directions of the planes.

PROP. XXXVII. If two bodies fall down two or more planes equally inclined, and proportional, the times of falling down theſe planes will be as the ſquare roots of their lengths.

Plate 1. Fig. 14.

Let the inclined planes be AB, BC, DE, EF: let AG, DH, be lines drawn parallel to the horizon; let AB, DE, be equally inclined to the plane of the horizon, and alſo BC, EF; let AB be to DE as AG to DH and as BC to EF, and draw GB, HE.

Becauſe ABG, DEH, are ſimilar triangles, AB is to DE (El. VI. 4.) as BG to EH, and $\sqrt{AB}$ to $\sqrt{DE}$ as $\sqrt{BG}$ to $\sqrt{EH}$: alſo AB is to DE as BG + BC is to HE + EF, and $\sqrt{AB}$ to $\sqrt{DE}$ as $\sqrt{BG + BC}$, or $\sqrt{GC}$ is to $\sqrt{HE + EF}$, or $\sqrt{HF}$.

And ſince (by conſtruction) AB is to DE as BC to EF, AB is to DE as AB + BC is to DE + EF, and $\sqrt{AB}$ to $\sqrt{DE}$, as $\sqrt{AB + BC}$ to $\sqrt{DE + EF}$. But AB, DE, being planes equally inclined, the accelerating force of gravitation will be the ſame upon each, and the bodies deſcending upon them may be conſidered as falling down different parts of the ſame plane. Hence (Prop. XXVI. Cor. 1. and XXIX. Cor.) the time of deſcent along AB is to that along DE, as $\sqrt{AB}$ to $\sqrt{DE}$; and the time of deſcent along GC is to that along HF, as $\sqrt{GC}$ is $\sqrt{HF}$, that is, as $\sqrt{AB}$ to $\sqrt{DE}$. Again, the time of deſcent along GB is to that along HE, as $\sqrt{BG}$ is $\sqrt{EH}$, that is, as $\sqrt{AB}$ to $\sqrt{DE}$. Since, therefore, the time of deſcent along the whole plane GC is to that along the whole plane HF, as $\sqrt{AB}$ to $\sqrt{DE}$, and that of the part GB is alſo to that of the part HE, as $\sqrt{AB}$ to $\sqrt{DE}$, the time of deſcent along the remainder BC is to that along the remainder EF (El. V. 19.) as $\sqrt{AB}$ to $\sqrt{DE}$. Conſequently, the time of deſcent down AB + BC is to that down DE + EF, as $\sqrt{AB}$ to $\sqrt{DE}$, that is, as $\sqrt{AB + BC}$ to $\sqrt{DE + EF}$.

Cor. Hence if bodies deſcend through arcs of circles, the times of deſcribing ſimilar arcs will be as the ſquare roots of the arcs. For ſuch ſimilar arcs may be conſidered as compoſed of an equal number of proportional ſides, or planes, having the ſame inclination to each other, and their elevations equal; whence by this propoſition, the times of deſcent will be as the ſquare roots of the lengths of the arcs.

PROP. XXXVIII. If a body be thrown up along an inclined plane, or the arc of a curve, it will, in the ſame time, riſe to the ſame height, from which with equal force it would have deſcended; and any velocity will be loſt in the ſame time in which it would, in deſcending, have been acquired.

For the ſame force of gravitation has, in every reſpect, the ſame efficacy to retard the motion of bodies aſcending, as to accelerate them deſcending on an inclined plane or curve.

SECT. III.

Of the Pendulum and Cycloid.

Def. V. A pendulum is a heavy body hanging by a cord or wire, and moveable with it upon a centre.

PROP. XXXIX. The vibrations of a pendulum are produced by the force of gravitation.

Plate I. Fig. 15. Let the ball A, ſuſpended from the centre B by the cord BA, be drawn up to C and let fall from thence: it will deſcend by the force of gravitation to A, from whence (being prevented from falling farther by the cord) it will proceed (by Prop. XXXVI. Cor.) with a velocity equal to that which it would have acquired in falling perpendicularly from E to A,

which will carry it on the oppoſite ſide to the height from which it fell. Being brought back again towards A by the force of gravitation, it will acquire a new velocity which will carry it towards C: and in this manner it will vibrate by the force of gravitation, till the reſiſtance of the air, and the friction of the ſtring, ſtop its motion.

PROP. XL. The ſame pendulum, vibrating in ſmall unequal arcs, performs its vibrations nearly in equal times.

In the circle CGA, the ſmall arcs CA, EA, will differ little from their reſpective chords in length or declivity. But (by Prop. XXXVI. Cor.) the times in which the chords are paſſed over are equal; therefore the times of deſcribing the arcs CA, EA, and alſo (by Prop. XXXVIII.) of deſcribing their doubles CAD, EAF, will be nearly equal.

Plate I. Fig. 16. and 17.

EXP. Two equal pendulums vibrating in ſmall, but unequal arcs, will, for a long time, keep pace in their vibrations.

PROP. XLI. If a pendulum vibrate through ſmall arcs of circles of different lengths, the velocity it acquires at the loweſt point, is as the chord of the arc which it deſcribes in its deſcent.

Let BA be the pendulum, and CAD, EAF, the arcs through which it vibrates; and draw the horizontal lines EK, CH. The velocity acquired in falling from H to A, is (by Prop. XXVI. Cor.) to that acquired by falling from G to A, as $\sqrt{HA}$, to $\sqrt{GA}$, that is, (by El. VI. 8. Cor.) as CA to GA. For the ſame reaſon, the velocity acquired in falling from G to A, is to that acquired in falling from K to A, as GA to EA. Conſequently, *ex æquali*, the velocity acquired in falling from H to A, is to that acquired by falling from K to A, as CA to EA. But (by Prop. XXXVI. Cor.) the velocity acquired in falling from H to A is equal to that from C to A; and the velocity acquired in falling from K to A is equal to that from E to A. Therefore the velocity acquired in deſcending through the arc CA, is to that through EA, as the chord CA to the chord EA: and the ſame may be ſhewn concerning the remaining half of the vibrations, AF, AD.

Plate I. Fig. 16. and 17.

COR. Hence the lengths of the chords of arcs through which pendulums move, are meaſures of velocity.

Fig. 17.

PROP. XLII. The time of the deſcent and aſcent of a pendulum, ſuppoſing it to vibrate in the chord of a circle, is equal to the time in which a body falling freely would deſcend through eight times the length of the pendulum.

For the time of the deſcent of a body upon the chord, is (by Prop. XXXV.) equal to that of the fall through the diameter of the circle, which is twice the length of the pendulum; but in double that time, that is, in the deſcent and aſcent, or whole vibration, the body would fall (by Prop. XXVII.) through four times the ſpace, that is, through eight times the length of the pendulum.

PROP. XLIII. The times in which pendulums of different lengths perform their vibrations, are as the ſquare roots of their lengths.

Plate I. Fig. 18. Let the two pendulums, AB, CD, be of different lengths. The time in which the firſt, AB, vibrates through a chord, is equal to that in which a body (Prop. XXXV.) would fall freely through twice AB, the diameter of the circle of which AB is radius: in like manner, the time in which CD vibrates, is equal to that in which a body would fall through twice CD. But the times in which a body would fall through theſe different ſpaces are (Prop. XXVI. Cor. 1.) as the ſquare roots of the ſpaces, that is, as the ſquare roots of AB and CD, the lengths of the pendulums: therefore the vibrations are in the ſame ratio.

Fig. 19. Cor. The times in which pendulums of unequal lengths vibrate, are as the ſquare roots of the ſimilar arcs through which they move. Let BA, BC, be pendulums of different lengths vibrating in the ſimilar arcs FG, DE. Since the times of vibration are as the ſquare roots of the lengths BA, BC, and that ſimilar arcs are as the diameters, the times of vibration, are as the ſquare roots of the arcs, FA, DC, or of their doubles, FG, DE.

Exp. Two pendulums, the lengths of which are as 1 to 4, will perform their vibrations in times as 1 to 2, that is, the ſhorter pendulum will make two vibrations, whilſt the longer makes one; for $T : t :: \sqrt{L} : \sqrt{l}$.

PROP. XLIV. The ſquares of the times in which a pendulum of a given length performs its vibrations, are inverſely as the accelerating forces, or gravities.

By Prop. XXVI. where the accelerating force is given, the ſpace deſcribed is as the ſquare of the time in which it is deſcribed. And ſince, in any given moving body, the velocity is as the accelerating force (Prop. A. p. 14.) where the ſquare of the time, or the time itſelf, is given (by Prop. II.) the ſpace deſcribed will be as the accelerating force. Conſequently, where neither the accelerating force, nor the ſquare of the time, is given, the ſpace deſcribed will be in the ratio compounded of both. If then the ſpace deſcribed be called S, the accelerating force A, and the ſquare of the time T^2, S will be as T^2A, whence $\frac{T^2A}{A}$, or T^2 is as $\frac{S}{A}$. But, when the ſpaces are equal, S is a given quantity:

whence (ſince fractions whoſe numerators are given, are inverſely as their denominators) $\frac{S}{A}$ is inverſely as A. But T^2 is as $\frac{S}{A}$; therefore where S is given T^2 is inverſely as A; that is, where the ſpaces deſcribed are equal, the ſquares of the times in which they are deſcribed are inverſely as the accelerating forces. And if the ſquares of the times of falling bodies are inverſely as their accelerating forces, the ſquares of the times in which pendulums vibrate, are in the ſame ratio, on account of the conſtant equality between the time of vibration and that of the deſcent through eight times the length of the pendulum, by Prop. XLII.

COR. 1. Hence, if the ſame pendulum, at different parts of the earth, performs its vibrations in different times, the forces of gravitation will, in thoſe places, be inverſely as the ſquares of thoſe times.

COR. 2. If the vibrations of pendulums of unequal lengths be performed in the ſame time, the accelerating forces will be as their lengths. For (by Prop. XLIII. and XLIV.) $T : t :: \frac{L}{A} : \frac{l}{a}$, therefore when $T = t$; $A : a :: L : l$. Hence, as it is known by experiment, that the lengths of pendulums that vibrate ſeconds are diminiſhed in approaching the equator, the force of gravity muſt alſo decreaſe.

EX. At the equator a pendulum vibrating ſeconds is $\frac{1}{10}$th of an inch ſhorter than ſuch a pendulum in the latitude of London, and the length of this pendulum in London is 39.2 inches, therefore gravity under the equator is to gravity here as 391 is to 392.

LEMMA I.

If, from X *as the centre, with any diſtance* XA, *a quadrant of a circle* ADB *be deſcribed, and in the right line* AX *a body deſcends with ſuch force, that its velocity in any points* M, N, &c. *ſhall be always as* MD, NP, &c. *the ſines of the arcs* AD, AP: *the time in which the body will deſcend from* A *to* X, *will be equal to the time in which it would deſcribe the whole arc* ADB *with the uniform velocity, expreſſed by* XB, *acquired by the falling body when it arrives at* X: *alſo, the time of the fall through any ſpace* AM, *will be to the time of the fall through any other ſpace* AO, *as the arc* AD *to the arc* AQ; *and the force with which the body is accelerated in any place* M, *will be as* MX *the diſtance of that place from the centre.*

Let DP be a part of the circumference taken indefinitely ſmall, and therefore not aſſignably differing from a right line; join DX; and draw DL perpendicular to NP. Becauſe the triangles MDX, LDP, are ſimilar, (having each a right angle, and the angles MDX, LDP, whoſe common complement is LDX, equal) MD will be to DX as LD or MN to DP. But, by the hypotheſis, MD is as the velocity of the deſcending body at the point M, that is, as the velocity with which the indefinitely ſmall line MN is deſcribed; and XD is as the velocity laſt acquired by the falling body at X, that is, as the uniform velocity with which the arc DP is deſcribed. The velocity therefore of the body deſcending through the indefinitely ſmall line MN, will be to the velocity of the body moving along the arc DP, as MN to DP. Wherefore, ſince the velocities are proportional to the ſpaces paſſed over, the times wherein thoſe ſpaces, MN, DP, are deſcribed, will be equal. After the ſame manner it may be proved, that any other indefinitely ſmall portion of the circumference, PQ, may be deſcribed with the velocity XB, in the ſame time in which the correſponding line NO will be deſcribed with the correſponding velocity NP: and conſequently, by compoſition, the falling body will deſcend through all the indefinitely ſmall portions of the perpendicular AX, that is, through the whole line, in the time in which all the correſponding parts of the circumference, that is, the whole quadrant ADB, is deſcribed with an uniform velocity as XB.

Moreover, the time in which the falling body deſcends from A to M, is equal to the time in which the arc AD is paſſed over; and the time in which it deſcends from A to O is equal to the time in which the arc AQ is deſcribed: but the time in which the arc AD is paſſed over, is to that in which the arc AQ is paſſed over (ſince they are both deſcribed with the ſame velocity) as the arc AD to the arc AQ: therefore the time of deſcent from A to M, will be to the time of deſcent from A to O, as the arc AD to the arc AQ: and conſequently, by diviſion, the time of deſcent through AM will be to the time of deſcent through MO, as the arc AD to the arc DQ.

Laſtly, let the arcs DP, PQ, be equal; join XP, and from P let fall PS perpendicular to OQ: the time of deſcent through MN will be equal to that through NO: and, ſince the triangles LDP, MDX, are ſimilar, and alſo SPQ, NPX; LP will be to DP or PQ, as MX to XD or XP; alſo PQ is to SQ as XP to XN; and conſequently, (El. V. 11.) LP will be to SQ as XM to XN. But LP is as the increment of the velocity acquired while the body is paſſing over MN, and SQ is as the increment of the velocity acquired in paſſing over in an equal time the indefinitely ſmall line NO; and the forces with which the body is accelerated at M and N, are as the increments of the velocities generated in equal times: the accelerating forces at M and N will therefore be as the lines LP, SQ; that is, the force with which the body is impelled at M is to that at N, as the diſtance XM to the diſtance XN, or the accelerating forces are as their diſtances from the centre.

Cor. Hence, converſely, if a body, deſcending from A to X is impelled by a force which is as its diſtance from the centre X, and that force at the beginning of the motion is expreſſed by the right line CE (the arc AE being taken indefinitely ſmall) the velocities of the ſame body in any places M, O, will be expreſſed by the ſines MD, OQ; and the times by the arcs AD, AQ; and the increments of the velocities, or, if the arcs increaſe equally, the accelerating forces, will be expreſſed by the increments of the ſines.

LEM. II. *If a body, moving along the line* AX, *be impelled by forces proportional to its diſtance from the point* X; *from whatever height it falls, it will arrive at the point* X *in the ſame time; and this time will be to the time in which it would move over the whole line* AX *with the velocity which it acquires by falling from* A *to* X, *as half the circumference of a circle to its diameter.*

Plate 2. Fig. 2.

Let two bodies be let fall from the points A and P at the ſame time; and let them be impelled by forces proportional to their diſtances from the point X; theſe bodies will come to X at the ſame time. From X as a centre, with the radii XA and XP, deſcribe the two quadrants AB and PQ; and let the force by which the body A is impelled, or, which is the ſame thing, its velocity at the beginning of motion, be repreſented by RS, the ſine of the indefinitely ſmall arc AS: it is manifeſt (from the Cor. of the preceding Lemma) that its velocity, after the fall to X will be properly expreſſed by XB. But, by hypotheſis, the force by which the body at A is accelerated, is to that by which the body at P is accelerated, as AX is to PX, that is, (ſince the arcs AS and PN are ſimilar) as RS to MN. As therefore RS expreſſes the firſt velocity of the body moving from A, MN will expreſs the firſt velocity of the body moving from P: and conſequently (by the Cor. to the laſt Lemma) XQ will expreſs the velocity of the body moving from P, when it arrives at X. Farther, the time of the fall from A to X (by Lemma I.) is equal to the time in which the arc AB would be deſcribed with a velocity as XB: and the time of the fall from P to X is equal to the time in which the arc PQ would be deſcribed with a velocity as XQ. But (becauſe the line XQ is to the line XB as the arc PQ to the arc AB, and that the ſpaces paſſed over are proportional to the times) the time in which the arc AB is deſcribed with the velocity XB is equal to the time in which the arc PQ is deſcribed with the velocity XQ. Wherefore the time of the fall from A to X will be equal to the time of the fall from P to X.

Again, ſince (by Lem. I.) the time in which a body would fall from A to X is equal to the time in which it would move over the arc AB, with its laſt acquired velocity at X; and ſince it is evident, that the time in which a body would move over the arc AB with the velocity at X is to the time in which it would move over AX with the ſame velocity, as AB is to AX; the time in which a body would fall from A to X is to the time in which it would move over AX with the laſt acquired velocity, as AB to AX. But AB is to AX, as twice AB to twice AX, that is, as half the circumference of a circle is to its diameter,

diameter. Therefore the time in which a body would fall from A to X, is to the time in which it would move over AX with its laſt acquired velocity, as half the circumference of a circle is to its diameter.

Plate 2. Fig. 3.

DEF. VI. If a circle, as FCH, be rolled along the line AB, till it has turned once round; the point C in its circumference, which at firſt touched the line at A, will deſcribe the curve line ACXB, which curve is called a *Cycloid.* The right line AB is its *baſe:* the middle point X is its *vertex:* a perpendicular, as XD, let fall from thence to the baſe, is its *axis:* and the circle FCH, or any other as XGD, equal thereto, is called the *generating circle.*

LEM. III. *If on* XD, *the axis of the cycloid, as a diameter, the generating circle* XGD *be deſcribed; and if from a point in the cycloid, as* C, *the line* CIK *be drawn parallel to the baſe, the portion of it* CG, *will be equal to the arc* GX.

Becauſe the generating circles FCH, DGX, are equal, (the diameter HF being drawn) KG is equal to CI; whence, adding GI to both, KI will be equal to CG: and KI, by the conſtruction, is equal to DF; therefore CG is equal to FD. By the deſcription of the cycloid, the arc CF is equal to the line AF: and by the conſtruction the arc CF is equal to DG: therefore AF is equal to DG: but, by the deſcription of the cycloid, AFD is equal to DGX; conſequently, FD is equal to GX: and CG was proved to be equal to FD: therefore CG is equal to GX.

LEM. IV. *A tangent to the cycloid at the point* C *is parallel to* GX *a chord of the circle* DGX.

Draw ck, parallel to the baſe and indefinitely near to CK meeting the cycloid in c, the axis in k, and the circle in g. Let Cu and Gn, parallel to the axis, meet ck in u and n, and from T, the centre of the circle XGDM, draw the radius TG. Since cg is equal (Lem. III.) to gX, gk being added to both, ck will be equal to $Xg+gk$: therefore cu the exceſs of ck above CK is equal to $Gg+gn$, the exceſs of $Xg+gk$ above $XG+GK$. And, if we ſuppoſe ck to approach towards CK, as Gg and gn vaniſh, the triangles Ggn and GTK become ſimilar; for the angle Ggn is then equal to the angle TGK, ſince both have the ſame angle nGT, or its alternate GTK, as their complement. Whence Gg is to gn as TG to TK, and (El. V. 18.) $Gg+gn$ to gn, as TG+TK or DK to TK; but Gn is to gn as GK to TK; therefore $Gg+gn$ is to Gn as DK is to GK, that is (El. VI. 8.) as

GK to XK. And conſequently cu (ſhown to be equal to $Gg + gn$) is to Gn, or Cu, as GK to XK: and if the chord Cc be drawn, the triangles Cuc, XKG, will be ſimilar: ſo that the chord Cc (as the points C and c coincide) becomes parallel to XG; therefore the tangent of the cycloid at C is parallel to XG.

LEM. V. *If from a point of the cycloid, as* L, *the line* LMK *be drawn parallel to the baſe* AB, *the arc* XL *of the cycloid, will be double of* XM *the chord of the circle correſponding thereto.* Plate 2. Fig. 3.

Draw the line Sk parallel and indefinitely near to LK croſſing the circle in R, and the chord XM produced, in P: join the points X and R; on MP let fall the perpendicular RO; and draw MN, XN, tangents to the circle at M and X. Then will the lines XN and kS, being each perpendicular to the diameter DX, be parallel: and the triangles MNX, MPR, having their angles at M vertical, and at P and X alternate, will be ſimilar. But the tangents NX and NM are equal; (El. III. 36.) whence the lines PR and RM are alſo equal: the triangle RMP is therefore iſoſceles; and RO being perpendicular to its baſe MP, MO (El. I. 26.) is equal to OP; whence MP is equal to twice MO. The indefinitely ſmall arc LS of the cycloid will not aſſignably differ from a portion of a tangent drawn through the point L. LS may therefore (Lem. IV.) be ſaid to be parallel to MP, and conſequently (from the paralleliſm of ML and PS) equal to it: it is therefore equal alſo to twice MO. But LS is the difference between the cycloidal arcs XL and XS; and MO is the difference between the chords XM and XR: for ſince XO and XR are indefinitely near to each other, RO which is perpendicular to one of them, may be conſidered as perpendicular to both: the indefinitely ſmall difference therefore between any two arcs of the cycloid is twice that which is between the two correſponding chords of the circle; and the ſame is true when the magnitude of the difference is aſſignable, becauſe ſuch difference is compounded of indefinitely ſmall parts. Now, any arc whatſoever may be conſidered as a difference between two arcs, and conſequently any arc, as XL, is double of the correſponding chord XM.

Cor. Since when the arc XL becomes XB, the correſponding chord XM becomes XD the diameter of the circle DMX; it is obvious, that the ſemicycloid BX, or AX, is equal to twice DX the diameter of the generating circle DMX.

LEM. VI. *If a body deſcends in a cycloid, the force of gravity, ſo far as it acts upon the body in cauſing it to deſcend along the cycloid, will be proportional to the diſtance of the body from the loweſt point of the cycloid.*

Let the cycloid be AXB, whoſe baſe is AB, and its axis DX; on which laſt, as a diameter, deſcribe the generating circle DQX: draw the chords OX and QX; through Plate 2. Fig. 4.

the points O and Q, and parallel to the baſe AB, draw the lines LS and MR; draw alſo the tangents LV and MY. Then becauſe (by Lem. IV.) the tangent LV is parallel to OX, and the tangent MY parallel to QX, it is obvious that gravity exerts the ſame power upon a body deſcending in the cycloid at L (becauſe it then deſcends in the tangent LV) as it would do upon the ſame body deſcending along the chord OX: and for the like reaſon, it exerts the ſame force upon it when it comes to M, that it would do if it were deſcending along QX: but (from Prop. XXXV.) the power or force of gravity upon bodies deſcending along the chords OX and QX, are as the lengths of thoſe chords; that is, by Lem. V. (halves being proportional to their wholes) as the length of the cycloidal arcs LX and MX. The force therefore of gravity upon a body deſcending in the cycloidal at the point L is to its force upon the ſame when at M (as may be ſaid of any other correſponding points) as the ſpace or diſtance it has to move over in the former caſe, before it reaches the loweſt point X, to that which it has to paſs over in the latter, before it arrives at the ſame point.

PROP. XLV. If a pendulum be made to vibrate in a cycloid, all its vibrations, however unequal in length, will be performed in equal times.

The force of gravity, (by Lem. VI.) ſo far as it cauſes a body to deſcend in a cycloid, is proportional to the diſtance of that body from the loweſt point: imagine then that body to be a pendulum vibrating in the cycloid, and from whatever point it ſets out, it will (by Lem. II.) come to the loweſt point in the ſame time: and conſequently, ſince the ſame may be eaſily inferred in its aſcending from that point, all its vibrations, be they large or ſmall, will be performed in the ſame time.

Schol. This propoſition is demonſtrated only on the ſuppoſition that the whole maſs of the pendulum is concentrated in a point, for it cannot otherwiſe take place, becauſe as the ſtring varies in its length, the centre of oſcillation of a body will vary. On this account, therefore, pendulums vibrating in circular arcs are now always uſed, for the ſame arcs will be always deſcribed in the ſame time.

PROP. XLVI. To make a pendulum vibrate in a given cycloid.

Plate 2. fig. 4.

Let AXB be the given cycloid; its baſe AB, its axis DX, and its generating circle DQX, as before: produce XD to C, till DC is equal to DX: through C draw the line EF parallel to AB, and take CE and CF, each equal to AD or DB; and on the line CE as a baſe, and with the generating circle AGE equal to DQX, deſcribe the ſemicycloid CTA, whoſe vertex will therefore touch the baſe of the given cycloid in A. And on the line CF alſo as a baſe, deſcribe an equal ſemicycloid CB. Let the ſemicycloids CA, CB, repreſent thin plates of metal bent to their figure, and on the point C, hang the pendulum

CTP by a flexible line equal in length to the line CX. The upper part of its ſtring (as CT, in its preſent ſituation in the figure) as it vibrates, will then apply itſelf to the cycloidal cheeks CA and CB, and a ball at P will oſcillate in the given cycloid AXB.

Draw TG and PH each parallel to the baſe AB, and draw AG, and DH. Then (Lem. V. Cor.) AC is equal to twice AE; and by conſtruction, twice DC, that is, twice AE, is equal to CX; therefore AC is equal to CX. Alſo, by conſtruction, CTP is equal to CX, that is, to ATC: whence, taking away CT, AT is equal to TP. By Lem. IV. GA is parallel to TP; and, by conſtruction, AK is parallel to GT; therefore GA is equal to TK, and GT to AK; but (Lem V.) GA is half TA; therefore TK is equal to half TA: ſince therefore it has been proved that TA is equal to TP, TK is equal to half TP, that is, to KP. Hence it is manifeſt, that the parallel lines GT, PH are equally diſtant from AD, the arc GA equal to the arc DH, the chords GB and DH parallel, and GE equal to HX. And, becauſe GA has been ſhewn to be parallel to TK, and alſo to DH, KP and DH are parallel; whence KD is equal to PH. But (Lem. III.) GT, that is, AK is equal to the arc AG: and by the deſcription of the ſemicycloid CTA, AKD is equal to AGE; therefore KD is equal to EG, that is, PH is equal to HX. And (by Lem. III.) if PH be equal to HX, P is a point in the cycloid AXB. The ball of the pendulum therefore being at that point, is in the given cycloid.

SCHOL. 1. It is eaſy to conceive, that in a pendulum there muſt be ſome one point, on each ſide of which the momenta of the ſeveral parts of the pendulum will be equal, or in which the whole gravity of the pendulum might be collected without altering the time of its vibrations. This point, which is called the *centre of oſcillation*, is different from the centre of gravity: for if a plane, perpendicular to the ſtring of the pendulum AB be conceived to paſs through the centre of the ball B, biſecting it; the velocity of the lower half, and conſequently its momentum, will, in vibration, be greater than that of the upper half: conſequently the centre of oſcillation muſt be farther from A than the centre of gravity is; and a plane paſſing through the centre of oſcillation will divide the ball into two unequal parts, ſo that the greater quantity of matter above it, ſhall compenſate for the greater velocity below it, and the momenta on each ſide be equal. If the pendulum be an inflexible rod every where of equal ſize, it is found, that the diſtance of the centre of oſcillation from the point of ſuſpenſion is two-thirds of the length of the rod.

Plate 1. Fig. 18.

If, whilſt a pendulum is in motion, it meets with an obſtacle at its centre of oſcillation ſufficient to ſtop it, the whole motion of the pendulum will ceaſe at once, without any jarring: for the obſtacle reſiſts equal momenta above and below this point; which is therefore alſo called the *centre of percuſſion.*

SCHOL. 2. The vibrations of pendulums are ſubject to many irregularities, for which no effectual remedy has yet been deviſed. Theſe are owing partly to the variable denſity and temperature of the air, partly to the rigidity and friction of the rod by which they are ſuſpended, and principally to the dilatation and contraction of the materials, of which they are formed. The metalline rods of pendulums are expanded by heat, and contracted by cold; therefore clocks will go ſlower in ſummer, and faſter in winter. The common remedy for this inconvenience is the raiſing or lowering the bob of the pendulum (by means of a ſcrew) as the occaſion may require. By the laſt ſcholium it appears, that a pendulum conſiſting of a tube of glaſs or metal, every where uniform, filled with quickſilver, and 58.8 inches long will vibrate ſeconds; for $\frac{2}{3}$ of 58.8 is equal to 39.2. Such a pendulum will be expanded and contracted at the ſame time; for when the tube is extended by heat, the mercury will alſo be expanded, and by riſing in the tube, will raiſe the centre of oſcillation, ſo that its diſtance from the point of ſuſpenſion will be diminiſhed, and the vibrations of the pendulum which would have been rendered ſlower by the expanſion of the tube, will become quicker by the expanſion of the mercury: and by adjuſting the tube and mercury in ſuch a manner, that theſe contrary effects may be the ſame, a clock with ſuch a pendulum would admit of little or no variation for a long time. Phil. Tranſ. N°. 392. p. 40.

SECT. IV.

Of the Centre of Gravity.

PROP. XLVII. In every body there is a *centre of gravity*, or a point about which all its parts balance each other.

Plate 2. Fig. 5. Let AB be an inflexible rod, throughout uniform and of the ſame denſity: let it be ſupported at the point C, equally diſtant from its extreme points A and B, by the prop C. Let A and B be indefinitely ſmall and equal portions of the rod AB. Theſe portions, A and B, tend towards the centre of the earth with equal forces of gravitation. They would likewiſe, without obſtruction, move with equal velocities: Fig. 6. for if the rod AB be moved on its prop till it come into the poſition DE, the velocities of the parts A, B, or F, will be as the ſpaces over which they paſs in the ſame time; that is, as the arcs AD, EB or FG; which arcs are as their reſpective circumferences, or as their diameters or *radii*: whence the velocity of the part B is to the velocity of the part A, or F, as BC is to AC, or FC. And the quantities of matter in A and B are by ſuppoſition equal. Therefore, if the parts A and B were in motion, they would have equal momenta; that is, the efforts which A and B make to deſcend towards the earth, are equal. But theſe efforts counteract each other: for, whilſt the portion A endeavours with a certain force to draw down one arm of the rod, the other portion B endeavours with the ſame force to draw down the other arm, that is, ſince the rod is inflexible, to raiſe the portion A. Therefore the portion A is acted upon by two equal forces in contrary directions, and conſequently muſt be at reſt. For the ſame reaſon, the portion B will be at reſt. And the ſame may be ſhewn concerning any other equal portions, at equal diſtances from C, in the rod AB. Therefore the rod

will be at reſt ; that is, the parts on each ſide of the point will balance each other, and C will be the *centre of gravity*.

If the rod were placed oblique to the prop C, indefinitely ſmall and equal parts being taken, as before, at equal diſtances from C, and reſolving each oblique force into an horizontal and perpendicular force (as in Prop. XVI.) it might be ſhewn, by a ſimilar manner of reaſoning, that they would tend towards the earth with equal forces, and conſequently, that an equilibrium would be produced.

And if, inſtead of equal portions of the rod, portions of matter were placed at different diſtances, which ſhould be to each other inverſely as thoſe diſtances, as at F and B, the equilibrium would ſtill be preſerved : for the forces with which ſuch portions of matter, ſo ſituated, would endeavour to deſcend, would be equal, when the quantities of matter, multiplied into the velocities with which they are endeavouring to move, that is, into their diſtances (Prop. XI. Cor.) are equal ; as will be more fully ſhewn, in treating of the Mechanical Powers. Fig. 6.

Since, therefore, all the parts of any irregular body may be referred to ſome one of the above caſes, it is manifeſt, that there is in every body a certain point, the parts on each ſide of which, balance each other.

PROP. XLVIII. If the centre of gravity in any body be ſupported, the whole body is ſupported ; if this centre be not ſupported, the body will fall.

For, when the centre of gravity is ſupported, the body reſts on a prop on which the parts on each ſide, acting with equal force againſt each other, will (Prop. XLVII.) be in equilibrio, and neither ſide will move ; but when this centre is not ſupported, but the body has a prop under ſome other point, the parts of the body on one ſide of that other point will over-balance the parts on the other ſide, and the body will fall.

COR. Whenever a body moves by the power of gravitation, or falls, its centre of gravity deſcends : for if this centre do not deſcend, it muſt be ſupported ; and if the centre be ſupported, the whole body is ſuſtained or kept from falling.

EXP. 1. Let a board of a circular form be ſuſtained perpendicularly on its centre of gravity, it will be at reſt in any poſition.

2. A beam turning on an axis which paſſes through its centre of gravity will reſt in the ſame manner.

3. A beam, whoſe axis paſſes through a point which is directly above the centre of gravity, will be at reſt only when the beam is parallel to the plane of the horizon, becauſe the centre of gravity will be then fallen as low as poſſible.

4. A cylinder, which has its centre of gravity near one of its ſides, will roll up an inclined plane, if the ſide neareſt the centre of gravity be placed towards the upper part of

the plane; for this centre, endeavouring to deſcend, will carry the cylinder forward in the aſcending direction of the plane.

5. Let a body conſiſting of two equal and ſimilar cones united at their baſes, be placed upon the edges of two ſtraight and ſmooth rods, which at one end meet in an angle, and reſt upon an horizontal plane, and at the other are raiſed a little above the plane, the body will roll towards the elevated end of the rules, and appear to aſcend, while its centre of gravity deſcends; as may be ſeen by applying a ſtring horizontally above the path of the baſe of the cones.

PROP. XLIX. If the line of direction comes within the baſe on which any body is placed horizontally, the body will be ſuſtained; otherwiſe it will fall.

Plate 2. Fig. 7. In the body ABDE let C be the centre of gravity. The line of direction CO (that is, the line drawn from the centre of gravity towards the centre of the earth) being within the baſe DE, the body will be ſupported, becauſe the weight preſſes upon the baſe. Alſo ſince the body cannot fall towards K without turning round on the point E, the point C muſt in the motion aſcend towards F, contrary to Prop. XLVIII. Cor. But in the poſition of the body *abde*, *co* the line of direction falling out of the baſe, *c* in its motion towards *k* deſcends, and the body will fall.

Our own motions and actions are ſubject to this rule. When a man ſtands upright, his centre of gravity falls between his feet, and he is ſupported; but if he lean forward he throws the line of direction without his baſe, and he would fall if he did not put forward one of his feet ſo as to cauſe it to fall within. Hence a porter with a load on his back leans forward that the load may not throw the line of direction out of his baſe behind; and by an artful adjuſting of this point it is that ſuch wonders are performed in horſemanſhip, and on the tight and ſlack rope, &c.

Exp. 1. Any body of a cylindrical or other regular form, ſo placed upon its baſe, that its line of direction does not come within the baſe (which may be ſeen by a cord and weight ſuſpended from the centre of gravity) will fall; otherwiſe it will not fall.

2. Let two bodies be laid upon an inclined plane, the one a cube, the other a figure with many ſides, and let the line of direction of the former fall within the baſe, and that of the latter without the baſe, the former body will *ſlide*, the latter *roll* down the plane.

Def. VII. The centre of motion is the point about which a body moves.

PROP. L. A heavy body ſuſpended on a centre of motion will be at reſt, if the centre of gravity is directly under, or above, the centre of motion; otherwiſe it will move.

Plate 2. Fig. 8. If a heavy body E, hangs by a ſtring on a centre of motion C, the action of gravitation at E, is in the direction EL, contrary to the direction in which the ſtring acts to prevent the body from falling. In this poſition, therefore, the oppoſite forces being equal and in contrary directions, deſtroy each other, and the body is at reſt. But if the body is at *p*, one of the forces acts in the direction *p*C, and the other in the direction *p*L, that is, in directions oblique to each other, whence the body will move in the diagonal of the parallelogram formed by *p*C, *p*L. And in all caſes, ſince (without the aid of mechanical powers afterwards explained) the force which ſuſtains any body muſt be equal to its weight, the centre of gravitation can only be at reſt when theſe forces are in the ſame line of direction, that is, when the centre of gravity is directly under, or directly above, the centre of motion.

Exp. A circular board ſuſtained at a point above or below the centre of gravity, will only be at reſt when the centre of gravity is at the loweſt point, that is, in the line of direction; or when the centre of gravity is in the ſame line above the centre of motion.

Schol. If two or more bodies be united, they may be conſidered as one, and have a common centre of gravity.

Exp. 1. Let two unequal balls be fixed upon the ends of a wire, they will have a common centre of gravity.

2. A board, which of itſelf would fall from a table (its centre of gravity lying beyond the edge of the table) may be made, in the ſame poſition, to ſupport a veſſel of water, hanging upon it *near the table;* if a ſtick, fixed with one end at the bottom of the veſſel, and the other in a hole in the horizontal board, be long enough to puſh the veſſel a little out of the perpendicular, that is, to bring the centre of gravity of the whole under the table.

Plate 2. Fig. 4. Schol. The common centre of gravity of any number of bodies may be thus found. Let C be the common centre of gravity of two bodies, dividing their diſtances (ſee Prop. XLVII.) in ſuch a manner, that AC is to CB, as B to A; whence $A\times AC=B\times BC$, and conſequently, if the point C is ſupported, the bodies A and B balance each other. Suppoſe a third body, equal to the ſum of A and B, placed in their common centre of gravity C; from the point C draw a right line to the centre of a third body D, which divide in O, ſo that OD may be to OC, as $A+B$ is to D; then is O the common centre of the three bodies, A, B, D. In the ſame manner may be found the common centre of any number of bodies.

PROP. LI. If any number of bodies move uniformly in right lines, whether in the ſame or different directions, their common centre of gravity is either at reſt, or moves uniformly in a right line.

Plate 2. Fig. 5. If two bodies, A and B, move towards each other in the ſame right line, having their common centre of gravity C, and their momenta equal, the velocity of A will be to that of B, as the body B to the body A, that is (as was ſhewn, Prop. XLVII.) as AC to BC. Whence (Prop. VI.) whilſt A paſſes through AC, B will paſs through BC, and the bodies will meet in C, which is their centre of gravity during their motion, and at the time of concourſe: therefore the point C remains at reſt.

In the ſame manner, it may be ſhewn, that if the bodies recede from each other with uniform motions, the centres of gravity will be at reſt.

Plate 2. Fig. 9. Next, ſuppoſe that two bodies, A and B, move in different directions AC, BD, deſcribing equal ſpaces AC, CE, and BD, DF, in equal times; their common centre of gravity L, will move uniformly in a right line. Produce CA, DB, till they meet in G: Make AG to GH, as AC is to BD; draw the right line AH; and through C and E draw CI, EK, parallel to AH. AC is to HI (El. VI. 2.) as AG to GH, that is, as AC to BD: Therefore (El. V. 9.) HI is equal to BD, and adding IB to each, HB is equal to ID. In like manner, CE is to IK, as AG to GH, that is, as AC to BD, or CE to DF : Therefore (El. V. 9.) IK is equal to DF, and, adding KD to each, ID is equal to KF: But ID was proved to be equal to HB; therefore, KF is equal to HB. From L, the common centre of gravity of the bodies A and B, draw LM parallel to BD: Draw GM, and produce it till it cut CI, EK, in the points N and O; and through theſe points draw NP, OQ, parallel to BD. AL is to LB (El. VI. 2.) as AM to MH; and CP to PD, (as CN to NI, that is) as AM to MH; therefore (El. V. 11.) CP is to PD, as AL to LB, that is, (becauſe L is the common centre) as B to A. Conſequently, P will be the common centre of the bodies when they are found in C and D: And in like manner, it may be ſhewn, that Q will be their common centre, when they are in E and F. But, ſince ML is to HB, as AM to AH, that is, as CN to CI, that is, as NP to ID, and that HB has been proved to be equal to ID, ML is equal to NP; and, in like manner, NP equal to OQ: Whence (ML, NP, OQ, being parallel to one another) the line LPQ is equal to the line MNO, and, the points P, Q, (any points of the line in which the common centre of gravity is found as the bodies are moving from A to E, and from B to F) will be in a right line. Moreover, ſince (El. VI. 2.) AC is to CE, as MN to NO, or LP to PQ, and that AC is equal to CE, LP will be equal to PQ. Therefore the common centre of gravity of the bodies A and B, is always in the ſame right line, and moves uniformly, or paſſes over equal ſpaces in equal times.

In like manner, the common centre of theſe two bodies and any third body, or of the three bodies and a fourth, &c. being found, it may be proved that it moves uniformly in a right line.

COR. 1. Hence it is manifeſt, that any forces acting upon a ſyſtem of bodies, muſt affect the motion of the common centre of gravity of that ſyſtem, in the ſame manner as if the ſame force were ſimilarly applied to a body equal to the ſum of all the bodies, placed in the common centre of gravity. And the mutual actions of the parts of a ſyſtem upon each other, producing (by Prop. III.) equal momenta in contrary directions, cannot change the ſtate of motion or reſt of their common centre of gravity. Conſequently, the law of a ſyſtem of bodies, as to motion or reſt, is the ſame as that of one body, and is rightly eſtimated from the motion of its centre of gravity.

COR. 2. Hence the centre of gravity of a ſyſtem of bodies, will not be diſturbed by their mutual attractions, as the motions thus communicated are always equal and oppoſite. Hence the centre of gravity of our ſyſtem of planets is either at reſt, or moves uniformly in a ſtraight line. The latter is ſuppoſed by Dr. Herſchel to be the caſe.

CHAP. VI.

Of Motion as directed by certain Inſtruments called MECHANICAL POWERS.

DEF. VIII. That body which communicates motion to another, is called *the Power*.

DEF. IX. That body which receives motion from another, is called *the Weight*.

DEF. X. The *Lever* is a bar moveable about a fixed point, called its *fulcrum*, or prop. It is in theory conſidered as an inflexible line without weight. It is of three kinds: the firſt, when the prop is between the weight and the power; the ſecond, when the weight is between the prop and the power; the third, when the power is between the prop and the weight.

EXP. Let the three kinds of the lever be ſhewn, as in Plate 2, Fig. 12, 13, 14.

PROP. LII. A power and weight acting upon the arms of a lever will balance each other, when the diſtance of the point at which the power is applied to the lever from the prop, is to the diſtance of the point at which the weight is applied, as the weight is to the power.

Plate 2. Fig. 11.

Let C be the centre of motion in the lever KL; let A and B be any two powers applied to it at K and L, acting in the directions KA and LB. From the centre of motion C, let CM and CN be perpendicular to thoſe directions in M and N; ſuppoſe CM to be leſs than CN, and from the centre C, at the diſtance CN, deſcribe the circle NHD, meeting KA in D. Let the power A be repreſented by DA, and let it be reſolved into the power DG acting in the direction CD, and the power DF perpendicular to CD, by completing the parallelogram AFDG. The power DG, acting in the direction CD from the centre of the circle, or wheel, DHN towards its circumference, has no effect in turning it round the centre, from D towards H, and tends only to carry it off from that centre. It is the part DF only that endeavours to move the wheel from D towards H and N, and is wholly employed in this effort. The power B may be conceived to be applied at N as well as at L, and to be wholly employed in endeavouring to turn the wheel the contrary way, from N towards H and D. If, therefore, the power B be equal to that part of A which is repreſented by DF, theſe efforts being equal and oppoſite, muſt deſtroy each other's effect; that is, when the power B is to the power A, as DF to DA, or, (becauſe of the ſimilarity of the triangles AFD, DMC) as CM to CD, or as CM to CN, then the powers muſt be in equilibrio; and thoſe powers will ſuſtain each other, which are inverſely as the diſtances of their directions from the centre of motion.

Plate 2. Fig. 10.

Otherwiſe; Caſe 1. When the power acts perpendicularly: Let AB be the lever, C the prop, P the power, W the weight. The force with which any body moves being as its momentum, (Prop. XIII.) and its momentum as the quantity of matter multiplied into the velocity, (by Prop. XI. Cor.) the force with which the weight W would move in the firſt inſtant of its motion, if no other body counteracted it, would be as its quantity of matter multiplied into its velocity. But becauſe the weight W is ſuſpended from the lever AB at the point B, it would move with the ſame velocity as this point; which (as was ſhewn in Prop. XLVII.) is as the diſtance of the point B from the prop C, or D. The force therefore with which the weight W would move without any counteracting force, is as its quantity of matter multiplied into the diſtance of the point of ſuſpenſion B from the prop C, or D. But the weight will be prevented from deſcending, if a force equal to that with which it would deſcend without obſtruction, acts upon it in the contrary direction; that is, if a force be applied to raiſe the point B of the lever AB, equal to that with which the weight W would draw it downwards. Let the power P be ſuſpended from the other extremity of the lever at the point A; and let the quantity of matter in the power P multiplied into the diſtance of A, its point of ſuſpenſion from C, or D, be equal to the quantity of matter in the weight W multiplied into the diſtance of B from C, or D: it appears from what has been ſaid concerning the weight, that the force with which the power P, without obſtruction, would deſcend and draw down the point A, is equal to the force with which the weight W would deſcend and draw down the point B. But, as much force as the power P exerts to draw down the point A, it exerts to raiſe the point B. Therefore equal and oppoſite forces are exerted to raiſe and depreſs the point B ; and conſequently it will continue at reſt, and the weight and power will balance each other.

Caſe 2. When the power acts obliquely :

Let the weight A hang freely from one end of a balance, ſo as to have its line of direction DA perpendicular to the arm of the balance ; and let another weight as B, be hung at the other end E, in ſuch manner that its line of direction EC, by paſſing over a pulley at C, may be oblique to the arm of the balance. If the whole force of gravity in the weight B acting in the direction EC, be denoted by the line EC, it may be reſolved into two forces denoted by EF and FC, acting in the directions of theſe lines ; of which two forces, the latter only, which acts in the direction FC perpendicular to the arm of the balance, reſiſts the force of gravity in the weight A, the other force FE acting in the direction of the line of the lever. Since, therefore, that part of the weight B which acts in oppoſition to the weight A, is to the whole weight B, as FC to EC ; it is manifeſt, that in order to make the weight B balance the weight A, it muſt exceed the weight A, in the ſame ratio that the line EC exceeds the line FC. If from G the centre of motion be let fall GH perpendicular to EC produced, that line will be the perpendicular diſtance of the direction EC from G ; and EG, equal to DG, the perpendicular diſtance of the direction DA : but the triangles EFC and EHG are ſimilar, conſequently (El. VI. 4.) as EC is to CF, ſo is EG to HG ; but the weight B is to the weight A, as EC to FC ; B is therefore to A, as EG, or DG, to HG. Plate 3. Fig. 1.

SCHOL. It is evident that the force of the power is increaſed by the two firſt kinds of lever, but not by the laſt. The former are adapted to produce a ſlow motion by a ſwift one ; and the latter ſerves to produce a ſwift motion of the weight, by a ſlow motion of the power. See Fig. 12, 13, & 14.

To the *firſt* kind of lever may be reduced ſeveral ſorts of inſtruments ; ſuch as the ſteelyard, whoſe arms are unequal ; the falſe balance, whoſe arms are imperceptibly unequal ; the common balance, whoſe accuracy depends on its poſſeſſing the following properties : (1.) The arms muſt be equal in length and weight. (2.) The centre of motion muſt be a little above, and directly over the centre of gravity. (3.) The points from which the ſcales are ſuſpended ſhould be in a right line, paſſing through the centre of gravity of the beam. And (4.) the friction of the beam on the centre of motion ſhould be as little as poſſible. Sciſſars, pincers, ſnuffers, &c. are formed of two levers, the fulcrum of which is the pin which rivets them.

To the *ſecond* kind of lever may be reduced oars, and rudders of ſhips ; cutting knives fixed at one end ; doors moving on hinges, &c.

To the *third* kind, we may refer the action of the muſcles of animals, ladders fixed at one end, and raiſed againſt a wall by a man's arms, &c.

Def. XI. The *Wheel* and *Axis*, is a wheel turning round together with its axis; the power is applied to the circumference of the wheel, and the weight to that of the axis, by means of cords.

PROP. LIII. An equilibrium is produced in the wheel and axis, when the weight is to the power, as the diameter of the wheel to the diameter of the axis.

Plate 3. Fig. 2.

Let AB be the diameter of the wheel, DE, that of the axis, W the weight, and P the power, ſuſpended from the points D and B. When the wheel has performed one revolution, the power P has drawn off as much cord from the wheel as is equal to its circumference, and has therefore moved through a ſpace equal to that circumference. In the ſame time the weight W is raiſed through a ſpace equal to the circumference of the axis, upon which the cord, by which the weight is ſuſpended, is once turned round. Therefore the velocity of the power exceeds the velocity of the weight as much as the circumference, that is, the diameter of the wheel exceeds that of the axis. If then the weight exceeds the power as much as the velocity of the power exceeds that of the weight, that is, as much as the diameter, or ſemidiameter of the wheel, AB, or CB, exceeds the diameter, or ſemidiameter of the axis, DE, or CE, the momenta will be equal, and the power and weight will balance each other.

Or thus; The axis and wheel is a lever of the firſt kind; in which the centre of motion is in C, the centre of the axis; the weight W, ſuſtained by the rope DW, is applied at the diſtance DC, the radius of the axis; and the power P, acting in the direction PB perpendicular to CB, the radius of the wheel, is applied at the diſtance of that radius: therefore, Prop. LII. there is an equilibrium, when the power is to the weight, as the radius of the roller to the radius of the wheel.

Cor. 1. Hence it is evident, that by increaſing the diameter of the wheel, or diminiſhing that of the axis, a leſs power may ſuſtain a given weight.

Cor. 2. The thickneſs of the rope to which the weight is ſuſpended, ought not to be neglected.

Schol. To the wheel and axle we may refer the capſtan, mills, cranes, &c. A drawing and deſcription of a ſafe and truly excellent crane, invented by Mr. James White, may be ſeen in the 10th volume of the Tranſactions of the Society for encouraging Arts and Sciences, in London.

Def. XII. The *Pulley* is a ſmall wheel moveable about its axis by means of a cord which paſſes over it.

PROP. LIV. When the axis of the pulley is fixed, the pulley only changes the direction of the power; if moveable pullies are uſed, an equilibrium is produced, where the power is to the weight as one to the number of ropes applied to them: If each moveable pulley has its own rope, each pulley will double the power.

Plate 3. Fig. 3. Fig. 4. If the pulley ED be fixed upon the beam A, the power and weight, in equilibrio, will be equal. But, if one end of the rope be fixed in B, and the other ſupported by the power P, it is evident, that in order to raiſe the weight W one foot, the power muſt riſe two; for both the ropes, BC and CP, will be ſhortened, a foot each; whence the ſpace run over by the power will be double of that of the weight; if therefore the power is to the weight as 1 to 2, their momenta will be equal: For the ſame reaſon, if there be four ropes paſſing from the upper to the lower pullies, the velocity of the power will be quadruple to that of the weight, or as 4 to 1, &c. In all caſes, therefore, when the power is to the weight, as 1 to the number of ropes paſſing from the upper to the lower pullies, there will be an equilibrium.

Fig. 5. Or thus; Every moveable pulley hangs by two ropes equally ſtretched, which muſt bear equal parts of the weight; and therefore when one and the ſame rope goes round ſeveral fixed and moveable pullies, ſince all its parts on each ſide of the pullies are equally ſtretched, the whole weight muſt be divided equally amongſt all the ropes by which the moveable pullies hang: Conſequently, if the power which acts on one rope be equal to the weight divided by that number of ropes, the power muſt ſuſtain the weight.

Fig. 6. If each moveable pulley has its own cord, the firſt, as appears from what has been ſaid, doubles the velocity of the power; and therefore if the power be half of the weight, the momenta will be equal, and the balance will be produced. In like manner, the ſecond pulley cauſes the weight to move with half the velocity with which it would move, if ſuſpended from the firſt moveable pulley, that is, makes the velocity of the power quadruple of that of the weight; and ſo of the reſt.

Plate 12. Fig. 1. If in the ſolid block A, grooves be cut whoſe radii are 1, 3, 5, 7, &c. and in the block B other grooves be cut, whoſe radii are 2, 4, 6, 8, &c. and a ſtring be paſſed round theſe grooves, the grooves will anſwer the purpoſe of ſo many diſtinct pullies, and every point in each, moving with the velocity of the ſtring in contact with it, the whole friction will be removed to the two centres of motion in the blocks A and B, which is a great advantage over the common pullies. This pulley was invented by Mr. James White.

PROP. LV. In the Inclined Plane the power and weight balance each other, when the power is to the weight, as the ſine of the inclination of the plane is to the ſine of the angle which the line of the direction of the power makes with the perpendicular to the plane.

Let

Plate 3. Fig. 7. Let a weight be ſupported on the inclined plane CA by a power acting in any given direction PD. Let the whole force, whereby the weight would deſcend perpendicularly be repreſented by PB : and reſolving PB into two forces, one of which, BD, is perpendicular to the plane CA, and the other, PD, is in the direction of the power ; the force BD is deſtroyed by the re-action of the plane, and the force PD will be ſuſtained by an equal power, acting in the direction PD. Therefore, when there is an equilibrium, the power is to the weight, as PD to PB ; that is, as the ſine of the angle PBD, or (El. VI. 8.) its equal CAB, to the ſine of the angle PDB.

When PD is in the direction of the plane, this ratio becomes that of CD to CB, or of the height of the plane CB, to CA its length.

When the direction of the power PD is parallel to the baſe of the plane, the ratio of the power to the weight becomes that of ED to EB ; or (El. VI. 8. Cor.) of CB, the height of the plane, to BA the baſe.

When the direction of the power coincides with the perpendicular BD, the ratio of the power to the weight becomes that of the ſine of a finite angle, to the ſine of an angle indefinitely diminiſhed. From which it appears, that no finite power is ſufficient to ſupport a weight upon an inclined plane, if that power acts in a direction perpendicular to the plane.

Def. XIII. The *Screw* is a cylinder which has either a prominent part, or a hollow line, paſſing round it in a ſpiral form, ſo inſerted in one of the oppoſite kind, that it may be raiſed or depreſſed at pleaſure, with the weight upon its upper, or ſuſpended beneath its lower, ſurface.

PROP. LVI. In the ſcrew the equilibrium will be produced, when the power is to the weight, as the diſtance between two contiguous threads, in a direction parallel to the axis of the ſcrew, to the circumference of the circle deſcribed by the power in one revolution.

Plate 3. Fig. 8. While the ſcrew is made to perform one revolution, the weight W may be conſidered, as raiſed up an inclined plane *cq*, whoſe height *cp* is the interval between two contiguous ſpirals, whoſe baſe *pq* is the periphery of the cylinder, and whoſe length *cq* is the ſpiral line, by a power acting parallel to the baſe of the plane : for ſuch an inclined plane, involved about a cylinder, will form the ſpiral line of the ſcrew. A power at *p*, acting parallel to the baſe, is in equilibrio with the weight W to be raiſed, when the power is to the weight, as the height of the inclined plane, to the baſe ; or, in this caſe as *pc*, the interval between the ſpirals, to the circumference deſcribed by *p* : but a power applied at P, which is to that applied at *p*, as the circumference deſcribed by *p*, to the circumference deſcribed by P, has the ſame effect ; therefore there is an equilibrium, when the power applied at P, is to the

weight to be raiſed, as *pc*, the interval between two contiguous ſpirals, to the circumference deſcribed by the power P.

DEF. XIV. The *Wedge* is compoſed of two inclined planes, whoſe baſes are joined.

PROP. LVII. When the reſiſting forces, and the power which acts on the wedge, are in equilibrio, the weight will be to the power, as the height of the wedge to a line, drawn from the middle of the baſe to one ſide, and parallel to the direction in which the reſiſting force acts on that ſide.

Let the equilateral triangle ABC repreſent a wedge, whoſe baſe, or back, is AC, whoſe ſides are the lines AB and CB, and whoſe height is the line BP, which biſects the vertical angle ABC, and alſo the baſe perpendicularly in P. Let E and F repreſent two bodies, or two reſiſting forces acting on the ſides of the wedge perpendicularly, and whoſe lines of direction EP and FP meet at the middle point of the baſe, on which the power P acts perpendicularly, then will EP and FP (El. I. 5. and 26.) be equal: Let the parallelogram ENFP be completed; its diagonals PN and EF will biſect each other perpendicularly in H. Now when theſe forces (which act perpendicularly on the ſides and baſe of the wedge) are in equilibrio, they will be to each other (Prop. XIV.) as the ſides and diagonal of this parallelogram, that is, the ſum of the reſiſting forces will be to the power of P, as the ſides EP and FP to the diagonal PN, or as one ſide EP to half the diagonal PH, that is, (from the ſimilarity of the right-angled triangles BEP, EHP) as BP, the height of the wedge, to EP the line which is drawn from the middle of the baſe to the ſide AB, and is the direction in which the reſiſting force acts on that ſide. Plate 3. Fig. 9.

From the demonſtration of this caſe, in which the reſiſting forces act perpendicularly on the ſides of the wedge, it appears that the reſiſtance is to the power which ſuſtains it, as one ſide of the wedge AB is to the half of its breadth AP; becauſe AB is to AP, (El. VI. 8.) as BP is to EP.

It appears alſo from hence, that if PN be made to denote the force with which the power P acts on the wedge, the lines PE and PF which are perpendicular to the ſides, will denote the force with which the power P protrudes the reſiſting bodies in directions perpendicular to the ſides of the wedge.

Let us now ſuppoſe, in the ſecond caſe, that the reſiſting bodies E and F act upon the wedge in directions parallel to the lines DP and OP, which are equally inclined to its ſides, and meet in the point P. Draw the lines EC and FK perpendicular to DP and OP; then making PN denote the force with which the power P acts on the wedge, PE and PF will denote the forces with which it protrudes the reſiſting bodies in directions perpendicular to the ſides of the wedge, as was obſerved before; now each of theſe forces may be reſolved into two, denoted reſpectively by the lines PG and GE, PK and KF, of which GE and KF will be loſt, as they act in directions perpendicular to thoſe of the reſiſting bodies; and PG and PK will denote the forces by which the power P oppoſes the reſiſting bodies, by protruding them in directions contrary to thoſe in which they act on the wedge; therefore when the reſiſting forces are in equilibrio with the power P, the former muſt be to the latter, as the ſum of the lines PG and PK is to PN, or as PG is to PH. But (El. VI. 4.) PG is to PE, as PE to PD; and PH is to PE, as PE to PB: whence (El. VI. 16.) both the rectangle $PG \times PD$, and the rectangle $PH \times PB$, are equal to the ſquare of PE; theſe rectangles are therefore equal to one another: whence their ſides (El. VI. 14.) are reciprocally proportional, that is, PG is to PH, as PB to PD. Whence it follows from what was ſhewn above, that, in equilibrio, the reſiſting forces are to the power, as PB to PD; that is, as the height of the wedge to the line drawn from the middle of the baſe to one ſide of the wedge, and parallel to the direction in which the reſiſting force acts on that ſide.

From what has been demonſtrated, we may deduce the proportion of the power to the reſiſtance it is able to ſuſtain in all the caſes in which the wedge is applied. Firſt, when in cleaving timber the wedge fills the cleft, then the reſiſtance of the timber acts perpendicularly on the ſides of the wedge, therefore in this caſe, when the power which drives the wedge, is to the coheſive force of the timber, as half the baſe, to one ſide of the wedge, the power and reſiſtance will be in equilibrio.

Secondly; When the wedge does not exactly fill the cleft, which generally happens, becauſe the wood ſplits to ſome diſtance before the wedge. Let ELF repreſent a cleft into which the wedge ABC is partly driven; as the reſiſting force of the timber muſt act on the wedge in directions perpendicular to the ſides of the cleft, draw the line PD in a direction perpendicular to EL the ſide of the cleft, and meeting the ſide of the wedge in D; then the power driving the wedge and the reſiſtance of the timber, when they balance, will be to each other as the line PD to PB the height of the wedge.

Thirdly; When a wedge is employed to ſeparate two bodies that lie together on an horizontal plane, for inſtance, two blocks of ſtone; as theſe bodies muſt recede from each other in horizontal directions, their reſiſtance muſt act on the wedge in lines parallel to its baſe CA; therefore the power which drives the wedge, will balance the reſiſtance when they are to each other as PA, half the breadth of the wedge, to PB its height.

Schol. 1. Since in all the mechanical powers, an equilibrium is produced when the power is to the weight as the velocity of the weight is to the velocity of the power, in all compound machines there will be an equilibrium, when the ſum of the powers are to the weight, as the velocity of the weight is to the ſum of the velocities of the powers.

Schol. 2. In the theory of the mechanical powers, we ſuppoſe all planes and bodies perfectly ſmooth; levers to have no weight; cords to be perfectly pliable, and the parts of machines to have no friction. (See Schol. 3.) Allowances, however, muſt be made for the difference between theory and practice. Mr. Ferguſon obſerves, that there are but few compound machines, but what, on account of friction, will require a third part more to work them, when loaded, than what is ſufficient to conſtitute an equilibrium between the weight and the power.

Exp. 1. Let A, B, C, be a compound lever, conſiſting of three levers, in the firſt of which, A, the velocity of the weight is to that of the power, as 1 to 5; in the ſecond, B, as 1 to 4; in the third, C, as 1 to 6. The velocity of the weight will be to that of the power, as 1 to $5 \times 4 \times 6 = 120$; and if the power be to the weight, as 1 to 120, they will balance each other. Plate 2. Fig. 15.

2. Let GC and LF be the levers fixed to the ſupporters RA, SE, and let their ſhorter arms be kept in equilibrio with the longer reſpectively by the weights fixed at G and L. Let NH be a bar ſcrewed to the fixed parts to keep them ſteady. If the power C be ten times farther from A the prop, than the weight P, they will be in equilibrio when the power C is to the weight P, as 1 to 10. In like manner, the diſtance ME being ten times DE, if the power M be $\frac{1}{10}$ of the weight C ſuſpended from D, they will be in equilibrio: whence M, 1, will balance P, 100. Fig. 16.

3. Exhibit models or draughts of different compound machines, as mills, cranes, the pile-driver, &c.

Schol. 3. The inequality of the ſurface on which any body moves occaſions an attrition, called friction, which prevents the accurate agreement of many experiments in mechanics with theory. On this ſubject the very accurate experiments of Mr. Vince ſhould be conſulted, the object of which was, to determine, (1.) Whether friction be an uniformly retarding force. (2.) The quantity of friction. (3.) Whether friction varies in proportion to the preſſure or weight. And (4.) whether the friction be the ſame, on whichever of its ſurfaces a body moves. After a great variety of experiments made with the utmoſt care and attention, Mr. Vince deduces the following concluſions, which may be conſidered as eſtabliſhed facts.

I. That friction is an uniformly retarding force in hard bodies, not ſubject to alteration by the velocity; except when the body is covered with woollen cloth, &c. and in that caſe the friction increaſes a little with the velocity.

II. Friction increaſes in a leſs ratio than the weight of the body, being different in different bodies. It is not yet ſufficiently known for any one body, what proportion the increaſe of friction bears to the increaſe of weight.

III. The ſmalleſt ſurface has the leaſt friction; the weight being the ſame. But the ratio of the friction to the ſurface is not accurately known.

See a full account of theſe experiments, Vol. LXXV. Phil. Tranſ.

Schol. 4. Wheel carriages are uſed, to avoid friction as much as poſſible. A wheel turns round upon its axis, becauſe the ſeveral points of its circumference are retarded in ſucceſſion by attrition, whilſt the oppoſite points move freely. Large wheels meet with leſs reſiſtance than ſmaller from external obſtacles, and from the friction of the axle, and are more eaſily drawn, having their axles level with the horſes. But in uneven roads, ſmall wheels are uſed, that in aſcents the action of the horſe may be nearly parallel with the plane of aſcent, and therefore may have the greateſt effect : ſmall wheels are alſo more conveniently turned. The greater part of the load ſhould be laid on the hinder part of a wheel carriage.

CHAP. VII.

Of Motion as produced by the united Forces of PROJECTION *and* GRAVITATION.

SECT. I.

Of Projectiles.

PROP. LVIII. Bodies thrown horizontally or obliquely, have a curvilinear motion, and the path which they deſcribe is a parabola; the air's reſiſtance not being conſidered.

Plate 3. Fig. 10, 11.

If a body be thrown in the direction AF, and acted upon by the projectile force alone, it will continue to move on uniformly in the right line AF, and would deſcribe equal parts of the line AF in equal times, as AC, CD, DE, &c. But if, in any indefinitely ſmall portion of time, in which the body would by the projectile force move from A to C, it would, by the force of gravity, have fallen from A to G; by the compoſition of theſe forces (Prop. XVI.) it will, at the end of that time, be found in H, the oppoſite angle of the parallelogram ACGH. In two ſuch portions of time, whilſt it would have moved from A to D by the projectile force, it would (Prop. XXVI.) by gravitation fall through four times AG, that is, AM; and therefore, theſe forces being combined, it will be found at the end of that time in I, the opposite angle of the parallelogram DM. In like manner, at the end of the third portion of time, it would by the projectile force be carried through three equal divisions to E, and by the force of gravitation over nine times AG to N; and conſequently, by both theſe forces acting jointly, it will be carried to K, the oppoſite angle of the parallelogram EN. Therefore the lines CH, DI, EK, that is, AG, AM, AN, which are to each other as the numbers 1, 4, 9, are as the ſquares of the lines AC, AD, AE, that is, GH, MI, NK, which are as 1, 2, 3. And becauſe the action of gravitation is continual, the body in paſſing from A to H, &c. is perpetually drawn out of the right line in which it would move if the force of gravitation were ſuſpended, and therefore moves in a curve. And H, I and K, are any points in this curve in which lines let fall from points equally diſtant from A in the line AB meet the curve. Therefore the body moves in a parabola, the property of which is, (*Simſon's Conic Sections*, Book I. Prop. XII. Cor.) that the *abſciſſa* AG, AM, AN, are to each other as the ſquares of the ordinates GH, MI, NK.

REMARK. Very denſe bodies moving with ſmall velocities deſcribe the parabolic track ſo nearly, that any deviation is ſcarcely diſcoverable; but with very conſiderable velocities the reſiſtance of the air will cauſe the body projected to deſcribe a path altogether different from a parabola, which will not appear ſurpriſing when it is known that the reſiſtance of the air to a cannon ball of two pounds weight, with the velocity of 2000 feet per ſecond, is more than equivalent to 60 times the weight of the ball. See Hutton's Dict. Art. Reſiſtance.

PROP. LIX. The path which a body thrown perpendicularly upwards deſcribes in riſing and falling is a parabola.

A ſtone lying upon the ſurface of the earth, partaking of the motion of the earth (here ſuppoſed) round its axis, this motion which it has with the earth will not be deſtroyed by throwing it in a direction perpendicular to the ſurface of the earth. After the projection, therefore, the ſtone will be moved by two forces, one horizontal, the other perpendicular, and will riſe in a direction which may be ſhewn, as in the laſt propoſition, to be the parabolic curve; in which it will continue till it reaches the higheſt point, from whence it might be ſhewn, as in the laſt propoſition, that it will deſcend through the other ſide of the parabola.

PROP. LX. The velocity with which a body ought to be projected to make it deſcribe a given parabola, is ſuch as it would acquire by falling through a ſpace equal to the fourth part of the parameter belonging to that point of the parabola from which it is intended to be projected.

The velocity of the projectile at the point A (by Prop. LVIII.) is ſuch as would carry it from A to E, in the ſame time in which it would deſcend by its gravity from A to N. Plate 5, Fig. 10, 11. And the velocity acquired in falling from A to N (by Prop. XXVII.) is ſuch as in the ſame time by an uniform motion would carry the body through a ſpace double of AN. Therefore the velocity which is acquired by the body in falling to N is to that with which the body is projected at A, and uniformly carried forwards to E, as twice AN is to AE. But ſince, from the nature of the parabola, (*Simſon's Conic Sections*, Book I. Prop. XIII.) $\frac{AE^2}{AN}$ is equal to the parameter of the point A, one fourth part of this parameter will be expreſſed by $\frac{\frac{1}{4}AE^2}{AN}$. And becauſe the velocities acquired by falling bodies are (by Prop. XXVI. Cor. 1.) as the ſquare roots of the ſpaces they fall through, the velocity acquired by a body in falling through AN is to the velocity acquired in falling through $\frac{\frac{1}{4}AE^2}{AN}$ or one fourth part of the parameter of A, as the ſquare root of AN to the ſquare root of $\frac{\frac{1}{4}AE^2}{AN}$; that is, as $\surd$ AN to $\frac{\frac{1}{2}AE}{\surd AN}$, or AN to $\frac{1}{2}$AE, or twice AN to AE. Therefore the velocity acquired by a body in falling from A to N has the ſame ratio to the velocity with which the body is projected or the line AE deſcribed, and to the velocity acquired by a body in falling through a fourth part of the parameter belonging to the point A: conſequently (El. V. 11.) theſe velocities are equal.

Plate 3. Fig. 13.

Cor. Hence may be determined the direction in which a projectile from a given point, with a given velocity, muſt be thrown to ſtrike an object in a given ſituation.

Let A be the place from which the body is to be thrown, and K the ſituation of the object. Raiſe AB perpendicular to the plane of the horizon, and equal to four times the height from which a body muſt fall to acquire the given velocity. Biſect AB in G: through G draw HG perpendicular to AB: at the point A raiſe AC perpendicular to AK, and meeting HG in C: on C as a centre with the radius CA deſcribe the circle ABD; and through K draw the right line KEI perpendicular to the plane of the horizon, and cutting the circle ABD in the points E and I. AE, or AI, will be the direction required.

For, drawing BI, BE, ſince AK is a tangent to the circle, and BA, IK, are parallel to each other, the angle ABE (El. III. 32.) is equal to the angle EAK; and the alternate angles BAE, AEK, are equal: Therefore the triangles ABE, AEK, are ſimilar; and AB is to AE, as AE to EK. Therefore $AB \times EK = AE^2$; and $AB = \frac{AE^2}{EK}$. In like manner, the triangles BAI, KAI, being ſimilar, BA is equal to $\frac{AI^2}{IK}$. Since, then, AB is equal to four times the height from which a body muſt fall to acquire the velocity with which it is to be thrown; $\frac{AE^2}{EK}$ (or $\frac{AI^2}{IK}$ its equal) is the ſame. Conſequently (by this Prop.) the point K will be in the parabola which the body will deſcribe, which is thrown with the given velocity in the directions AE, or AI, and the body will ſtrike an object placed at K.

Schol. If the velocity with which a projectile is thrown be required, it may be determined from experiments in the following manner. By the help of a pendulum or any other exact chronometer, let the time of the perpendicular flight be taken; then, ſince the times of the aſcent and deſcent are equal, the time of the deſcent muſt be equal to one half of the time of the flight, conſequently, that time will be known: and, ſince a heavy body deſcends from a ſtate of reſt at the rate of 16.1 feet in the firſt ſecond of time, and that the ſpaces through which bodies deſcend are as the ſquares of the times; if we ſay, as one ſecond is to 16.1 feet, ſo is the ſquare of the number of ſeconds which expreſs the time of the deſcent of the projectile, to a fourth proportional, we ſhall have the number of feet through which the projectile fell, which being doubled, will give us the number of feet which the projectile would deſcribe in the ſame time with that of the fall, ſuppoſing it moved with an uniform velocity, equal to that which it acquired by the end of the fall; which laſt found number of feet, being divided by the number of ſeconds which expreſs the time of the projectile's deſcent, will give a quotient, expreſſing the number of feet, through which the projectile would move in one ſecond of time with a velocity equal to that which it acquired in its deſcent, which velocity is equal to the velocity with which the projectile was thrown up; conſequently, this velocity is diſcovered.

PROP. LXI. The ſquares of the velocities of a projectile in different points of its parabola, are as the parameters belonging to thoſe points.

For (by the laſt Prop.) the velocities in the ſeveral points of the parabola, are equal to the velocities acquired in falling through the fourth parts of the parameters of the points. Therefore the ſquares of theſe velocities being (by Prop. XXVI.) as the ſpaces deſcribed, the ſquares of the velocities in the ſeveral points of the parabola are as the fourth parts of the parameters of thoſe points: but the whole parameters are as their fourth parts: therefore the ſquares of the velocities at the ſeveral points of the parabola are as the parameters of thoſe points.

COR. Hence, ſetting aſide any difference which may ariſe from the reſiſtance of the air, a projectile will ſtrike a mark as forcibly at the end as at the beginning of its courſe, if the two points be equally diſtant from the principal vertex: for, the parameters belonging to theſe points being equal, the velocities in theſe points muſt alſo be equal.

PROP. LXII. When a body is thrown obliquely with a given velocity, if the ſpace through which it muſt have fallen perpendicularly to acquire that velocity is made the diameter of a circle, the height to which the body will riſe is equal to the verſed ſine of double the angle of elevation.

Let a body be thrown in the direction BE, with the ſame velocity which any body would acquire by falling perpendicularly through AB; if AB is made the diameter of a circle, the greateſt height to which the body will riſe will be BD. Plate 3. Fig. 12.

Let IL be a right line drawn in the plane of the horizon, touching the circle in B, and making with the line BE, which is the direction in which the body is thrown, the angle IBE, or angle of elevation. Becauſe IL touches the circle, and EB drawn in the circle meets it in the point of contact, (El. III. 32.) the angle EBI is equal to the angle EAB. And ECB is double of EAB, (El. III. 20.) therefore ECB is double of EBI, the angle of elevation. And BD is the verſed ſine of ECB, that is, of double the angle of elevation.

Let BE repreſent the velocity with which the body is thrown. Then ſince this velocity is, by ſuppoſition, ſuch as might be acquired by falling down AB, if the body was thrown perpendicularly upwards with the ſame velocity BE, it would riſe to the height BA. Let the oblique motion BE be reſolved into two others, one in the direction BD perpendicular to the horizon, and the other in the direction DE parallel to it: then the aſcending velocity will be to the horizontal velocity, as BD to DE, and to the whole velocity, as BD to BE. But the part of the velocity BD is the only part which is employed in raiſing the body, ſince the other part DE is parallel to the plane of the horizon. Now, the height of a body aſcending perpendicularly with the whole velocity BE, will be to the height when it aſcends with the part BD (compare Prop. XXVI. and Prop. XXVIII.) as the ſquare of BE to the ſquare of BD. But becauſe (El. VI. 8.) the triangle EDB is ſimilar to the triangle AEB, BD is to EB, as EB is to BA; and BD, BE, BA, being continued proportionals, BD is to BA, as the ſquare of BD is to the ſquare of BE. And the perpendicular heights to which the velocities BE and BD will make the body aſcend have been ſhewn to be as the ſquare of BE to the ſquare of BD; the heights are therefore as BA to BD. Since therefore the firſt velocity BE would make the body aſcend through BA, the other velocity BD, which is the part of the whole velocity which acts to make the body thrown in the direction DE to aſcend, will carry it to the height BD, which is the verſed ſine of double the angle of elevation. The ſame might be ſhewn in any other direction of the body, as BF, or BG.

Def. XV. The *Random* of a projectile is the horizontal diſtance to which a heavy body is thrown.

PROP. LXIII. When a body is thrown obliquely with a given velocity, if the ſpace through which it muſt have fallen perpendicularly to acquire that velocity is made the diameter of a circle, the random will be equal to four times the ſine of double the angle of elevation.

Plate 3. Fig. 12. If EBI be the angle of elevation, and ECB double that angle, DE will be the ſine of double the angle of elevation. Let a body be thrown from the point B in the direction BE, with the velocity which it would acquire in falling through AB; the random, or horizontal diſtance at which the body will fall, is equal to four times DE.

For, ſince (as in the laſt Prop.) the velocity BE being reſolved in BD, DE, the aſcending velocity is BD, and the horizontal DE, if theſe two velocities were to continue uniform, the ſpaces deſcribed in equal times (Prop. V.) would be as the velocities, and in the ſame time in which the body by the aſcending velocity would riſe through BD, by the horizontal velocity it would be carried forwards through DE. Of theſe velocities, the horizontal one DE is uniform, becauſe the force of gravity can neither accelerate nor retard a motion in this direction; but the aſcending velocity is uniformly retarded; and therefore the body (compare Prop. XXIII. and XXVIII.) will be twice as long in aſcending to its greateſt height BD, as it would have been if the firſt aſcending velocity had continued uniform: but on this ſuppoſition, the body would have been carried through BD and DE in the ſame time: therefore in double the time, that is, in the time of aſcent through BD with an uniformly retarded velocity, it would be carried forward through twice DE: conſequently, in the times of deſcent and aſcent together it would move forwards through four times DE. Therefore a body thrown from B in the direction BE with ſuch a velocity as might be acquired by falling down AB, the diameter of a circle, will fall at the diſtance of four times the ſine of double the angle of elevation.

PROP. LXIV. The random of a projectile will be the greateſt poſſible, with a given velocity, when the angle of elevation is an angle of forty-five degrees.

The velocity being given, the height from whence the body muſt have fallen to acquire that velocity, or (Prop. XXXV.) the diameter of the circle AB, is a given quantity. And in a given circle the greateſt ſine is the radius or ſine of a right angle: therefore four times the radius is greater than four times any other ſine; and conſequently, the random which is equal to four times the radius, (which, by Prop. LXII. will be the caſe when the double angle of elevation is a right one, or the angle of elevation forty-five degrees) will be the greateſt poſſible random.

EXP. This propoſition, and the two following, may be illuſtrated by water ſpouting from a pipe.

PROP. LXV. The random of a projectile, whoſe velocity is given, will be the ſame at two different elevations, if the one be as much above forty-five degrees as the other is below it.

If EBI be an angle of 30 degrees, and GBI an angle of 60 degrees, becauſe EBI falls ſhort of half a right angle as much as GBI exceeds it, the double of EBI will fall ſhort of a right angle as much as the double of GBI will exceed it, therefore, from the definition of a ſine, Plate 3. Fig. 12.

a ſine, theſe doubles will have the ſame ſine. Conſequently, four times their ſines, that is, (by Prop. LXIII.) their randoms will be equal.

PROP. LXVI. The greateſt random of a projectile, whoſe velocity is given, is double the height to which it would riſe if it were thrown perpendicularly with the ſame velocity.

Plate 3. Fig. 12. If a body be projected in the direction BF, at an angle of forty-five degrees, and its velocity be equal to that which a body would acquire in falling down AB (by Prop. LXIV.) the random will be the greateſt poſſible, and will be equal to four times CF, or twice BA. But the body caſt perpendicularly upwards with the ſame velocity would (by Prop. XXVIII.) riſe to the height BA. Therefore the greateſt random, with a given velocity, is double the height to which the body, thrown perpendicularly with the ſame velocity, would riſe.

PROP. LXVII. The randoms of projectiles, whoſe elevations are given, are as the ſquares of their velocities.

Plate 3. Fig. 12. If a body be thrown in any direction BE, its random (Prop. LXIII.) will be equal to four times DE, or four times the ſine of double the angle of elevation, in a circle whoſe diameter AB is the height from which the body muſt fall to acquire the velocity with which it is projected. But the velocities being ſuppoſed variable, AB the diameter will be directly as the velocity; ſince the greater velocity a body moves with, the greater ſpace it will fall through in a given time. And becauſe, in the triangle EDC the angle at D being a right angle is always invariable, and that the angle ECD, which is double of EAD, that is (El. III. 32.) of the given angle of elevation EBI, is given, the triangle ECD in every variation of AB, is always equiangular and ſimilar to itſelf, and ED is always as EC: but EC being a radius, is as AB: therefore ED, the ſine of the given angle of elevation, is as AB the diameter. Conſequently four times the ſine ED, that is, the random is as AB. But the height AB from which a body muſt fall to acquire any velocity, is (by Prop. XXVI.) as the ſquare of that velocity. Therefore the random is as the ſquare of the velocity.

SECT. II.

Of Central Forces.

PROP. LXVIII. A body which is conſtantly drawn or impelled towards any point, may be made to deſcribe, round that point as a centre, a curve returning into itſelf.

Let T be the centre of the earth, and GDEI its ſurface. Let a body be projected in any Plate 3. Fig. 14. direction, GH, which does not paſs within the ſurface of the earth. The projectile force, together with the force of gravity, will make it deſcribe a curve, which, as the projectile force is increaſed, will recede farther from the perpendicular GE, as GB, GC, GD. It is manifeſt that the projectile force may be increaſed, till the body ſhall paſs beyond the ſurface CDKE, and move in the path GML, GNV, or ſome larger curve.

Firſt; Suppoſe the projectile force to be ſuch, that the body will be carried in the ſemicircle GN, it will continue in the curve of that circle till it returns to G. For, when a body moves in the circumference of a circle, (as in Fig. 15.) the projectile force, acting in a line which is a tangent to the circle, as GB, acts (El. III. 18.) in a direction which is Plate 3. Fig. 15. perpendicular to the direction BA, in which it is impelled towards the centre. And ſince if the force which impels the body towards the centre ceaſed to act in any point, as C the body would move forwards in the right line CF, the projectile force in every point of the circumference, acts in a direction perpendicular to the force of gravitation: conſequently, theſe two forces remaining the ſame, and acting always in the ſame direction with reſpect to each other, the velocity of the body muſt remain the ſame: whence, at the point M, it Fig. 14. will have the ſame power to recede from the centre as at G; and, retaining this power through every remaining part of its courſe, it will proceed in the circumference, till it arrive at G, and will continue to revolve in the circle.

Next; Let the body be projected from G with a force greater than that which is required to carry it round in the circumference of the circle GNV; and let the curve in which it moves be an ellipſe, having the earth in its remoter focus. Becauſe the force of projection, as the body proceeds in the firſt half of its orbit, acts in the direction of a tangent to the curve, whilſt the force of gravitation acts in the direction of a right line from the body to the centre of the earth, the directions of theſe two forces make an acute angle with one another, and conſequently, through this part of the courſe of the body, the force of gravitation *conſpiring with* the force of projection, the velocity of the body muſt be increaſed, and at the ſame time it muſt be continually *drawn downwards* towards the earth. At the point in which the forces act in directions perpendicular to each other, the force of gravitation does not conſpire with that of projection to bring the body towards the earth: and afterwards, in the latter half of its courſe, the directions of the forces making an obtuſe angle with each other, the force of gravitation is *oppoſed* by that of projection in the ſame degree in which the former was before aided by the latter; and therefore the body in paſſing towards G will fly off from the earth or *riſe*, as much as it before approached to the earth or *deſcended*, and thus will return to the point G with the ſame velocity with which it ſet out at firſt, having loſt as much velocity by receding from the earth in the latter part of its courſe, as it had gained by falling towards the earth in the former part.

Laſtly; Let the body be projected from G with a force which is leſs than ſufficient to carry it round in the circle GNV; and let it perform its revolutions in an elliptic curve, whoſe greater axis is leſs than the radius of the circle GNV, ſetting out from G, and having the earth in the nearer focus: the effect will be the ſame as in the laſt caſe, except that the projectile force will oppoſe the force of gravitation in the *firſt* half of the revolution; and conſpire with it in the *latter*.

Exp. Let a ball revolve round the central point of a Whirling Table. Concerning the conſtruction and uſe of this machine, ſee Ferguſon's Lectures, Lect. II.

PROP. LXIX. A body revolving in an orbit, endeavours in every point of its courſe to fly off from the centre in a right line which is a tangent to the orbit.

Plate 3. Fig. 15. Let BCDL be a circle in which a body is revolving: when it is arrived at the point B, let the force which impels it towards the centre be withdrawn, and the body (by Prop. I.) would fly off from the point B in the direction BG; in like manner at C, it would fly off in the right line CF; at D, in DH; and at L, in LK. The ſame is manifeſtly true in an elliptical orbit. Now the ſame force with which it would fly off, if no other cauſe prevented it, muſt make it endeavour to fly off in the ſame manner in every point of the orbit.

Exp. Whilſt a ball is revolving on a whirling table, if the cord which retains it, be ſuddenly cut, the ball will fly off in a right line, which will be a tangent to the orbit in which it moved.

Cor. A body revolving about a centre endeavours to recede from that centre: for every point of the tangent in which it endeavours to move out of the circle, is farther from the centre, than the point in which the tangent meets the curve.

Def. XVI. The force which impels a body towards the centre, when it revolves in an orbit, is called the *centripetal* force: that by which it endeavours to recede from the centre, is called the *centrifugal* force; and theſe two forces are called jointly, *central* forces.

Schol. The projectile and centrifugal forces differ from each other, as the whole from the part. The projectile force is that with which a body would move forwards in a tangent to its orbit, if there were no centripetal force to prevent it: the centrifugal force is that part of the projectile force which carries the body off from the centre while it is deſcribing the tangent. Thus, if the body revolved in the orbit BD, the projectile force is that which would make it deſcribe the tangent BA, if the centripetal force were to ceaſe acting. But in the mean time, the whole force BA does not carry the body off from the centre C: when it is arrived at A, it is farther from the centre than it was at B, only by the length AN, and it is that part of the projectile force which, when the whole is reſolved into

Plate 3. Fig. 16.

into two forces, may be conſidered as acting in this line AN, which carries the body off from the centre, and is called the centrifugal force.

PROP. LXX. When bodies revolve in a circular orbit about a centre, the centripetal and centrifugal forces are equal.

Plate 3. Fig. 16. If a body revolve in the circle BD, in the time in which it deſcribes the arc BN, it will have been impelled towards the centre through the ſpace AN; for, by the projectile force alone it would have been carried from B to A. The line AN is then the ſpace deſcribed by means of the centripetal force, and this force is proportional to AN. But if, when the body was at B, no centripetal force had acted upon it, inſtead of deſcribing the arc BN, it would have moved along the tangent BA, and the line NA would have been the ſpace through which it would have departed from the centre: therefore the centrifugal force is proportional to NA. Both theſe forces being then proportional to the ſame line NA, they are equal to one another.

LEMMA I.

Quantities, and the ratios of quantities, which, in any finite time, tend continually to equality, and, before the end of that time, approach nearer to each other than by any given difference, become ultimately equal.

If you deny it, let them be ultimately unequal; and let their ultimate difference be D. Therefore they cannot approach nearer to equality than by that given difference D; which is contrary to the ſuppoſition. If a ſtraight and a curve line, continually diminiſhing, perpetually approach towards equality, and at the end of any finite time would vaniſh together, at the inſtant in which they are vaniſhing they are equal.

Plate 3. Fig. 18. LEM. II. *If in any figure* AacE, *terminated by the right lines* Aa, AE, *and the curve* acE, *there are inſcribed any number of parallelograms* Ab, Bc, Cd, *&c. contained under equal baſes* AB, BC, CD, *&c. and the ſides*, Bb, Cc, Dd, *&c. parallel to* Aa *the ſide of the figure; and the parallelograms* aKbl, bLcm, cMdn, *&c. are completed: then, if the breadth of theſe parallelograms be diminiſhed, and their number augmented continually, the ultimate ratios, which the inſcribed figure* AKbLcMdD, *the circumſcribed figure* AalbmcndoE, *and the curvilinear figure* AabcdE, *have to each other, are ratios of equality.*

For the difference of the inſcribed and circumſcribed figure is the ſum of the parallelograms K*l*, L*m*, M*n*, D*o*, that is, (becauſe of the equality of all their baſes) the rectangle under one of their baſes K*b*, and the ſum of their altitudes A*a*; that is, the rectangle AB*la*. But this rectangle, becauſe its breadth AB is diminiſhed indefinitely, becomes leſs than any given rectangle. Therefore (by Lem. I.) the inſcribed and circumſcribed, and much more the intermediate curvilinear figure become ultimately equal.

Plate 3. Fig. 18. LEM. III. *The ſame ultimate ratios are alſo ratios of equality, when the breadths* AB, BC, CD, *&c. of the parallelograms are unequal, and are all diminiſhed indefinitely.*

For, let AF be equal to the greateſt breadth: and let the parallelogram FA*af* be completed. This will be greater than the difference of the inſcribed and circumſcribed figures; but, becauſe its breadth AF is diminiſhed indefinitely, it will become leſs than any given rectangle.

Cor. Hence the ultimate ſum of the evaneſcent parallelograms coincides in every part with the curvilinear figure. Much more does the rectilinear figure, which is comprehended under the chords of the evaneſcent arcs *ab*, *bc*, *cd*, &c. ultimately coincide with the curvilinear figure. As alſo the circumſcribed rectilinear figure, which is comprehended under the tangents of the ſame arcs. And therefore, theſe ultimate figures (as to their perimeter *ac*E) are not rectilinear, but curvilinear limits of rectilinear figures.

PROP. LXXI. The plane of an orbit in which a body revolves paſſes through the line of projection, and through the centre towards which the centripetal force is directed.

Plate 4. Fig. 2. Let ABCF be the orbit in which the body revolves; S the centre, or point towards which the centripetal force is directed; and AV the line of projection: the plane of the orbit will paſs through AV and S; or the orbit lies in the ſame plane, with the lines AV, B*d*, &c. (lines in which the projectile force acts in different parts of the orbit) and with the centre S.

For, let ABCD repreſent a part of the orbit deſcribed by a body impelled towards S. The body beginning to move by the projectile force from A in the direction ABV, would, by that force alone, be carried on uniformly in that direction. Suppoſe the centripetal force to act upon it by ſeparate impulſes after equal intervals of time, and that, when the body is carried by the projectile force to B, it receives one impulſe from the centripetal force, drawing it out of its courſe towards S, ſo that by the action of both forces together at B, it will (by Prop. XIV.) be made to deſcribe BC in the ſame time in which the projectile force alone would have made it deſcribe B*c*. The ſame will take place after equal intervals at C and D. At B, the projectile force is in the direction BV, the centripetal force in the direction BS. Let B*c* and BG, taken in the direction of theſe forces, repreſent their ratio to each other, and the projectile force be to the centripetal as B*c* to BG. The body (by Prop. XIV.) will deſcribe BC, the diagonal of a parallelogram of which B*c* and BG are the ſides. But (El. XI. 1 and 2.) BC is in the ſame plane with B*c* and BG, that is, with AV, the line of projection, and with BS, in which is the centre or point S. The ſame may be proved concerning the lines CD, and DE. And if the projectile force acts continually, and not by interrupted impulſes, the diagonals AB, BC, &c. will be diminiſhed indefinitely, and the ultimate perimeter ADF (by Lem. III. Cor.) will become a curve line, which, from what hath been ſhewn, muſt be always in the ſame plane with the line of projection, and with the centre.

PROP. LXXII. A body revolving in an orbit deſcribes, by a radius drawn to the point towards which the centripetal force acts, equal areas in equal times, and in unequal times areas proportional to the times.

Let ABCD be part of an orbit deſcribed by a body which revolves round the point S, towards which it is impelled by a centripetal force. If this force be ſuppoſed to act upon the body by ſeparate impulſes, as at B, C, D, when the body receives the impulſe at B, it will be drawn out of its courſe towards S, and (by Prop. XIV.) will deſcribe the diagonal BC in the ſame time in which the projectile force alone would have made it deſcribe B*c*. After equal intervals, the ſame will take place at C, and at D. Plate 4. Fig. 2.

Since AB, BC, &c. are the lines deſcribed in equal times by the body, the areas deſcribed round S by a radius drawn from the body to S, are ASB, BSC, &c. Now AB, B*c*, expreſſing ſpaces paſſed over in equal times by the uniform motion of the body acted upon by the projectile force alone, are equal baſes of the triangles ASB, BS*c*, which, being terminated by the ſame point S, are of the ſame altitude: theſe triangles are therefore (El. I. 38.) equal. And, becauſe the body at B, is by the joint action of the projectile and centripetal forces carried forwards in the diagonal BC, of the parallelogram G*c*, the oppoſite ſides thereof, GB, C*c*, are parallel; and C*c* is parallel to BS. But BS is the common baſe of the two triangles BSC, BS*c*. Therefore theſe triangles, being upon the ſame baſe, and between the ſame parallels, (El. I. 37.) are equal. Conſequently, ASB, which has been proved equal to BS*c*, is likewiſe equal to BSC; that is, the areas deſcribed in equal times are equal. And, by compoſition, any ſums of theſe areas ASC, ASE, are to each other as the times in which they are deſcribed; that is, univerſally, the areas are as the times.

Let the number of theſe triangles be augmented, and their breadth diminiſhed indefinitely, and (by Lem. III. Cor.) their ultimate perimeter will be a curve line; and therefore the centripetal force will act continually: and, the above reaſoning ſtill being applicable to thoſe triangles whoſe breadth is indefinitely diminiſhed, the areas will be as the times.

Cor. 1. The velocity of a body revolving freely about an immoveable centre is inverſely as a perpendicular let fall from that centre on a right line that touches the orbit. For ſince the lines AB, BC, are deſcribed in equal times, the velocities will be as theſe lines, which being (by this Prop.) the baſes of equal triangles, muſt be inverſely as the heights of the triangles; therefore the velocities are inverſely as theſe heights, which are meaſured by perpendiculars let fall from the common vertex, the centre S, to the baſes, or the baſes produced, that is, when AB, BC, &c. are indefinitely ſmall, to a tangent to the orbit.

Cor. 2. If the chords AB, BC, of two arcs ſucceſſively deſcribed in equal times by the ſame body moving freely, are completed into a parallelogram ABCG, and the diagonal BG, in the poſition which it ultimately acquires when theſe arcs are diminiſhed indefinitely, be produced towards S, it will paſs through S, the centre of the centripetal force: for BG is an indefinitely ſmall part of the radius SB.

Cor. 3. AB, BC, and DE, EF, of arcs deſcribed in equal times be completed into the parallelograms ABCG, DEFZ, the centripetal forces at B and E will be to each other in the ultimate ratio of the diagonals BG, EZ, when thoſe arcs are indefinitely diminiſhed. For, the motions of the body, BC, EF, (by Prop. XVI.) are compounded of the motions B*c*, BG, and E*f*, EZ; but BG and EZ are equal to C*c* and F*f*, which, as appears from this propoſition, are generated by the impulſes of the centripetal force in B and E, and are therefore proportional to thoſe impulſes.

Cor. 4. The forces with which bodies are drawn into curvilinear orbits are to each other as the verſed ſines, $\frac{1}{4}$GB, $\frac{1}{4}$ZE, of the indefinitely ſmall arcs AC, DF, deſcribed in equal times, which verſed ſines converge to the centre S, and biſect the chords when thoſe arcs are diminiſhed indefinitely: for ſuch verſed ſines are half the diagonals of a parallelogram, BG, EZ, being biſected by the diagonals AC, FD.

PROP. LXXIII. When a body deſcribes equal areas in equal times about an immoveable point, or proportional areas in unequal times, it is impelled towards that point by the centripetal force which retains it in its orbit.

Plate 4. Fig. 2. Let AB, BC, &c. be lines deſcribed by the revolving body in equal times; and it may be proved as before, that the triangles ASB, and BS*c*, being of the ſame height, and having equal baſes AB, B*c*, are equal. But, by ſuppoſition, ASB, BSC, are equal; therefore BS*c* and BSC are equal. And theſe equal triangles, being upon the ſame baſe SB, (El. I. 39.) are between the ſame parallels; therefore C*c* and BS are parallel. Becauſe the body at B is acted upon by two forces, the projectile force in the line B*c*, and the centripetal force, and, by ſuppoſition, theſe two forces together make it deſcribe BC; BC is the diagonal of a parallelogram of which B*c* is one ſide, and the direction of the centripetal force at B is in

the other ſide. Now, in the parallelogram whoſe diagonal is BC, and one of its ſides B*c*, C*c* muſt be another; whence the oppoſite ſide, that is, the direction of the centripetal force, muſt be parallel to C*c*: but C*c* and BS have been proved to be parallel, when equal areas are deſcribed in equal times about the point S. Therefore, on this ſuppoſition, the body at B is acted upon by the centripetal force in the direction BS. The ſame may be ſhewn at every other point C, D, &c. Therefore the centripetal force tends to that point round which the body, by a radius drawn thither, deſcribes equal areas in equal times.

LEM. IV. *If any arc* ACB, *of a finite curvature, is ſubtended by its chord* AB, *and a ſtraight line* AD *produced both ways touch the arc in the point* A, *the arc, the chord, and the tangent, in their ultimate vaniſhing ſtate, will be equal.* Plate 4. Fig. 1.

The arc being ſuppoſed of a finite curvature, or ſuch as may be meaſured by a circle of a finite diameter; let BAC be the circle of the curvature; draw the line AC in that circle parallel to the ſubtenſe BD, and complete the triangle BAC. Becauſe the angle DAB (El. III. 32.) is equal to the angle ACB, and the alternate angles CAB, ABD, are equal, the triangles, CAB, DAB, are ſimilar. Hence, (El. VI. 4.) AB is to AD, as AC is to BC. The point B approaching continually to A, let BA become leſs than any aſſignable quantity; then the finite lines AC, BC, approach nearer to the ratio of equality, than by any given diſtance: therefore likewiſe AB, AD, which are proportional to AC, BC, and much more the intermediate arc, are ultimately equal.

LEM. V. *The naſcent or evaneſcent ſubtenſe of the angle of contact, in circles and in all curves which have a finite curvature, is as the ſquare of the conterminous arc.*

Let AD, in the ſemicircle ADC, be the given arc, AB the tangent, and the angle BAD the angle of contact. Draw BD, HG, parallel to AC the diameter; theſe lines, ſubtending the angle BAD, are called the ſubtenſes of the angle of contact. The arcs AD and AG, having the common term or limit the point A, are called conterminous arcs. Draw the lines DC, GC. If theſe lines be conceived to turn round upon the point C as a centre, ſo that the two points D, G, and with them the two ſubtenſes BD, HG, may approach towards A; it is manifeſt, that as theſe ſubtenſes come nearer to A, they will diminiſh, and at laſt will vaniſh in A. At the inſtant of their vaniſhing, BD will be to HG, as the ſquare of the arc AD is to the ſquare of the arc AG. Plate 3. Fig. 17.

Let ED be drawn parallel to AB, and FG to AH. Then, becauſe AB is a tangent at the point A, and conſequently (El. III. 18.) perpendicular to the diameter AC, ED which is parallel to AB is likewiſe perpendicular to AC: for the ſame reaſon FG is perpendicular to AC. And ADC, AGC (El. III. 31.) are right angles. Therefore (El. VI. 8.) AED is ſimilar to ADC, and AE is to AD, as AD to AC. Therefore (El. VI. 17.) the rectangle of AE, AC, is equal to the ſquare of AD. But AE is equal to BD: therefore the rectangle BD, AC, is equal to the ſquare of AD. For the ſame reaſon, the rectangle of HG, AC, is equal to the ſquare of the chord AG. Conſequently, the ſquare of the chord AD is to the ſquare of the chord AG, as the rectangle BD, AC, is to the rectangle HG, AC, that is, (El. VI. 1.) as BD to HG. But (by Lem. IV.) the arc AD and the chord AD, are ultimately in the ratio of equality, and alſo the arc AG and the chord AG. Therefore the ſquare of the arc AD is to the ſquare of the arc AG, at the inſtant in which they vaniſh, as BD to HG; that is, the evaneſcent ſubtenſe of the angle of contact is as the ſquare of the conterminous arc.

Fig. 16. Next; Let the ſubtenſes be not parallel to the diameter, but parallel to one another. Let MN, FG, be the ſubtenſes parallel to the diameter; and AN, OG, two ſubtenſes parallel to each other, but not to the diameter. Becauſe BA is a tangent at the point B, BD (El. III. 16.) is perpendicular to BA: ſince, therefore, MN and FG are parallel to BD, they are alſo perpendicular to BA, and the angles OFG, AMN, are equal. But becauſe OG and AN are parallel by conſtruction, the angle FOG (El. I. 29.) is equal to the angle MAN. Therefore the triangles FGO, MNA, are ſimilar, and (El. VI. 4.) AN is to OG, as MN is to FG. But it has been proved that MN is ultimately to FG, as the ſquares of the conterminous arcs: therefore AN is ultimately to OG, as the ſquares of the conterminous arcs BN, BG.

Plate 3. Fig. 16. Laſtly; Suppoſe both AN and OG directed to C, the centre of the circle. In this caſe, each of theſe would be a ſemidiameter, continued from G and N reſpectively to the tangent BA. In their ultimate ſtate theſe lines AN, OG, muſt coincide in the point B, and in the ſame right line BC; and therefore will become parallel, and will be, from what has been ſhewn, ultimately as the ſquares of the conterminous arcs.

Fig. 17. If GC, DC, be beginning to move from A, they are in their naſcent ſtate; and it is manifeſt, that the ſubtenſes in this ſtate are the ſame, and therefore have the ſame ratio, as in the evaneſcent ſtate.

Cor. 1. Hence, becauſe the tangents AB, AH, the arcs, AD, AG, and their ſines ED, FG, became ultimately equal (by Lem. IV.) to the chords AD, AG, their ſquares, alſo will be ultimately as the ſubtenſes BD, HG.

Cor. 2. The ſame ſquares are alſo ultimately as thoſe verſed ſines of the arcs, which biſect the chords and converge to a given point. For by the ſecond caſe of this propoſition theſe verſed ſines AE, AF, are as the ſubtenſes BD, HG, or AN, OG.

Cor. 3. Hence theſe verſed ſines AE, AF, are as the ſquares of the times in which a body deſcribes the arcs AD, AG, with given velocities. For the ſpaces AD, AG, deſcribed with given velocities, are as the times, and the ſquares of the ſpaces as the ſquares of the times; but (by laſt Cor.) the ſquares of theſe ſpaces are as the verſed ſines AE, AF; therefore theſe verſed ſines are as the ſquares of the times in which the arcs AD, AG, are deſcribed.

LEM. VI. *The naſcent or avaneſcent ſubtenſe of the angle of contact is equal to the ſquare of the conterminous arc divided by the diameter.*

It has been ſhewn, in the preceding Lemma, that BD is to AD, as AD is to AC. Therefore BD multiplied into AC, is equal to the ſquare of AD, and BD $=\frac{AD^2}{AC}$. But by Lem. IV. the arc AD is ultimately equal to the chord AD: therefore the naſcent or evaneſcent ſubtenſe BD is equal to the ſquare of the arc AD divided by the diameter AC. Plate 3. Fig. 17.

PROP. LXXIV. The centripetal forces of bodies revolving in different circular orbits about the ſame centre towards which they tend, are as the ſquares of the arcs deſcribed in the ſame time, divided by the radii of the circles.

In the circular orbits BND, RLE, let bodies revolve about the centre C, towards which they tend. Let them in the ſame time deſcribe the indefinitely ſmall arcs BG, RL. Then, becauſe the projectile forces would carry them in the ſame time through the tangents BF, RH, and the ſpaces through which, at the points G and L, they have been drawn from the tangents towards the centre by the centripetal force, are FG, HL; the centripetal forces muſt be as FG and HL. And by (Lem. VI.) the evaneſcent, or naſcent, ſubtenſe FG is equal to the ſquare of the arc BG divided by BD; and the evaneſcent, or naſcent, ſubtenſe HL is equal to the ſquare of the arc RL divided by RE. Therefore the ſubtenſe FG is to the ſubtenſe HL as the ſquare of the arc BG divided by BD, or its half BC, is to the ſquare of the arc RL, divided by RE, or its half RC. Therefore the centripetal forces, when the arcs are naſcent, are in the ſame ratio, that is, as the ſquares of the arcs divided by the radii. Plate 3. Fig 16.

And this is true, whatever arcs BG and RL be taken, if they be deſcribed in the ſame time: for the naſcent arcs will be as the velocities; and any other arcs BND, RLE, deſcribed in any given time, will be alſo as the velocities; therefore, the arcs BND, RLE, are as the naſcent arcs BG, RL, and their ſquares are likewiſe proportional. But the centripetal forces are as the ſquares of the naſcent arcs BG, RL, divided by the radii BC, RC; therefore theſe forces are as the ſquares of any other arcs, BND, RLE, divided by the radii of their circles.

PROP. LXXV. The centripetal forces of equal bodies revolving in circular orbits, are as the ſquares of the velocities directly, and the radii of the orbits inverſely.

Becauſe arcs deſcribed in the ſame time are as the velocities, and that the centripetal forces are (by Prop. LXXIV.) as the ſquares of the arcs deſcribed in the ſame time divided by the radii, theſe forces are alſo as the ſquares of the velocities divided by the radii, that is, as the ſquares of the velocities directly, and the radii of the orbits inverſely.

Cor. Hence the centripetal forces of equal bodies, at equal diſtances from the centre, are as the ſquares of the number of revolutions in any given time; for this number is as the velocity with which the body moves.

PROP. LXXVI. The centripetal forces of equal bodies revolving in equal circular orbits are inverſely as the ſquares of their periodical times.

The circular orbits or ſpaces being equal, the times in which theſe are deſcribed, or the *periodical times*, are (by Prop. V.) inverſely as the velocities; and therefore the ſquares of the periodical times are inverſely as the ſquares of the velocities, or the ſquares of the velocities are inverſely as the ſquares of the periodical times: But (by Prop. LXXV.) the centripetal forces are as the ſquares of the velocities: Therefore theſe forces are inverſely as the ſquares of the periodical times.

PROP. LXXVII. The centripetal forces of equal bodies revolving in unequal circular orbits, if the periodical times are equal, are as the radii of the circles.

Plate 3. Fig. 16. Let one body revolve in the circular orbit BND, and another, in the ſame time, in the circular orbit RLE. Becauſe the periodical times are equal, each body in any given part of its periodical time will deſcribe an equal number of degrees in its reſpective orbit, that is, will deſcribe ſimilar arcs. The arcs BN, RL, being ſimilar, will be deſcribed in equal portions of the periodical time: Therefore (by Prop. LXXIV.) the centripetal forces will be as the ſquares of the ſimilar arcs BN, RL, divided by the radii BC, RC, that is, as $\frac{BN^2}{BC}$ to $\frac{RL^2}{RC}$. But becauſe ſimilar arcs are to each other as the circumferences, or radii, of circles BN is to RL, as BC to RC, and conſequently, BN^2 to RL^2, as BC^2 to RC^2: Therefore $\frac{BN^2}{BC}$ is to $\frac{RL^2}{RC}$, as $\frac{BC^2}{BC}$ is to $\frac{RC^2}{RC}$, that is, as BC to RC. But the centripetal forces (Prop. LXXIV.) are as $\frac{BN^2}{BC}$ to $\frac{RL^2}{RC}$; therefore theſe forces are as BC to RC, that is, as the radii of the orbits in which the bodies move.

PROP. LXXVIII. The centripetal forces of equal bodies revolving in circular orbits, are as the radii of the orbits directly, and the ſquares of the periodical times inverſely.

If the periodical times are equal, and the radii unequal, the forces are (by Prop. LXXVII.) as the radii. If the radii are equal, and the periodical times unequal, the forces (by Prop. LXXVI.) are inverſely as the ſquares of the periodical times. Therefore, if both the radii and periodical times are unequal, the forces will be in the compound ratio of both, or as the radii directly, and the ſquares of the periodical times inverſely.

PROP. LXXIX. When bodies revolve round the ſame centre, if the ſquares of their periodical times are as the cubes of their diſtances from the centre, the centripetal forces will be inverſely as the ſquares of their diſtances.

Let the diſtances of the two bodies be expreſſed by D, d; and the periodical times by P, p: then, by the ſuppoſition $P^2 : p^2 :: D^3 : d^3$.

By Prop. LXXVIII. the centripetal forces are as the diſtances directly, and the ſquares of the periodical times inverſely, that is, (taking C, c, for the centripetal forces) $C : c :: \frac{D}{P^2} : \frac{d}{p^2}$; and by ſuppoſition $P^2 : p^2 :: D^3 : d^3$: therefore, ſubſtituting D^3, d^3, for P^2, p^2, $C : c :: \frac{D}{D^3} : \frac{d}{d^3}$, that is, $C : c :: \frac{1}{D^2} : \frac{1}{d^2}$; and, becauſe where the dividend is given, the quotient is inverſely as the diviſor, $\frac{1}{D^2}$ is to $\frac{1}{d^2}$ inverſely as D^2 to d^2. Therefore $C : c :: d^2 : D^2$, that is, the centripetal forces are inverſely as the ſquares of the diſtances.

SCHOL. 1. Let C, c, expreſs the central forces; A, a, the arcs deſcribed; V, v, the velocities with which the bodies move; P, p, the periodical times of their revolution; D, d, the radii or diſtances from the centre; and N, n, the number of revolutions in a given time:—the preceding Propoſitions may be thus expreſſed.

The bodies being equal,

PROP. LXXIV.	$C : c :: \frac{A^2}{R} : \frac{a^2}{r}$.
LXXV.	$C : c :: \frac{V^2}{D} : \frac{v^2}{d}$.
COR.	$C : c :: N^2 : n^2$.
LXXVI.	$C : c :: \frac{1}{P^2} : \frac{1}{p^2}$ or $p^2 : P^2$.
LXXVII.	$C : c :: D : d$.
LXXVIII.	$C : c :: \frac{D}{P^2} : \frac{d}{p^2}$.
LXXIX.	If $P^2 : p^2 :: D^3 \; d^3$, $C : c :: \frac{1}{D^2} : \frac{1}{d^2}$ or $d^2 : D^2$.

Schol. 2. Since it was proved (Prop. LXX.) that the centripetal and centrifugal forces are, in circular orbits, equal to one another, the preceding Propoſitions, being demonſtrated reſpecting the centripetal force, are alſo true of the centrifugal force; and it may be aſſerted univerſally, that the *central forces* are in the ratios above expreſſed.

Theſe propoſitions may be confirmed by the following experiments, on the Whirling Tables.

Exp. 1. Let two equal balls, placed at equal diſtances from the centre of motion on the whirling tables; and let one table revolve twice whilſt the other revolves once: the ball on the table whoſe number of revolutions is, with reſpect to that of the other in the ſame time, as 2 to 1 (or the periodical times as 1 to 2) will raiſe 4 times the weight raiſed by the other ball; that is, according to Prop. LXXV. and Cor. the radii being equal, $C : c :: V^2 : v^2 :: N^2 : n^2$; or (by Prop. LXXVI.) $:: p^2 : P^2$.

2. Let two equal balls be placed on tables whoſe number of revolutions in the ſame time are as 2 to 1: let the ball on the table whoſe number of revolutions is 2, be placed at half the diſtance from the centre, at which the ball on the table, whoſe number of revolutions is 1, is placed; whence their velocities will be equal. The ball at the diſtance 1, will raiſe double the weight raiſed by the ball at the diſtance 2; that is, according to Prop. LXXV. the velocities being equal, $C : c :: d : D$.

3. Let two equal balls revolve on tables whoſe periodical times are equal; and let the diſtances of the balls from the centre be to each other as 2 to 1: the ball which is at the diſtance 2 will raiſe double the weight raiſed by the ball which is at the diſtance 1; that is, according to Prop. LXXVII. $C : c :: D : d$.

4. Let equal balls be placed on tables whoſe periodical times are as 2 to 1; let the ball on the table whoſe periodical time is 2, be placed twice as far from the centre as the ball whoſe periodical time is 1; the ball whoſe diſtance is 2, and periodical time 2, will raiſe half the weight raiſed by the ball whoſe diſtance is 1, and periodical time 1; that is, according to Prop. LXXVIII. $C : c :: \frac{D}{P^2} : \frac{d}{p^2} :: \frac{2}{4} : \frac{1}{2}$.

5. Let the equal balls be ſo placed on different tables, that the diſtance of one from the centre may be to that of the other as 2 to $3\frac{1}{6}$; let that ball which is at the leaſt diſtance revolve twice in the ſame time in which the other ball revolves once; the periodical time of the ball at the leſs diſtance, is to that of the ball at the greater, as 1 to 2, and the ſquares of the periodical times will be as 1 to 4, and the cubes of the diſtances are 8, and 31.75: but 1 : 4 : 8 : 32, therefore the ſquares of the periodical times being in this caſe nearly as the cubes of the diſtances, the weight raiſed by the ball whoſe diſtance is 2, will be to that raiſed by the ball whoſe diſtance is $3\frac{1}{6}$ as the ſquare of $3\frac{1}{6}$ is to the ſquare of 2; that is, nearly as 10 to 4, or 5 to 2.

PROP. LXXX. The centripetal forces of revolving bodies are as their quantities of matter.

For the whole centripetal force of any body is made up of the centripetal forces of each particle of matter of which it conſiſts; and therefore the more numerous the particles of matter in any body are, the greater will be its centripetal force.

EXP. Let two glaſs tubes be half filled with water; into one put ſome leaden ſhot, and into the other a few ſmall round pieces of light wood; let the orifice of each tube be cloſed by a cork: faſten the tubes to an inclined plane, and let the lower end of it reſt upon the centre of the whirling table. On turning the table, the bodies will be carried by their centripetal forces from the centre; and the heavier bodies will recede farther from the centre than the lighter. See Ferguſon's Lectures.

COR. Hence, when the revolving bodies are not equal, the centripetal forces are in the ratios laid down in the preceding propoſitions multiplied into their quantities of matter. Thus Q, q, expreſſing the quantities of matter, and the other expreſſions remaining as in Prop. LXXIX. Schol.

$$C : c :: Q : q$$

$$C : c :: \frac{QV^2}{D} : \frac{qv^2}{d}$$

$$C : c :: QN^2 : qn^2$$

$$C : c :: \frac{Q}{P^2} : \frac{q}{p^2}$$

$$C : c :: QD : qd$$

$$C : c :: \frac{QD}{P^2} : \frac{qd}{p^2}$$

$$\text{If } P^2 : p^2 :: D^3 : d^3,\ C : c :: \frac{Q}{D^2} : \frac{q}{d^2}$$

COR. Hence the central forces will be equal, whenever the expreſſions proportional to them are equal; thus, $C = c$ if $QD = qd$.

Any of the above proportions may be confirmed by experiment; for example;

EXP. 1. Let the two balls A, B, be as 2 to 1; let the diſtance of the ball A be to that of the ball B from the centre, as 2 to 1, and the periodical time of the ball A be twice that of the ball B; their velocities will be equal; therefore the centrifugal force of A will be to that of B, as $\frac{Q}{D}$ is to $\frac{q}{d}$, that is, as 1 to 1, or A and B will raiſe equal weights.

2, 3. Let the ſame balls revolve about a fixed point, and have their diſtances reciprocally proportional to their quantities of matter, their centrifugal forces (compare Prop. LXXV.

and LXXX.) will be equal, and they will balance each other. This may be ſhewn by two balls ſuſpended freely and united by a cord, having the point of the cord which is directly above the centre of the table at diſtances from the balls reciprocally as their weight; or by two balls united by a wire, and reſting in equilibrio on a forked ſupport fixed in the centre of the tables, which will continue in equilibrio when the tables are turned.

In like manner other caſes may be confirmed by experiment.

LEM. VII. *If a body revolves freely in any orbit about an immoveable centre, and in an indefinitely ſmall time deſcribes any naſcent arc; and the verſed ſine of the arc be drawn which may biſect the chord, and being produced may paſs through the centre of force; the centripetal force, in the middle of this arc, will be as the verſed ſine directly, and the ſquare of the time inverſely.*

Plate 4. Fig. 3.

Let two bodies revolve round their centre of force S, *s*; let QPM, *qpm*, be the naſcent arcs deſcribed in any times, T, *t*; and let PB, *pb*, or QR, A*a*, be the verſed ſines biſecting the chords, and when produced, paſſing through S the centre of force. Suppoſing the arcs MQP, NA*p*, to be deſcribed in the ſame time with different forces, C, *c*; by Prop. LXXII. Cor. 4. $QR : Aa :: C : c$. Hence, ſuppoſing the forces to be equal, QR is equal to A*a* deſcribed in the ſame time; and (by Lem. V.) QR or $Aa : qr :: Ap^2 : qp^2$, that is, ſince the motion in the arcs is uniform, $Aa : qr :: T^2 : t^2$. Therefore ſuppoſing both the times and forces different, and compounding theſe ratios, $QR : qr :: C \times T^2 : c \times t^2$; whence $C : c :: \frac{QR}{T^2} : \frac{qr}{t^2}$.

Plate 4. Fig. 4.

Cor. 1. If a body P, revolving about the centre S, deſcribes a curve line APQ, and a right line ZPR touches that curve in any point P; and, from any other point Q of the curve, QR is drawn parallel to the diſtance SP, meeting the tangent in R; and QT is drawn perpendicular to the diſtance SP; the centripetal force will be reciprocally as the quantity $\frac{SP^2 \times QT^2}{QR}$, if this be taken of that magnitude which it ultimately acquires, ſuppoſing the points P and Q continually to approach to each other. For QR is equal to the verſed ſine of double the arc QP, in whoſe middle is P: and double the triangle SQP, or SP × QP (or ultimately QT) is proportional to the time, in which that double arc is deſcribed (by Prop. LXXII.) and therefore may be uſed for the exponent of the time. Whence $C : c :: \frac{QR}{SP^2 \times QT^2} : \frac{qr}{ſp^2 \times qt^2}$; that is, C is to *c* reciprocally as $\frac{SP^2 \times QT^2}{QR} : \frac{ſp^2 \times qt^2}{qr}$; or the centripetal forces are reciprocally as $\frac{SP^2 \times QT^2}{QR}$.

COR. 2. Hence, if any curvilinear figure APQ is given; and therein a point S is alſo given, to which a centripetal force is perpetually directed; the law of centripetal force may be found, by which the body P, continually drawn back from a rectilinear courſe, will be retained in the perimeter of that figure, and will deſcribe the ſame by a perpetual revolution. That is, we are to find the quantity $\frac{SP^2 \times QT^2}{QR}$, reciprocally proportional to this force.

PROP. LXXXI. If equal bodies, revolving in ellipſes, deſcribe equal areas in equal times, their centripetal forces are to one another inverſely as the ſquares of their diſtances from the *foci* of the ellipſe towards which they tend.

Let S be the focus: let a body P, tending towards S, deſcribe a part of the ellipſe PQ; join SP; draw QR to the tangent YZ, parallel to SP; join PC, and produce it to G. Complete the parallelogram $Q x P R$, produce $Q x$ to v, $Q v$ is ordinately applied to GP; draw DK, a diameter parallel to YZ, and draw IH from the other focus H to SP parallel to YZ; join HP, and draw QT perpendicular to SP, as alſo PF to DK. Plate 4. Fig. 4.

EP is equal to the greater ſemiaxis AC. For, becauſe CS is equal to CH, ES is equal to EI, (El. VI. 2.) whence EP is half the ſum of PS, PI, that is, of PS, PH, (for El. I. 13.) RPI, IPZ, are equal to HPR, HPZ; and taking away the common part HPI, the angle IPR is equal to HPZ; whence (El. I. 29.) the angle PIH is equal to PHI, and PI is equal to PH; and PS, PH, together, (Simſon's Conic Sect. II. 1.) are equal to the whole axis 2AC. EP therefore is equal to AC.

Putting L for the principal *latus rectum* of the ellipſe, L, (by definition) is equal to $\frac{2BC^2}{AC}$ (for $AC : CB :: CB : \frac{L}{2}$, whence $\frac{2CB^2}{AC} = L$.) And $L \times QR : L \times Pv :: QR : Pv$; and $QR = Px$; and $Px : Pv :: PE : PC$; whence $L \times QR : L \times Pv :: PE$ or $AC : PC$. And (El. VI. 1.) $L \times Pv : Gv \times Pv :: L : Gv$; and (Simſ. II. 15.) $Gv \times Pv : Qv^2 :: PC^2 : DC^2$. And (Lem. IV.) the points Q and P continually approaching, Qv^2 is to Qx^2 ultimately in the ratio of equality. And (ſince the triangles QTx, EPF, are ſimilar, for $QxT = PEF$, and QTx to EFP) Qx^2 or $Qv^2 : QT^2 :: EP^2$ or $AC^2 : PF^2$. But becauſe (Simſ. II. 20. Compare Vince's Con. Sect. II. 10. Cor. 1.) parallelograms about conjugate diameters are equal to the rectangle under the axes, the rectangle PF, DC, is equal to the rectangle ACB, whence $PF : AC :: CB : DC$, and $AC^2 : PF^2 :: CD^2 : CB^2$, wherefore $Qv^2 : QT^2 :: CD^2 : CB^2$. Compounding the following ratios,

$L \times QR : L \times Pv :: AC : PC$,
$L \times Pv : Gv \times Pv :: L : Gv$,
$Gv \times Pv : Qv^2 :: PC^2 : CD^2$,
$Qv^2 : QT^2 :: CD^2 : CB^2$;

And, ſtriking out the equal quantities, $L \times QR : QT^2 :: AC \times L \times PC : Gv \times CB^2$.

Then ſubſtitute for $AC \times L$ is equal $2CB^2$, and

$L \times QR : QT^2 :: 2BC^2 \times PC : Gv \times BC^2$

or $BC^2 \times 2PC : Gv \times BC^2$

or $2PC : Gv$

But the points Q and P continually approaching without end, 2PC and Gv are equal; wherefore $L \times QR$ and QT^2, proportional to theſe are alſo equal. Multiply theſe equals into $\frac{SP^2}{QR}$ and $L \times SP^2$ will become equal to $\frac{SP^2 \times QT^2}{QR}$.

Therefore (by Lem. VII. Cor. 1 and 2.) the centripetal force is reciprocally as $L \times SP^2$, that is, ſince L is a given quantity, as SP^2, or in a duplicate ratio of the diſtance SP.

BOOK

BOOK III.

OF HYDROSTATICS AND PNEUMATICS;

OR THE LAWS OF

INCOMPRESSIBLE AND COMPRESSIBLE FLUIDS.

PART I.

OF HYDROSTATICS.

CHAP. I.

Of the Weight and Preſſure of Fluids.

DEF. I. A FLUID is a body, the parts of which yield to any force impreſſed upon them, and eaſily move out of their places.

PROPOSITION I.

The weight of fluids is as their quantities of matter.

Since each particle of any fluid gravitates towards the earth, the greater is the number of particles, that is, the quantity of matter in any maſs of fluid, the greater will be the weight of that maſs.

EXP. 1. The different preſſures of different columns of fluid in the ſame veſſel at different depths, appear from the different quantities of fluid diſcharged, at different depths, in the ſame time, from orifices of the ſame bore.

2. If the air be exhauſted from a tube in part filled with water, and the tube be cloſed up, the ſolidity of the particles of water will be perceived from the ſound produced by ſuddenly lifting up the tube.

COR. Fluids gravitate in fluids of the ſame kind. For they cannot loſe the property of gravity which belongs to all bodies by ſuch a change of ſituation.

EXP. Suſpend a ſtopped phial from one arm of a balance, in a veſſel of water, and balance it by weights from the oppoſite arm of the balance: upon unſtopping the phial under water, a quantity of water will ruſh into it, by which the weight will be increaſed as much as the weight of the water in the phial.

PROP. II. When the ſurface of a fluid is level, the whole maſs will be at reſt.

Plate 5. Fig. 2. Let ABCD be a veſſel containing water, the level ſurface of which is EF. Conceive the whole maſs of fluid in the veſſel to be divided into thin *ſtrata*, or plates, RS, TV, XY, &c. lying horizontally one above another; and into ſmall perpendicular columns GH, IK, LM, &c. contiguous to each other. In the ſtratum XY, and the columns IK, let *m*, *n*, be two adjacent drops. Neither of theſe drops can move towards the column in which the other is, without driving that other out of its place, becauſe the fluid is ſuppoſed incompreſſible. But, with whatever force the particle *m* endeavours to diſplace the particle *n*, this force is counterbalanced by an equal and contrary effort on the part of *n*; becauſe (Prop. I.) they are equally preſſed by the equal columns above them: conſequently the particles will be at reſt.

PROP. III. Any part of a fluid at reſt preſſes, and is preſſed, equally in all directions.

For (Def. I.) each particle is diſpoſed to give way on the ſlighteſt difference of preſſure: and, by ſuppoſition, each particle is preſſed by the contiguous particles in ſuch manner as to be kept at reſt in its place: it is therefore preſſed with an equal degree of force on all ſides; and, conſequently, (Book II. Prop. III.) it preſſes equally in all directions.

COR. Hence the lateral preſſure of a fluid is equal to the perpendicular preſſure. This is one of the moſt extraordinary properties of fluids, and can be conceived to ariſe only from the extreme facility with which the component particles move among each other.

EXP. 1. Into ſeveral tubes, bent near their lower ends in various angles, pour a ſufficient quantity of mercury to fill the lower parts of their orifices; then dip them into a deep glaſs veſſel filled with water, keeping the orifice of the longer legs above the ſurface: whilſt the tubes are deſcending, the mercury will be gradually preſſed upwards in the tubes, and the preſſure will be equal at any given depth, whatever be the direction of the preſſing column of fluid in the ſhorter leg of the tube. Oil may be uſed inſtead of mercury.

2. Dip an open end of a tube, having a very narrow bore, into a veſſel of quickſilver; then, ſtopping the upper orifice with the finger, lift up the tube out of the veſſel: a ſhort

column of quickſilver will hang in the lower end, which, when dipped in water lower than 14 times its own length, will, upon removing the finger, be ſuſpended, and preſſed upwards.

3. Let a large open tube be covered at one end with a piece of bladder drawn tight: pour into the tube a quantity of coloured water ſufficient to preſs the bladder into a convex form; then, dip the covered end of the tube ſlowly into a deep veſſel of water; the bladder, by the upward preſſure, will become firſt leſs convex, then plane, and at laſt concave.

4. If the like be done with ſeveral tubes, whoſe covered orifices are cut obliquely at different angles, the lateral preſſure will be ſeen to increaſe with the depths to which the tubes are immerſed.

5. Let a circular piece of braſs, whoſe upper ſurface is covered with wet leather, be held cloſe to one orifice of a large open tube, by means of a cord or wire faſtened to the middle of the plate, and paſſing through the tube: let the plate, thus kept cloſe to the orifice of the tube, be immerſed with the tube into a large veſſel of water: when the plate is at a greater depth than 8 times its thickneſs in the water, the cord or wire may be left at liberty, and the upward preſſure of the fluid will keep the plate cloſe to the tube.

6. Let a ſmall bladder, tied cloſely about one end of an open tube having a large bore, be filled with coloured water till the water riſes above the neck of the bladder; upon immerſing the bladder into a veſſel of water, the bladder will be compreſſed on all ſides, and the coloured water will be raiſed up in the tube in proportion to the depth to which the bladder is ſunk.

PROP. IV. When a fluid flows through a tube which is wider in ſome parts than in others, the velocity of the fluid will, in every ſection of the tube, be inverſely as the area of the ſection.

Let ADMN, a bended tube larger at IL than at FG, be filled with water to the height ADFG. Let the water be forced downwards in the part ADBP, and conſequently be made to riſe in the other part KHMN. It is manifeſt, that the water which is forced out of one part of the tube, is driven into the other. Hence equal quantities paſs through every ſection of the tube in the ſame time: for if leſs, or more, water paſſed through the ſection FG than through IL in the ſame time, the quantity of water between FG and IL muſt be increaſed or diminiſhed, which cannot be, ſince no cauſe is ſuppoſed which could increaſe or diminiſh it. But if equal quantities paſs through unequal parts of the tube in the ſame time the water muſt run proportionally faſter where the tube is narrower, and ſlower where it is wider. If, for example, as much water runs through the ſection FG, as runs in the ſame time through the ſection IL, the water muſt move as much faſter at FG than it moves at IL, as the tube is narrower at FG than at IL; that is, the velocity is inverſely as the area of the ſection. Plate 5. Fig. 1.

Cor. The momentum will be the ſame in every ſection of the tube; for the quantity of water at each ſection is directly as the area of the ſection, and the velocity is inverſely as the area; therefore the velocity is inverſely as the quantity of matter: whence (Book II. Prop. XI.) the momentum is every where the ſame.

Schol. Hence we may account for the ſuſpenſion of the fluid in a tube the upper part of whoſe bore is capillary, and the lower of a much larger dimenſion, as was ſeen in the experiment, Book I. Prop. VII.

Plate 5. Fig 3. Let there be a tube conſiſting of two parts DR and RCK, of different diameters: DR the ſmaller part of the tube, is able (Book I. Prop. VIII.) to raiſe water higher than the other: let then the height to which the larger would raiſe it, be TC, and that to which it would riſe in the leſſer, if continued down to the ſurface of the fluid, be XH. If this compound tube be filled with water, and the larger orifice CK be immerſed in the ſame Fig. 4. fluid, the ſurface of the water will ſink no farther than XL, the height to which the leſſer part of the tube would have raiſed it. But if the tube be inverted, and the ſmaller orifice XL be immerſed, the water will run out till the ſurface falls to TF; the height to which the larger part of the tube would have raiſed it.

Fig. 3. Let the tube DR be conceived to be continued down to HI; and let it be ſuppoſed that the fluids contained in the tube XLHI, and the compound one XLKC, are not ſuſpended by the ring of glaſs at XL, but that they preſs upon their reſpective baſes, HI and CK. Let it farther be ſuppoſed that theſe baſes are each of them moveable, and that they are raiſed up or let down with equal velocities; then will the velocity with which XL the uppermoſt ſtratum of the fluid XLCK moves, exceed that of the ſame ſtratum, conſidered as the uppermoſt of the fluid in the tube XLHI, as much as the tube RCK is wider than DR, (by this Prop.) that is, as much as the ſpace MNKC exceeds XLIH. Conſequently, the effect of the attracting ring XL, as it acts upon the fluid contained in the veſſel XLCK, exceeds its effect, as it acts upon that in XLHI, in the ſame ratio. Since, therefore, it is able to ſuſtain the weight of the fluid XLHI by its natural power, it is able, under this *mechanical* advantage, to ſuſtain the weight of as much as would fill the ſpace MNKC: but the preſſure of the fluid XLCK is equal to that weight, as having the ſame baſe and an equal height (as will be ſhewn by Prop. VI.) Its preſſure, therefore, or the tendency it has to deſcend in the tube, is equivalent to the power of the attracting ring XL, for which reaſon it ought to be ſuſpended by it.

Fig. 4. Again, the height at which the attracting ring in the larger part of the tube is able to ſuſtain the fluid is no greater than NF, that to which it would have raiſed it, had the tube been continued down to MN. For here the power of the attracting ring acts under a like *mechanical* diſadvantage, and is thereby diminiſhed, as much as the capacity of the tube TFNM is greater than that of HIXL; becauſe, if the baſes of theſe tubes are ſuppoſed to be moved with equal velocities, the riſe or fall of the ſurface of the fluid TFXL, would be ſo much leſs than that of TFMN. And, ſince the attracting ring TF is able, by its natural power, to ſuſpend the fluid only to the height NF in the tube TFMN; it is in this

caſe able to ſuſtain no greater preſſure than what is equal to the weight of the fluid in the ſpace HIXL: but the preſſure of the fluid TFXL, which has equal height, and the ſame baſe with it, is equal to that weight; and therefore is a balance to the attracting power.

From hence we may clearly ſee the reaſon, why a ſmall quantity of water put into a capillary tube, which is of a conical form, and laid in an horizontal ſituation, will run towards the narrower end. For let AB be the tube, and CD a column of water contained within it; when the fluid moves, the velocity of the end D will be to that of the end C reciprocally as the cavity of the tube at D, to that at C, (by this Prop.) that is, (El. XII. 2.) reciprocally as the ſquare of the diameter at D, to the ſquare of the diameter at C; but the attracting ring at D is to that at C, ſingly as the diameter at D to the diameter at C. Now, ſince the effect of the attraction depends, as much upon the velocity of that part of the fluid where it acts, as upon its natural force, its effect at D will be greater than at C; for though the attraction at D be leſs in itſelf than at C, yet its loſs of force upon that account, is more than compenſated by the *mechanical* advantage it has ariſing from hence, that the velocity of the fluid in that part is more increaſed than the force itſelf is diminiſhed at C. The fluid will therefore move towards B. See on this ſubject Mr. Vince's Principles of Hydroſtatics, p. 65—9. Plate 5. Fig. 5.

PROP. V. In bended cylindrical tubes, fluids at reſt will be at the ſame height on each ſide.

In the tube ADMN, filled with water to the height AD, the water cannot deſcend from AD, without riſing towards MN. The water in each ſide of the veſſel may therefore be conſidered as two forces acting upon each other in contrary directions: and conſequently theſe two maſſes of fluid will only be at reſt when their momenta are equal, that is, (Book II. Prop. XI. Cor.) when the quantities of matter are inverſely as the velocities, or (Prop. IV.) directly as the area of the ſection through which it flows. Thus, at the ſections BP, KH, the momenta are equal, when the quantities of matter, or cylindrical maſſes of fluid are as the areas of the ſections, that is, as the baſes of the cylinders ADBP, FGHK. But cylinders are as their baſes (El. XII. 11.) only when their perpendicular heights are equal. Therefore the momenta of the two cylinders of fluid will be equal, and conſequently the maſs will be at reſt, only when the perpendicular heights of each column are equal. Plate 5. Fig. 1.

EXP. 1. In a bended tube of large but unequal bore, water will riſe to the ſame height on each ſide.

2. Let water ſpout upward through a pipe, having a ſmall orifice, inſerted into the bottom of a deep veſſel; it will riſe nearly to the height of the upper ſurface of the water in the veſſel. The reſiſtance of the air, and of the falling drops, prevents it from riſing perfectly to the level.

Cor. If, therefore, a pipe convey a fluid from a reſervoir, it can never carry it to a place higher than the ſurface of the fluid in the reſervoir.

Schol. In this demonſtration, we do not conſider the velocity with which the two columns of fluid are moving, but the velocity with which, if they move at all, they muſt begin to move. And ſince, if their perpendicular height is the ſame, the velocity with which they muſt begin to move will be inverſely as their reſpective quantities of matter, they cannot begin to move but with equal momenta; and their motions muſt be in contrary directions, becauſe one column cannot deſcend without making the other aſcend: therefore thoſe equal momenta would deſtroy each other. Theſe two columns then, making a continual effort to move with equal momenta in contrary directions, counterbalance each other.

PROP. VI. The preſſure of fluids is proportional to the baſe, and the perpendicular height of the fluid, whatever be the form of the veſſel or quantity of the fluid.

Caſe 1. Let the fluid be contained in a perpendicular cylindrical veſſel.

Plate 5. Fig. 2. In ſuch a veſſel, ABCD, becauſe the whole weight of the fluid, and no other force, preſſes directly upon the bottom CD, the preſſure (by Prop. I.) muſt be as the quantity, that is, (El. XII. 11, 14.) as the baſe and perpendicular height of the fluid.

Caſe 2. Let the fluid be contained in a perpendicular veſſel, the bottom of which is equal to that of the cylinder in the laſt caſe, but its top narrower than the bottom.

Plate 5. Fig. 6. Let the veſſel DBLP, have the portions of its baſe LA, CP, each equal to OR. From Prop. I. and III. it appears, that each of theſe portions are equally preſſed by the column DBOR, as the baſe OR. In like manner, every portion of the baſe LP equal to OR is as much preſſed as OR. Therefore the whole baſe LP is as much preſſed as if the veſſel was of the cylindrical form FHLP.

Or thus: Becauſe (by Prop. V.) if a tube were inſerted at NT, of the diameter OR, the water being at the height DB, would riſe to the level FE, there muſt at NT, be an upward preſſure towards F ſufficient to fill up the columns of fluid FELA, that is, equal to the weight of as much water as would fill the ſpace FENT. Conſequently the re-action, that is, the preſſure upon the baſe LA muſt be equal to the weight of as much water as would fill FENT. But the baſe LA ſupports this re-action, and likewiſe the weight of the water NTLA, which are together equal to the weight of DBOR. The baſe LA, therefore, ſuſtains a preſſure equal to the weight of the column DBOR. And every equal portion of the baſe may, in the ſame manner, be ſhewn to ſuſtain an equal preſſure. Therefore, the preſſure on the baſe is the ſame in veſſels of the form ſuppoſed in this caſe, as in cylinders of equal baſes, and of the ſame altitude with theſe veſſels. The ſame may be ſhewn with reſpect to a veſſel of the form of plate 5, fig. 7.

Caſe 3. Let the veſſel be of the ſame baſe and altitude, but have its top wider than the baſe.

Plate 5. Fig. 8.

Let the fluid of the veſſel be divided into ſtrata EF, GH, IK, &c. Let us alſo imagine the bottom of the veſſel C to be moveable, that is, capable of ſliding up and down the narrow part of the veſſel, from C to GH. Let it further be ſuppoſed that this moveable bottom, is drawn up or let down with a given velocity, while the veſſel itſelf is fixed and immoveable; it is evident the loweſt ſtratum, which is contiguous to the bottom, will be raiſed or let down with the ſame velocity, and will therefore have a momentum proportional to that velocity, and the quantity of matter it contains: but (by Prop. IV. Cor.) the reſt of the ſtrata will have the ſame momentum: conſequently, the momentum of all taken together, that is, of the whole fluid, is the ſame as if the veſſel had been no larger in any one part than it is at the bottom, for then the momentum of each ſtratum would alſo have been as great as that of the loweſt. The preſſure, therefore, or action of the fluid, with which it endeavours to force the bottom out of its place, is as the number of ſtrata, that is, the perpendicular height of the fluid, and the magnitude of the loweſt ſtratum, that is, the baſe.

Caſe 4. Let the fluid be in an inclined cylindrical veſſel.

Plate 5. Fig. 9.

In the inclined cylindrical veſſel ABNI, as much as the fluid is prevented from preſſing upon the baſe NI, by being in part ſupported by the ſide of the veſſel AN, ſo far is the preſſure upon the baſe increaſed by the re-action of the oppoſite ſide BI, which is equal to the action of the former, becauſe the fluid, preſſing every way alike at the ſame depth below the ſurface, exerts an equal force againſt both the ſides. The baſe NI is therefore preſſed with the ſame force with which it would be preſſed, if the fluid contained in the veſſel ABNI was included in the veſſel EDIO, having an equal baſe, and the ſame perpendicular height with the veſſel ABNI; that is, (by the firſt caſe) the preſſure is as the baſe NI and altitude CN.

Since then, the preſſure upon the baſe of veſſels, either wider or narrower at the top than the bottom, and likewiſe the preſſure upon the baſe of veſſels inclined to the horizon, is equal to that upon the baſe of a cylindrical veſſel of the ſame baſe and height, the ſides of which are perpendicular to the horizon; and ſince the preſſure upon the baſe of ſuch a cylinder is as its baſe and height; the preſſure upon the bottom of all veſſels filled with fluid, is proportional to their baſe and perpendicular height.

EXP. 1. Let two tubes of different forms be ſucceſſively applied to the ſame moveable circular baſe, ſuſpended by a wire, paſſing from the centre of the baſe through the tubes, to the beam of a balance: when the different tubes are filled to the ſame height, it will require the ſame weight at the oppoſite end of the balance to keep the baſe from ſinking. Hence any quantity of fluid, how ſmall ſoever, may be made to balance and ſupport any quantity, how great ſoever, which is called the *hydroſtatical paradox*.

2. Let two tubes, the one cylindrical, the other of the form of a ſpeaking trumpet, have their baſes of equal diameter, covered with bladder, and inſerted in a veſſel of water, as in Prop. III. Exp. 3. the bladder will become plane at the ſame depth in both: from whence it appears, that ſince the upward preſſures, at the ſame depth, are equal, the downward preſſures in the two tubes are alſo equal.

Cor. 1. Hence in different veſſels, containing different fluids, the preſſures are as the areas of the baſes multiplied into the depths, and ſpecific gravities.

Cor. 2. If a cone be filled with a fluid, and ſtanding on its baſe, the preſſure on its baſe will be equal to three times the weight of the fluid. Let B be equal the baſe, H equal the perpendicular height, then the ſolid content, or weight, will be equal $\frac{B}{3} \times H$, but the preſſure will be $B \times H$, therefore equal to three times its weight.

Cor. 3. A ſmall quantity of fluid may be made to preſs with a force ſufficient to raiſe a great weight.

Plate 5. Fig. 6. Since (as was ſhewn in Prop. V.) as much fluid as will fill the tube DBIV preſſes upwards againſt VM, with a force equal to the weight of as much fluid as would fill the ſpace BHVM; the baſe remaining the ſame, the ſpace BHVM, that is, the weight which may be raiſed will (by this Prop.) be as the height VB, which may be increaſed at pleaſure.

Exp. Let two circular pieces of wood be united by leather in the manner of a pair of bellows; in the upper board inſert a long tube with a large bore; through which pour water into the veſſel; the upward preſſure of the water as it is poured in, will raiſe a great weight.

Cor. 4. From hence it may be proved, independently of the reaſoning in Prop. V. that, in bended veſſels, or channels of any form, fluids riſe to the ſame height, whatever be the difference between the quantities of fluid on each ſide: for whatever be the form of the channels, the plane which is perpendicular to the loweſt point being conſidered as the common baſe, the preſſure upon it is equal, when the fluid on each ſide is of equal altitude; and the whole maſs can only be at reſt when the oppoſite preſſures are equal.

Schol. This preſſure of the fluid upon the baſe does not alter the weight of the veſſel and fluid conſidered as one maſs, becauſe the action and re-action which cauſe it, with reſpect to the weight of the veſſel, deſtroy one another; the veſſel being as much ſuſtained by the action upwards, as it is preſſed by the re-action downwards.

PROP. VII. The preſſure of a fluid upon any indefinitely ſmall part of the ſide of the veſſel which contains it, is equal to the weight of a column of the ſame fluid, whoſe baſe is the part preſſed, and whoſe height is the diſtance of that part from the ſurface of the fluid.

Let

Plate 5. Fig. 12. Let ABCD be a veſſel filled with fluid; AB its ſurface; and L a point in the ſide of the veſſel. The indefinitely ſmall drop which lies next to the point L is preſſed downwards (by Prop. I.) by a force equal to the weight of a column of water whoſe baſe is L, and height LA, the diſtance of that part from the ſurface. And (by Prop. III.) this drop is preſſed ſideways towards L with the ſame force with which it is preſſed downwards. Whence the propoſition is manifeſt concerning the point L. And the ſame may be proved concerning any other points M, N, C, equal to L. The ſame is evidently true in an inclined veſſel.

PROP. VIII. The preſſure of a fluid upon any plane, is equal to the weight of a body which has the ſame denſity with the fluid, and is formed by raiſing perpendiculars upon each indefinitely ſmall part of the plane, equal in height to the diſtance of that part from the ſurface of the fluid.

Plate 5. Fig. 12. It has been proved, in the laſt propoſition, that the preſſure upon each indefinitely ſmall part of the line AC, in the ſide of the veſſel ABCD, is equal to the weight of a column of fluid whoſe baſe is the part preſſed, and whoſe height is the diſtance of that part from the ſurface AB. Hence, if from the point L a perpendicular LO be raiſed whoſe baſe is L, and whoſe length LO is equal to LA the diſtance of L from the ſurface, if this perpendicular conſiſted of matter of the ſame denſity with the fluid in the veſſel, the weight of this perpendicular column would be equal to the preſſure upon the point L. If, in like manner, perpendiculars, conſiſting of matter of the ſame denſity with the fluid, were raiſed upon every point between A, C, they would together fill up the area of the triangle ACD; and the preſſure upon the whole line AC in the ſide of the veſſel ABCD, becauſe it is equal to the ſum of the preſſures upon all its parts, muſt be equal to the weight of this triangle ACD. The ſame may be proved concerning any other lines in the ſide of the veſſel, as HI, EF. Fig. 14. Conſequently, the preſſures upon the whole ſide will be equal to the weight of as many ſuch triangles as there can be lines drawn upon it in the ſame manner as AC, HI, EF are drawn. But all theſe triangles together would fill up the whole ſpace, or compoſe a ſolid, CFGDAE. Therefore the preſſure upon the ſide AECF will be equal to the weight of this ſolid conſiſting of matter which has the ſame denſity with the fluid in the veſſel; which ſolid is formed by raiſing perpendiculars upon each line of the ſide, reſpectively equal to the diſtance of that point from the ſurface of the fluid.

Plate 5. Fig. 13. In like manner, if AC is a line drawn in the inclined ſide of a veſſel, in which the water reaches to the level AB, the preſſure upon this line may be eſtimated as before. SL is the diſtance of L from the ſurface. Let therefore a perpendicular LO, equal in length to LS, be raiſed upon the point L; then, if this perpendicular was a column of matter of the ſame denſity with water, the weight of it would be equal to the preſſure upon L. For the ſame

N reaſon,

reaſon, if a perpendicular MP is raiſed upon the point M, and is made equal in length to MT the diſtance of M from the ſurface; ſuch a perpendicular, conſiſting of matter of the ſame denſity with water, and being of the ſame ſize, would have the ſame weight as the column of water MT. And ſince (by Prop. I.) the preſſure upon M equals the weight of the incumbent water MT, it likewiſe equals the weight of the perpendicular MP. In like manner, the points N and C are preſſed by the weight of the incumbent columns NV and XC, which is equal to the weight of the perpendiculars NQ, CR, ſuppoſing thoſe perpendiculars to be equal in height to NV and XC, and to conſiſt of matter whoſe denſity is the ſame with that of the columns NV and XC. Thus the preſſure upon the whole line, being made up of the preſſures upon all its parts, will be equal to the weight of as many perpendiculars, as can be raiſed in this manner between A and C. The ſum of all thoſe perpendiculars is the triangle ACR, whoſe weight therefore is equal to the preſſure upon the line AC. But if as many ſuch triangles were added together, as there are lines parallel to AC in the whole ſide of the veſſel, all theſe triangles together would form a ſolid. And ſince this ſolid is the ſum of all the preſſures upon each point of the ſide, the weight of it, ſuppoſing it to conſiſt of matter that has the ſame denſity as water, would be equal to the preſſure upon the whole ſide.

PROP. IX. The preſſure upon any one ſide of a cubical veſſel filled with fluid, is half the preſſure upon the bottom.

Plate 5. Fig. 14. The bottom ſuſtains a preſſure equal to the whole weight of the fluid in the veſſel. And the preſſure which the ſide ſuſtains is equal to the weight of the priſm CFGDAE, which (El. XI. 28.) is half the cube: therefore the ſide ſuſtains a preſſure equal to half the preſſure upon the bottom.

Or thus: Becauſe the preſſure upon every part of the veſſel at the bottom is equal to the weight of a column whoſe baſe is the part preſſed upon, and height that of a perpendicular from the bottom to the ſurface; if the preſſure were the ſame every where from the top to the bottom, it would be equal to the weight of as many ſuch columns as would correſpond to all the parts of the veſſel. But the preſſure every where diminiſhes as we approach towards the ſurface, where it is nothing; the preſſure on the ſide is therefore only half of that on the bottom of the veſſel: a number of terms in arithmetical progreſſion beginning from nothing being half the ſum of an equal number of terms, each of which is equal to the laſt in the progreſſion.

Cor. 1. The gravity of the fluid in a cubical veſſel producing upon each of the four ſides a preſſure equal to half that upon the bottom, and upon the bottom a preſſure equal to itſelf, produces on the whole a preſſure three times as great as itſelf.

Cor. 2. When the area of the part preſſed is given, the preſſure is as the perpendicular diſtance of that part from the ſurface: where the depth of the part is given, the preſſure is as the area; and where the diſtance from the ſurface is given, the preſſure is as the baſe.

SCHOL. There is a particular point in which the whole preſſure againſt the ſide acts: it is called the *centre of preſſure*, and is the ſame with the centre of *oſcillation* of the ſide vibrating on the upper line of it as an axis. See Prop. XLVI. Schol. 1. Book 2.

CHAP. II.

Of the MOTION *of Fluids.*

SECT. I.

Of Fluids paſſing through the Bottom or Side of a Veſſel.

PROP. X. The momentum with which any fluid runs out of a given orifice in the bottom or ſide of a veſſel, is proportional to the perpendicular depth of the orifice below the ſurface of the fluid.

The preſſure of a fluid againſt any given ſurface being (by Prop. I. and III.) proportional to the perpendicular height of the fluid above that part; if that given ſurface be removed, the fluid will be driven through the orifice by this preſſure. The force therefore with which the fluid paſſes through the orifice is as the perpendicular depth of the orifice below the ſurface of the fluid: but the momentum is always as the moving force: therefore the momentum is alſo as the perpendicular depth of the orifice.

PROP. XI. The momentum with which any fluid runs out of a given orifice in the bottom or ſide of a veſſel, is as the ſquare of its velocity, or as the ſquare of the quantity of matter.

The momentum (by Book II. Prop. XI.) is in the compound ratio of the quantity of matter and velocity. And it is manifeſt, that, ſince the orifice is given, the quantity of fluid diſcharged will always be as the velocity: therefore the momentum is as the ſquare of the velocity, or of the quantity of fluid.

PROP. XII. The velocity with which any fluid runs out of an orifice in the bottom or ſide of a veſſel, is as the ſquare root of the perpendicular depth of the orifice from the ſurface of the fluid.

Becauſe the momentum is as the ſquare of the velocity, (by Prop. XI.) and as the perpendicular depth of the orifice, (by Prop. X.) the ſquare of the velocities (El. V. 11.) is as the perpendicular depth, and, conſequently, the velocity as the ſquare root of the perpendicular depth.

Cor. 1. Hence a fluid running out of a veſſel which empties itſelf, flows with an uniformly retarded velocity: for the perpendicular depths are continually diminiſhing.

Cor. 2. Hence alſo the ſurface deſcends with an uniformly retarded velocity, and the ſpaces deſcribed by it, in equal portions of time, are (Prop. XXVIII. Book II.) as the odd numbers 1, 3, 5, 7, 9, &c. taken backwards.

Cor. 3. If therefore a cylindrical veſſel be divided into portions, continued to the ſurface of the fluid, which are as the odd numbers, 1, 3, 5, 7, &c. a clepſydra or hour-glaſs will be formed; for the ſurface will deſcend through theſe diviſions in equal times.

PROP. XIII. A fluid runs out of an orifice in the bottom or ſide of a veſſel, with the velocity which a heavy body would acquire in falling freely through a ſpace, equal to the perpendicular diſtance of the orifice from the ſurface of the fluid.

Plate 5. Fig 10. Let ABCD be a veſſel filled with any fluid, to the height FG. It is manifeſt, that at the beginning of the fall of each drop from the upper ſurface FG, it muſt be carried downward by its gravity with the ſame velocity with which any other heavy body would begin to deſcend. And, if an orifice be made in the veſſel at L, any point below the ſurface, the fluid which paſſes through that orifice, will (by Prop. XII.) move with a velocity which is as the ſquare root of the diſtance from the ſurface. But if a body were to fall from the ſurface to the point L, it would acquire a velocity which would be (by Book II. Prop. XXVI. Cor. 2.) as the ſquare root of this diſtance. Therefore, ſince the velocity with which the fluid moves, is, at the beginning of its motion, equal to that of a falling body, and ſince at every given diſtance, theſe velocities have the ſame ratio, namely, that of the ſquare root of the diſtance from the ſurface, that is, (El. V. 9.) are equal, the propoſition is manifeſt.

Cor. Suppoſing O, V, T, Q, to repreſent the area of the orifice, velocity, time, and quantity, flowing out in that time reſpectively; Q will vary as $O \times V \times T$, or as $O \times T \times \sqrt{H}$, (Prop. XII.) and when T is given as $O \times \sqrt{H}$.

Schol. When a fluid ſpouts from a veſſel, it ruſhes from all ſides towards the orifice, which is the cauſe of the contraction of the ſtream at the diſtance from the orifice equal to its diameter, and is called the *vena contracta*. Now the area of the orifice, is to the area of the ſmalleſt ſection of the ſtream, nearly as $\sqrt{2}$ to 1; hence (by Prop. IV.) the velocity at the vena contracta is to the velocity at the orifice as $\sqrt{2}$ to 1. Sir I. Newton found, that the velocity at the vena contracta was that which a body acquires in falling down the altitude of the fluid above the orifice. We muſt, therefore, diſtinguiſh between the velocity at the orifice, and at the vena contracta, and in the doctrine of ſpouting fluids, it is the latter velocity which muſt be conſidered, and the point of projection muſt be aſſumed from that point.

PROP. XIV. When two cylindrical veſſels have their baſes, heights and orifices equal, if one of them be always kept full, it will diſcharge double the quantity of fluid diſcharged in the ſame time by the other whilſt it empties itſelf.

For (by Prop. I.) the fluid will continue through the whole time, to run with the ſame velocity out of the veſſel that is kept full. But the fluid will run (Cor. 1. Prop. XII.) with an uniformly retarded velocity out of the veſſel which empties itſelf. And, ſince both veſſels are full at firſt, the velocity which continues uniform in one veſſel, will (by Prop. I.) be the ſame with the firſt velocity in the veſſel in which the fluid is uniformly retarded. Therefore the quantity diſcharged out of the former veſſel, will be to the quantity diſcharged in an equal time out of the latter, as the ſpace deſcribed by a body moving with an uniform velocity, to the ſpace deſcribed by a body which ſets out with the ſame velocity, and is uniformly retarded. But (by Book II. Prop. XXVII.) the ſpace deſcribed by the former will be double of the ſpace deſcribed by the latter. Therefore the quantity diſcharged out of the former veſſel, will be double of the quantity diſcharged out of the latter.

PROP. XV. A ſtream of any fluid which ſpouts obliquely forms a parabola.

Each drop in a ſtream of fluid ſpouting obliquely, is a heavy body projected obliquely by the force or preſſure which drives it out of the orifice. Therefore (by Book II. Prop. LVIII.) every drop of the ſtream, that is, the whole ſtream, forms a parabola.

Exp. Obſerve the figure formed by a fluid ſpouting obliquely.

Cor. Hence fluids ſpouting obliquely are ſubject to the laws of projectiles laid down, Book II. Ch. VII. Sect. I.

PROP. XVI. When a fluid ſpouts horizontally from an orifice in the ſide of a veſſel which is kept full, if the ſide of the veſſel is made the diameter of a circle, and a line is drawn perpendicular to the ſide from the orifice to the circumference, the diſtance to which the fluid will ſpout will be double of this perpendicular.

Let

Plate 5. Fig. 11. Let AB be the ſide of the veſſel ; C, E, or *e*, the orifice ; ADHB the ſemicircle drawn on the ſide, ED, CH, *de*, lines drawn perpendicular to the ſide from the orifice to the circumference. The fluid ſpouts at E (by Prop. XIII.) with the velocity which a heavy body would acquire in falling from A to E ; and this motion, being in an horizontal direction, can neither be accelerated nor retarded by the force of gravitation, and will therefore continue uniform. But beſides this, the fluid ſpouts with the velocity which it acquires in falling after it has paſſed the orifice. This velocity, when the fluid arrives at GB, is the ſame with that which any other heavy body would have acquired in falling through an equal ſpace from E to B. Let this velocity be called the deſcending velocity, and that with which the fluid ſpouts at E the horizontal velocity. Then, ſince the horizontal velocity is the ſame with that which a body would acquire by falling from A to E, and the deſcending velocity when the fluid arrives at the plane GB, is the ſame with that which a body would acquire by falling from E to B, and ſince (by Book II. Prop. XXVI.) the ſpaces AE, EB, deſcribed by falling bodies, are as the ſquares of the laſt acquired velocities of bodies falling through them ; that is, (inverting the terms) the ſquares of theſe laſt acquired velocities, or the ſquares of the horizontal and deſcending velocities, are as the lines AE, EB. But in the triangle ADB, right-angled (El. VI. 8.) at D, DE is a mean proportional between AE, EB, and the ſquare of AE is to the ſquare of ED, as AE is to EB. But the ſquare of the horizontal velocity is to the ſquare of the laſt deſcending velocity as AE to EB. Therefore the ſquare of the horizontal velocity is to the ſquare of the laſt deſcending velocity as the ſquare of AE to the ſquare of ED : whence the horizontal velocity is to the laſt deſcending velocity as AE to ED. Now the ſpaces deſcribed in the ſame time, in uniform motions, are (Book II. Prop. VI.) as the velocities. Conſequently, if the fluid had begun to fall from E with the velocity it has acquired at B, and had fallen uniformly, in the time of deſcent the ſpaces deſcribed by the horizontal and deſcending velocities would have been reſpectively as thoſe velocities, that is, as AE to ED. Thus while the fluid was deſcending till it reached the plane GB, the horizontal velocity would have carried it forward through a ſpace equal to ED, or the horizontal diſtance would be ED. But the deſcending velocity being at the firſt nothing, and continually increaſing, the time of deſcent (ſee Prop. IX.) is twice what it would have been upon the ſuppoſition that it began to deſcend with the laſt acquired velocity. And the horizontal velocity is uniform, and therefore in twice the time, or the true time of deſcent, the fluid will be carried horizontally to twice the diſtance ED. Conſequently, if BF be made equal to twice DE, whilſt the ſtream is deſcending from E to GB, it will be carried forwards to the point F. The ſame may be proved concerning any other points C, *e*.

PROP. XVII. If a fluid ſpouts horizontally out of orifices in the ſide of a veſſel which is kept full, it will ſpout to the greateſt diſtance from the orifice which is in the middle of the ſide, and to equal diſtances from orifices equally diſtant from the middle.

Let C be the orifice in the middle of the ſide, and E, *e*, equal orifices at equal diſtances from C. Plate 5. Fig. 11.

The diſtance to which the fluid will ſpout at C (by Prop. XVI.) is twice CH, and at E twice ED. But CH (El. III. 15.) is greater than DE, any line drawn from the diameter parallel to the radius: therefore twice CH is greater than twice ED.

Alſo, ſince the horizontal diſtances to which the fluid will ſpout at E and *e*, are twice ED, or *ed*; and that ED, *ed*, being equally diſtant from the centre, and parallel to the radius, (El. III. 14.) are equal; the horizontal diſtances from E, *e*, are equal.

Hence if in the plane of the horizon, GB be drawn perpendicular to the ſide AB, and GB be double of CH, and FB double of DE, or *de*, the fluid ſpouting from C will fall upon G, and from E and *e*, upon F.

COR. If the ſide of the jet be inclined, in any angle to the horizon, and the direction, and velocity of the ſpouting fluid be known, the amplitude, altitude, and time of flight, may be diſcovered by the rules inveſtigated in Book II. on Projectiles.

EXP. Let water ſpout from the middle orifice, and from orifices equally diſtant from the middle, the truth of the propoſition will be manifeſt.

REMARK. In all the propoſitions reſpecting the times in which veſſels empty themſelves, the orifice is ſuppoſed to be very ſmall in reſpect to the bottom of the veſſel, otherwiſe the experiments do not agree with the theory.

DEF. II. A *river* is a ſtream of water which runs by its own weight down the inclined bottom of an open channel.

DEF. III. A *ſection of a river* is an imaginary plane, cutting the ſtream, which is perpendicular to the bottom.

DEF. IV. A river is ſaid to *flow uniformly* when it runs in ſuch a manner, that the depth of the water in any one part continues always the ſame.

PROP. XVIII. If a river flows uniformly, the ſame quantity of water paſſes in an equal time through every ſection.

Let AB be the reſervoir, BC the bottom of the river, and ZX, QR, ſections of the river. Becauſe the river flows uniformly, the ſame quantity of water which paſſes through ZX in a given time muſt paſs through QR in the ſame time: otherwiſe the quantity of water Plate 5. Fig. 15.

water in the ſpace ZQXR, muſt in that time be increaſed or diminiſhed, and conſequently the depth of the water in that ſpace altered; contrary to the ſuppoſition.

Cor. Hence if V, B, D, be the velocity, breadth, and depth reſpectively, $V \times B \times D$ will be a given quantity, and V will vary as $\frac{1}{B \times D}$.

PROP. XIX. The breadth of the channel being given, the water in rivers is accelerated in the ſame manner with any body moving down an inclined plane.

For each drop of the water moves down upon the inclined plane of the bottom, or upon the inclined plane of the ſheet of water, next below it, parallel to the bottom.

PROP. XX. The breadth of the channel being given, the velocity of each drop of water in a river is the ſame that a body would acquire in falling from the level of the ſurface of the water in the reſervoir, to the place of the drop.

Let AB be the depth of the reſervoir, AP the level of its ſurface, and BC the bottom of the channel. Any drop at E, after it comes out of the reſervoir at K, (by Prop. XIX.) rolls down the inclined plane KE, parallel to the bottom. And this drop, when it comes out of the reſervoir AB at K (by Prop. XIII.) has the ſame velocity which a heavy body would acquire in falling from A to K: and, in rolling down the inclined plane KE, it acquires (by Book II. Prop. XXXIV.) the ſame velocity which any heavy body would acquire in falling down GE the perpendicular height of the plane. At E the drop will therefore have acquired a velocity equal to that which a body would acquire by falling through AK and GE, that is, through MGE, the perpendicular drawn from the level of the reſervoir to the place of the drop.

Cor. 1. Hence the breadth of the channel being given, the velocity of each drop of water in a river is as the ſquare root of its diſtance from the level of the ſurface of the reſervoir. For, if E and R be two drops in different parts of the river, and AP the level, the velocity of the drop E is the ſame that a body would acquire by falling down ME, and that of R the ſame which a body would acquire by falling down HR. Therefore (by Book II. Prop. XXVI. Cor. 2.) the velocity of the drop E is to the velocity of R, as the ſquare root of ME to the ſquare root of HR. Plate 5. Fig. 15.

Cor. 2. Hence the breadth of the channel being given, the water at the bottom of a river will run faſter than the water at the ſurface.

PROP. XXI. The breadth of the channel being given, the depth of the river continually decreaſes as it runs.

The ſame quantity of water (by Prop. XVIII.) paſſes through each of the ſections ZX, QR, in the ſame time. But (by Prop. XX. Cor. 2.) the water runs faſter at the lower ſection QR, than at the upper ZX. Therefore the area of the ſection QR muſt be as much leſs than the area of the ſection ZX, as the velocity at QR is greater than the velocity at ZX. But the breadth of the ſections are by ſuppoſition equal; therefore their areas are (El. VI. 1.) as their heights. Conſequently, the heights of the ſection QR, ZX, will be inverſely as the velocities at thoſe ſections; that is, the depth of the water at QR will be as much leſs than the depth at ZX, as the velocity at QR is greater than the velocity at ZX.

PROP. XXII. At a given diſtance from the reſervoir, if the river flows uniformly, the velocity of the water will be inverſely as the breadth of the channel.

Becauſe the river flows uniformly, the depth at any given ſection ZX is always the ſame: and in any given time, the ſame quantity of water muſt flow through the different ſections ZX, QR, as was ſhewn in Prop. XVIII. But a given quantity of water cannot flow in a given time through any ſection, unleſs as much as the area is increaſed, ſo much the velocity is diminiſhed, and the reverſe; that is, the velocity muſt be inverſely as the area of the ſection, or the depth being given, as its breadth.

PROP. XXIII. The depth of a river being given, the preſſure upon any part of the bank will be the ſame, whatever is the breadth of the river.

The preſſure upon any given part in the bank (by Prop. I. and III.) will be as the diſtance of that part from the ſurface; which remains the ſame whilſt the depth is the ſame, whatever be the breadth of the river: therefore the preſſure will remain the ſame.

PROP. XXIV. If the breadth of a river be given, the preſſure on any part of the bank will be as the depth of the river.

For the preſſure on any part of the bank is (by Prop. I. and III.) as the depth of that part below the ſurface, which depth will increaſe with the depth of the river.

PROP. XXV. The preſſure againſt any given ſurface in the bank of a river, if that ſurface reaches from the bottom to the top of the ſtream, is equal to the weight of a column of water whoſe baſe is the ſurface, and whoſe height is half the depth of the ſtream.

Plate 5. Fig. 15. Let ZQXR be a given ſurface in the bank, reaching from the bottom BC of the river to its top AD. The preſſure upon this is (from what was ſhewn in Prop. IX.) half the preſſure on an equal ſurface at the bottom XR; which preſſure (by Prop. I. and III.) is equal to the weight of a column of water whoſe baſe is the ſurface ZQ, and whoſe height is the depth of the ſtream. Therefore the preſſure againſt the ſurface ZQXR is equal to the weight of a column whoſe baſe is the ſurface ZQ, and its height half the depth of the ſtream.

PROP. XXVI. When a ſtream which moves with the ſame velocity in every part ſtrikes perpendicularly upon any obſtacle, the force with which it ſtrikes is equal to the weight of a column of the ſame fluid, whoſe baſe is the obſtacle, and whoſe height is the ſpace through which a body muſt fall to acquire the velocity of the ſtream.

Plate 5. Fig. 11. Let a ſtream of water flow horizontally out of the orifice *e*. If this ſtream were to ſtrike upon an obſtacle of the ſame breadth every way as the orifice or ſtream, placed perpendicular to the horizon, the ſtream muſt ſtrike upon the obſtacle with its whole force. But this force is equal to the weight of a column of water whoſe baſe is *e*, and height *Ae*. And (by Prop. XIII.) *Ae* is the height from which a body muſt fall to acquire the velocity with which the ſtream ſpouts from *e*. Therefore the force with which this ſtream would ſtrike ſuch an obſtacle is equal to the weight of a column of water whoſe baſe is *e*, and height that from which a body muſt fall to acquire the velocity of the ſtream. And becauſe no part of the ſtream, however broad, can ſtrike the obſtacle except ſo much as is contained within a ſection equal to the ſurface of the obſtacle, no other part of the ſtream is to be conſidered in eſtimating this force. It is alſo manifeſt, that if the ſtream flow horizontally with the ſame velocity, in any other manner than through an orifice, as in the current of a ſtream, it will ſtrike an obſtacle with the ſame force.

PROP. XXVII. When the obſtacle is given, the force with which a ſtream ſtrikes upon it, will be as the ſquare of the velocity with which the ſtream moves.

If any ſtream ſtrikes upon a given obſtacle, the force will (by Prop. XXVI.) be equal to the weight of a column of water whoſe baſe is the obſtacle, and whoſe height is equal to the ſpace through which a body muſt fall to acquire the velocity of the ſtream. Since then the baſe is given, the weight will be as the height of ſuch a column. But the ſpaces through which bodies fall to acquire different velocities are (by Book II. Prop. XXVI.) as the ſquares of thoſe velocities. Therefore the height of this column, and its weight, and conſequently the force of the ſtream, which is equal to this weight, will be as the ſquare of the velocity with which the ſtream moves.

CHAP. III.

Of the RESISTANCE *of Fluids.*

PROP. XXVIII. If a ſpherical body is moving in a given fluid, the reſiſtance which ariſes from the re-action of the particles of the fluid is, within certain limits of the velocity, as the ſquare of the velocity with which the body moves.

A ſpherical body moving in a given fluid, the number of particles which it will meet with in a given time will be as its velocity; for the ſpace through which it will paſs will be as its velocity, and the number of particles it will meet with will be as the ſpace through which it paſſes. But the re-action of the particles of the fluid, and conſequently the reſiſtance, is as the number of particles or quantity of matter by which the reſiſtance is made. Again, if a given quantity of matter is to be moved, the moving force is (by Book II. Prop. IX.) as the velocity communicated; and the reſiſtance of that given quantity of matter is as the moving force. Therefore the reſiſtance ariſing from re-action in a given number of particles of fluid is as the reſpective velocities with which they are moved; that is, as the velocities with which the bodies which paſs through the fluid move. The reſiſtance of the fluid being then as the velocity on a double account, firſt, becauſe the number of particles moved are as the velocity of the moving body, and ſecondly, becauſe the reſiſtance of a given number of particles is as the velocity of the moving body; the reſiſtance will be in the duplicate ratio, or as the ſquare of this velocity.

SCHOL. In very ſwift motions, the reſiſtance of the air increaſes in a greater ratio; (ſee Remark to Prop. LVIII. Book II.) and in other fluids, the ſame conſequence would follow for the ſame reaſon, with reſpect to projected bodies. Beſides, the greater the velocity is, the leſs will be the preſſure againſt the back of the body which will cauſe a deviation in the law of reſiſtance.

PROP. XXIX. When a ſpherical body moves with a given velocity in any fluid, the reſiſtance of the fluid ariſing from its re-action, will be as the ſquares of the diameter of the ſpherical body.

A ſpherical body, in moving through a fluid, diſplaces a cylindrical column of that fluid, the height of which is the ſpace which the ſphere deſcribes, and its baſe a great circle of the ſpherical body. Becauſe the velocity is given, the ſpace deſcribed in a given time, that is, the length of the column is given: whence, the quantity of fluid in the column, that is, the column will be as its baſe, a great circle of the ſphere. And the reſiſtance which the column of fluid makes by re-action to the motion of the ſphere will be as its quantity of matter: it will therefore be as the baſe of the column, or as the great circle of the ſphere, or (El. XII. 2.) as the ſquare of its diameter.

PROP. XXX. If two unequal homogeneous ſpheres are moving in the ſame fluid with equal velocities, the greater ſphere will be leſs reſiſted in proportion to its weight, than the leſſer ſphere.

The weights of ſpheres, or their ſolid contents, are (El. XII. 18.) as the cubes of their diameters: but their reſiſtances (Prop. XXIX.) are as the ſquares of their diameters: and the cubes of any numbers have a greater ratio to each other than their ſquares. Therefore the ratio of the weights of ſpherical bodies is greater than that of their reſiſtances in a given fluid: that is, the weight of the greater ſphere exceeds the weight of the leſſer, more than the reſiſtance of a given fluid againſt the former exceeds the reſiſtance againſt the latter, provided the ſpheres are moving with equal velocities.

Schol. Hence the reſiſtance of the air may be able to ſupport ſmall particles of fluid, but unable to ſupport them when they are collected into larger drops.

PROP. XXXI. The reſiſtance of a fluid ariſing from its re-action, is as the ſide of the body perpendicularly oppoſed to it.

The reſiſtance is as the column, or quantity of fluid removed in a given time, which, as was ſhewn, Prop. XXIX. is as the baſe of the column, that is, as the ſide of the body perpendicularly oppoſed to it.

PROP. XXXII. When equal ſpheres move with the ſame velocity in different fluids, the reſiſtances will be as the denſities of the fluids.

The reſiſtances ariſing from re-action are as the momenta communicated to the fluid in a given time, that is, ſince the ſpheres move with equal velocities, as the quantities of matter moved. But, becauſe the ſpheres are equal, the baſes of the columns to which they communicate motion, are equal; and becauſe the ſpheres move with equal velocity, the length of the columns to which they communicate motion are equal. Hence the columns to which motion is communicated, having their baſes and heights equal, are of equal magnitude: and conſequently, their quantities of matter are as their denſities. But it has been ſhewn, that their momenta and reſiſtances are as their quantities of matter: therefore their reſiſtances are as their denſities.

Schol. Hence drops of water may be ſuſtained in the lower parts of the atmoſphere, which cannot be ſuſtained in the higher.

PROP. XXXIII. The retardation of bodies in a reſiſting fluid, where the weights of the bodies are given, is as the reſiſtance of the fluid.

The more a body is reſiſted by any fluid in which it moves, the greater portion of its momentum is deſtroyed; but, becauſe the weight of the body is given, its momentum is as its velocity; therefore the greater the reſiſtance of the fluid, the greater portion of its velocity is deſtroyed, that is, the more it is retarded.

PROP. XXXIV. When the reſiſtance is given, the retardation is inverſely as the weights.

The ſame reſiſtance will deſtroy an equal portion of momentum whatever is the weight of the moving body. But when the momentum is the ſame, the velocity is (by Book II. Prop. XII.) inverſely as the quantity of matter. Therefore the velocity deſtroyed, on the retardation, will be inverſely as the quantity of matter in the body in which the momentum is deſtroyed: and the weight is as the quantity of matter: therefore the retardation is inverſely as the weight.

PROP. XXXV. The retardation of ſpherical bodies moving with equal velocities in the ſame fluid, is inverſely as their diameters.

The reſiſtance which ſpherical bodies meet with in a given fluid is (by Prop. XXIX.) as the ſquare of their diameter. The retardation, when the weight is given, is (by Prop. XXXIII.) as the reſiſtance: and when the reſiſtance is given, the retardation (by Prop. XXXV.) is inverſely as the weight, that is, (El. XII. 18.) inverſely as the cubes of the diameter. Now, when unequal ſpheres move with the ſame velocity in the ſame fluid, the retardations will be unequal, both becauſe the reſiſtances are unequal, and becauſe the weights are unequal. The retardations will therefore be directly as the ſquares of the diameters, and inverſely as the cubes of the diameters: that is (compounding theſe ratios) inverſely as the diameters.

PROP. XXXVI. When a body moves in an imperfect fluid which has tenacity, or the parts of which cohere, the reſiſtance of any given portion of the fluid from this cauſe, is inverſely as the velocity of the body: the reſiſtance, when the velocity is given, is as the quantity of fluid through which the body paſſes: and the reſiſtance is always as the time during which the body moves in the fluid.

Caſe 1. Suppoſe ſuch an imperfect fluid, as ſoft clay, divided into thin plates; each plate having a certain portion of tenacity will continue to reſiſt the body during the whole time in which it is paſſing through it: the reſiſtance therefore will be leſs, the ſhorter time the body takes in paſſing through it, that is, the greater velocity the body moves with. And this is true concerning every plate which compoſes the fluid. Therefore the reſiſtance ariſing from tenacity in a given quantity of fluid, is inverſely as the velocity of the body which paſſes through it.

Caſe 2. Again, the velocity of the body being given, the reſiſtance which the body meets with, from what has been ſaid, is alſo given, and will be as the number of plates or quantity of the fluid.

Caſe 3. Laſtly, when a body moves for a given time, the reſiſtance (by the ſecond caſe) is as the number of plates, that is, as the ſpace through which it paſſes in a given time, that is, (by Book II. Prop. VI.) as the velocity directly. And (by the firſt caſe) the reſiſtance is, on account of the tenacity, inverſely as the velocity. Therefore as much as the reſiſtance is increaſed on account of the velocity in one reſpect, ſo much it is diminiſhed on account of the velocity in another; and conſequently, whatever be the velocity of a body in ſuch a fluid, the reſiſtance which it meets with in a given time will be the ſame: whence this reſiſtance will be as the time in which the body moves in the fluid.

CHAP. IV.

Of the Specific Gravities of Bodies.

Def. V. The *denſity* of a body, is its quantity of matter when the bulk is given.

Def. VI. The *ſpecific gravity* of a body, is its weight compared with that of another body of the ſame magnitude.

Cor. 1. The ſpecific gravity of a body is as its denſity. For the ſpecific gravity of a body is the weight of a given magnitude, and the weight of a body (by Book II. Prop. XXIV. Cor.) is as its quantity of matter; therefore the ſpecific gravity of a body is as the quantity of matter contained in a given magnitude, that is, as its denſity.

Cor. 2. The ſpecific gravities of bodies are inverſely as their magnitudes when their weights are equal. For by the laſt Cor. the ſpecific gravities of bodies are as their denſities, and their denſities (from Def. I.) are inverſely as their magnitudes when their weights are equal. Therefore the ſpecific gravities are alſo inverſely as their magnitudes when their weights are equal.

PROP. A. The weight of a body varies as its magnitude, and ſpecific gravity conjointly.

For if the magnitude of any body is varied, its ſpecific gravity remaining the ſame, the weight muſt be altered in the ſame ratio. And if the ſpecific gravity vary while its magnitude continues the ſame, the weight muſt alſo vary in the ſame ratio. Therefore the weight muſt vary as the magnitude and ſpecific gravity conjointly.

PROP. XXXVII. A fluid ſpecifically lighter than another fluid will float upon its ſurface.

For (by Book II. Prop. XXIV.) the lighter fluid will be leſs powerfully acted upon by the force of gravitation than the heavier; whence, the heavier will take the lower place.

EXP. 1. Let a ſmall and open veſſel of wine be placed within a large veſſel of water; the wine will aſcend.

2. Let mercury, water, wine, oil, ſpirits of wine, be put into a phial in the order of their ſpecific gravities; they will remain ſeparate.

PROP. XXXVIII. The heights to which fluids, which preſs freely upon each other, will riſe, are inverſely as their ſpecific gravities.

Since (by Prop. VI.) the oppoſite parts of an homogeneous maſs of fluid, in a curved tube or channel, preſs equally againſt each other when they riſe to the ſame height; in order to preſerve the preſſure equal when the fluids on each ſide are different, that which has the leaſt ſpecific gravity, muſt proportionally riſe above the level to preſerve the balance; and the reverſe.

EXP. Into the longer arm of a recurved tube, of equal bore throughout, and open at each end, pour ſuch a quantity of mercury, that it ſhall riſe in each arm about half an inch; then pour water into the longer arm till the mercury is raiſed one inch above its former height; the ſpecific gravities of theſe fluids will be inverſely as the heights to which they riſe.

PROP. XXXIX. The force with which a body lighter than any fluid endeavours to aſcend in that fluid, is as the exceſs of the ſpecific gravity of the fluid above the ſolid.

Since ABCD, the fluid in a veſſel, will be at reſt (Prop. III.) when every part of an imaginary plane SQ, under the ſurface of the floating body *ptei*, ſuſtains an equal preſſure; if the ſolid body be of equal ſpecific gravity with the fluid, that is, weighs as much as a quantity of the fluid equal to it in bulk, and whoſe place it takes up, this imaginary plane being equally preſſed by the ſolid, as if the ſame ſpace were filled with fluid, the fluid will be at reſt, and the ſolid will neither aſcend nor deſcend. Conſequently, if the body be ſpecifically heavier than the fluid, that part of the plane which is directly under the ſolid being ſo much more preſſed than the other equal parts of the ſame plane as the ſolid body is ſpecifically heavier than the fluid, the body muſt deſcend with a force equal to that exceſs; and, on the contrary, if the body be ſpecifically lighter than the fluid, that part of the plane which is directly under the ſolid being ſo much leſs preſſed than the other equal parts of the ſame plane, as the body is ſpecifically lighter than the fluid, it muſt be buoyed up with a force equivalent to the difference of their ſpecific gravities.

Plate 5. Fig. 19.

PROP. XL. Any fluid preſſes equally againſt the oppoſite ſides of a ſolid body immerſed in it.

Plate 1. Fig. 18. The oppoſite ſides of the ſolid are at the ſame depth; and fluids at the ſame depth preſs equally. Thus the oppoſite ſides RM, SN, of any body immerſed in a veſſel of water ABCD, are preſſed equally by the ſurrounding fluid.

Cor. No motion of the ſolid will be produced by theſe oppoſite lateral preſſures.

PROP. XLI. A body immerſed in a fluid is preſſed more upwards than it is downwards, and the difference of theſe two preſſures is equal to the weight of as much of the fluid as would fill the ſpace which the body fills.

Plate 5. Fig. 18. The body MRNS being immerſed in the veſſel of water ABCD, its lower part MN muſt be preſſed upwards juſt as much as the water itſelf at the ſame depth MNT would be if no ſolid were immerſed. Now the water at any depth (by Prop. III.) is preſſed as much upwards as it is preſſed downwards. And at the depth MNT, the portion of this ſtratum MN, would, if the ſolid were away, be preſſed downwards by a force equal to the weight of the incumbent column of water EMNH. Therefore the force with which MN, that is, the lower part of the ſolid, is preſſed upwards, is equal to the weight of as much water as would fill the whole ſpace EHMN. But the ſolid body RSMN is preſſed downwards by the weight of the column above it EHRS. Therefore the difference between the two preſſures, is the difference of the weights of the two columns of water EH MN, and EHRS; that is, the upward preſſure upon the ſolid body RSMN exceeds the downward preſſure, by a force equal to the weight of as much water as would fill the ſpace RSMN, taken up by the ſolid body. The caſe will be the ſame whatever is the figure of the body immerſed.

PROP. XLII. A body immerſed in a fluid, if it is ſpecifically heavier than the fluid, will ſink.

Plate 5. Fig. 18. If the body RSMN is ſpecifically heavier than the fluid, it weighs more than a quantity of the fluid of the ſame bulk with it. Hence the column EHMN, conſiſting of the column of fluid EHRS and the ſolid body RSMN, is heavier than the ſame column would be if it conſiſted wholly of water. But the upward preſſure againſt MN is (by Prop. III.) equal to the downward preſſure of the column of water EHMN, and therefore only ſufficient to ſupport the weight of that column. It cannot then ſupport the weight of the heavier column, conſiſting of a fluid and a ſolid, EHMN; and that part of this column which is ſpecifically heavier than the fluid, that is, the ſolid, will ſink, with a force equal to the difference of the weights of the column of fluid EHMN, and the mixed column EHRS, RSMN.

PROP. XLIII. A body ſpecifically lighter than the fluid in which it is immerſed, will riſe to the ſurface and ſwim.

If the ſolid RSMN be a body ſpecifically lighter than water, the column EHMN will weigh leſs as it conſiſts of the column of water EHRS and the ſolid RSMN, than if it conſiſted entirely of water. Conſequently, the upward preſſure upon MN, which is equal to the weight of the column of water EHMN, will be equal to more weight than that of the mixed column EHRS, RSMN: and therefore the lighter part of this column, that is, the ſolid body, will be carried upwards with a force equal to the difference of the weights of the column of fluid EHMN, and the mixed column EHRS, RSMN.

Plate 5. Fig. 18.

PROP. XLIV. A body which has the ſame ſpecific gravity with the fluid in which it is immerſed, will remain ſuſpended in any part of the fluid.

The body RMNS being of the ſame ſpecific gravity with the fluid, the column EHMN preſſes downwards with the ſame force, whilſt this body makes a part of it, as if the column conſiſted wholly of water, that is, with a force equal to the upward preſſure againſt MN. Therefore the body RSMN, having its lower ſurface MN, and in like manner all its parts, preſſed by equal forces in oppoſite directions, will remain at reſt.

Plate 5. Fig. 18.

Exp. Let ſmall glaſs images made hollow, and of ſpecific gravity ſomewhat leſs than water, having a ſmall orifice to receive water, ſwim in a large glaſs veſſel nearly filled with water and covered over cloſely with a piece of bladder: by preſſing the bladder with the hand, the air on the ſurface is compreſſed: this preſſure is communicated to the air in the images, which conſequently receive a larger portion of water, and become in ſpecific gravity as heavy as the water, or heavier, and either float in the water, or ſink.

PROP. XLV. A body ſpecifically heavier than the fluid in which it is immerſed, may be ſupported in it by the upward preſſure, if the preſſure downward be taken away: and a body ſpecifically lighter than the fluid in which it is immerſed, will not riſe in the fluid, if the upward preſſure be taken away.

For, in the firſt caſe, the preſſure which the ſolid RSMN ſuſtains from the weight of the fluid being removed, the ſolid may preſs downwards with a force equal to, or leſs than, that of the column of fluid EHMN, that is, than that with which it is preſſed upwards, according to the degree of depth in the fluid at which the ſolid is placed.

Plate 5. Fig. 18.

In the ſecond caſe, as the upward preſſure againſt MN is diminiſhed, the downward preſſure of the mixed column EHMN becomes equal to, or greater than, the upward preſſure, and the ſolid will either float in the fluid, or ſink.

Exp. For the firſt part of the propoſition ſee Prop. III. Exp. 2 and 5. The ſecond part may be thus confirmed. If a plane and ſmooth piece of hard wood, or of cork, be cloſely preſſed down by the hand upon the plane and ſmooth bottom of any veſſel, whilſt mercury is pouring into the veſſel; upon removing the preſſure of the hand, the downward preſſure of the mercury will prevent the wood from riſing.

PROP. XLVI. If a body floats on the ſurface of a fluid ſpecifically heavier than itſelf, it will ſink into the fluid till it has diſplaced a portion of fluid equal in weight to the ſolid.

Plate 5. Fig. 19. Let *ptei* be a body, floating on a liquor ſpecifically heavier than itſelf, it will ſink into it, till the immerſed part, *rnei*, takes up the place of ſo much fluid as is equal to it in weight. For, in that caſe, *ei*, that part of the ſurface of the ſtratum upon which the body reſts, is preſſed with the ſame degree of force, as it would be, was the ſpace *rnei* full of the fluid; that is, all the parts of that ſtratum are preſſed alike, and therefore the body, after having ſunk ſo far into the fluid, is in equilibrio with it, and will remain at reſt.

Exp. 1. Place a cube of wood on a ſmall jar, exactly filled with water; a part of its bulk will be immerſed, and will diſplace a quantity of the water: take the cube out of the water, and put it into a ſcale, with which an empty veſſel in the other ſcale ſtands balanced; then pour water into that veſſel till the equilibrium is reſtored; that portion of water will fill up the jar in which the cube was placed.

2. Let a glaſs jar, with a weight ſufficient to make it ſink in water to about two-thirds of its length, be placed firſt in a large veſſel of water, and afterwards in one which is very little wider than the jar, and which has in it a ſmall quantity of water; the jar will ſink to the ſame depth in both veſſels, that is, till ſo much of the veſſel is under water as is equal in bulk to a quantity of the fluid whoſe weight is equal to that of the whole veſſel.

Cor. Hence ariſes a rule for eſtimating the ſpecific gravities of fluids or ſolids. For, ſince (by this Prop.) the weights of the water diſplaced and of the ſolid are equal, their ſpecific gravities are inverſely as their magnitudes; that is, the magnitude of the water diſplaced is to that of the ſolid, as the ſpecific gravity of the ſolid is to the ſpecific gravity of the fluid: or (ſince the part immerſed is equal in magnitude to the fluid diſplaced) the part immerſed is to the whole, as the ſpecific gravity of the ſolid to the ſpecific gravity of the fluid. Conſequently, the greater portion of any given ſolid is immerſed in any fluid, the leſs is the ſpecific gravity of the fluid: and with reſpect to ſolids, inverting the propoſition, as the whole is to the part immerſed, ſo is the ſpecific gravity of the fluid to that of the ſolid: whence, the greater portion of any ſolid is immerſed in a given fluid, the greater is its ſpecific gravity.

PROP. XLVII. A ſolid weighs leſs when immerſed in a fluid than in open air, by the weight of a quantity of the fluid equal in bulk to the ſolid.

If the body immerſed were of the ſame ſpecific gravity with the fluid (by Prop. XLIV.) it would be ſupported in the fluid by the upward preſſure. The fluid therefore ſuſtains ſo much of the gravity of the body, or takes away ſo much of its weight, as is equal to the weight of that quantity of fluid which would fill the place taken up by the body.

Or thus; A body endeavours to deſcend by its whole weight; but (as was ſhewn, Prop. XLI.) when it is immerſed in a fluid, it is ſupported by a force equal to the weight of an equal bulk of that fluid. And ſince theſe two forces act in contrary directions, the weight which the body retains in the fluid will be the difference between them; that is, it weighs as much leſs in the fluid than as in the air, as the weight of a quantity of the fluid equal in bulk to the ſolid.

Exp. Having provided a ſolid cylinder of lead which exactly fills a hollow cylinder of braſs, place in one ſcale the hollow cylinder; under the ſame ſcale ſuſpend by a ſtring the ſolid cylinder, and balance the whole by weights; then immerſe the ſolid cylinder in water, and the equilibrium will be reſtored by filling up the hollow cylinder.

Remark. In ſtrictneſs both the ſolid and fluid ſhould be weighed in vacuo. The error, however, ariſing from the preſſure of the air is very ſmall, and may be neglected, unleſs where the body to be weighed is very light, and alſo where great preciſion is required.

Cor. 1. Hence the ſpecific gravities of different fluids may be compared, by obſerving how much the ſame ſolid (ſpecifically heavier than the fluids) loſes of its weight in each fluid; that fluid having the greateſt ſpecific gravity in which it loſes moſt of its weight.

Exp. Let a cubic inch of wood, made ſufficiently heavy to ſink in water, be immerſed ſucceſſively in different fluids; it will diſplace a cubic inch of the fluid in which it is immerſed; and ſince the cube (by Prop. XLVIII.) weighs leſs in the fluid, by the weight of a quantity of the fluid equal in bulk to the cube, its loſs of weight will be the weight of a cubic inch of the fluid.

Cor. 2. The weights which bodies loſe in any fluid are proportional to their bulks.

Exp. 1. Two balls of equal bulk, one of ivory, the other of lead, will loſe equal weight in water.

2. A piece of copper and a piece of gold being of equal weight in air, the gold outweighs the copper in water.

Cor. 3. If it be known what a cubic inch of any body loſes in water, the ſolid contents of any irregular maſs of the ſame kind may be known, by obſerving how much more or leſs it loſes, than a cubic inch would loſe.

Exp. Weigh a cubic inch and any irregular piece of wood of the ſame kind, and obſerve the difference of their weights.

Cor. 4. The weight of a ſolid body of the ſame ſpecific gravity with the fluid, or of a portion of the fluid itſelf, ſuſpended in the fluid, is not perceived, becauſe this weight is ſupported, and not becauſe the gravity of the body is loſt or deſtroyed.

PROP. B. If a and b be the ſpecific gravities of two fluids which are to be mixed together; A and B their magnitudes, and c the ſpecific gravity of the compound; then $A : B :: b-c : c-a$, provided the magnitude of the compound is equal to the ſum of the magnitudes of the parts when ſeparate.

Since (by Prop. A.) the weights of bodies are as their magnitude and ſpecific gravities conjointly, the weight of $A = A\times a$, and that of $B = B\times b$; and the weight of the compound $= \overline{A+B}\times c$; but the weight of the compound muſt be equal to the ſum of the weights of the two parts, $\overline{A+B}\times c = A\times a + B\times b$, therefore $Ac+Bc=Aa+Bb$ and $Ac-Aa=Bb-Bc$, conſequently $A : B :: b-c : c-a$.

Exp. Let the ſpecific gravity (to avoid fractions) of gold be 19; of ſilver 11, and of the compound 14; then the magnitude of the ſilver in the mixture, is to that of the gold as $19-14 : 14-11 :: 5 : 3$.

Cor. Hence the ſolution of that problem which was inveſtigated by Archimedes, in order to detect the fraud of the artiſt, who, inſtead of gold, was ſuſpected of having ſubſtituted ſilver, in the crown of Hiero, king of Syracuſe. If the proportion of the weights of each body is required, the ratios of their magnitudes, and of their ſpecific gravities, muſt be taken conjointly: in this caſe the weights of A and B are as $a\times\overline{b-c} : b\times\overline{c-a}$, that is, the weight of the ſilver, is to that of the gold as $11\times 5 : 19\times 3 :: 55 : 57$.

Schol. We eaſily deduce from this chapter the methods of obtaining the ſpecific gravities of any bodies, taking rain water as a ſtandard, a cubic foot of which being uniformly found to weigh 1.000 avoirdupoiſe ounces.

The weight which a body loſes in a fluid, is to its whole weight, as the ſpecific gravity of the fluid is to that of the body; where three terms of the proportion being given, the fourth is eaſily found. Ex. If a guinea weigh in air 129 grains, and in being immerſed in water loſe $7\frac{1}{4}$ of its weight, the proportion will be $7\frac{1}{4} : 129 :: 1000$ to the ſpecific gravity of a guinea. By this method the ſpecific gravities of all bodies that ſink in water may be found.

Cor. 1. Hence if different bodies be weighed in the ſame fluid, their ſpecific gravities will be as their whole weights directly, and the weights loſt inverſely.

If a body to be examined conſiſt of ſmall fragments, they may be put into a ſmall bucket and weighed; and then if from the weight of the bucket and body in the fluid, we ſubtract the weight of the bucket in the fluid, there remains the weight of the body in the fluid.

Cor. 2. If the ſame body be weighed in different fluids, the ſpecific gravity of the fluids will be as the weights loſt.

Exp. The loſs of weight ſuſtained by a glaſs ball in water and milk is reſpectively 803 and 831 grains, therefore the ſpecific gravity of water is to that of milk as $803 : 831$, that is, as $1.000 : 1.034$. By the ſame method, the ſpecific gravity of water is to that of ſpirits of wine as 1.000 to .857.

TABLE OF SPECIFIC GRAVITIES.

Platina (pure)	23.000	Ivory	1.825
Fine gold	19.640	Sulphur	1.810
Standard gold	18.888	Chalk	1.793
Mercury	14.019	Calculus humanus	1.542
Lead	11.325	Lignum vitæ	1.327
Fine ſilver	11.091	Coal	1.250
Standard ſilver	10.535	Mahogany	1.063
Copper	9.000	Milk	1.034
Gun metal	8.784	Brazil wood	1.031
Fine braſs	8.350	Box wood	1.030
Steel	7.850	Rain water	1.000
Iron	7.645	Ice	.908
Pewter	7.471	Living men	.891
Caſt iron	7.425	Aſh	.800
Loadſtone	4.930	Maple	.755
Diamond	3.517	Beech	.700
White lead	3.160	Elm	.600
Marble	2.705	Fir	.550
Green glaſs	2.600	Cork	.240
Flint	2.570	Common air	.001$\frac{7}{50}$

Remark 1. The above table ſhews the ſpecific weights of the various ſubſtances contained in it, and the abſolute weight of a cubic foot of each body is aſcertained in avoirdupoiſe ounces, by multiplying the number oppoſite to it by 1000: thus the weight of a cubical foot of mercury is 14019 ounces avoirdupoiſe, or 876lb.

Remark 2. If the weight of a body be known in avoirdupoiſe ounces, its weight in Troy ounces will be found by multiplying it into .91145. And if the weight be given in Troy ounces it will be found in avoirdupoiſe by multiplying it into 1.0971.

Remark 3. Mr. Robertſon, late librarian to the Royal Society, was the gentleman who inveſtigated the ſpecific gravity of living men, in order to know what quantity of timber would be ſufficient to keep a man afloat in water, ſuppoſing that moſt men were ſpecifically heavier than river water; but the contrary appeared to be the caſe from trials which he made upon ten different perſons, whoſe mean ſpecific gravity was, as expreſſed in the table, 0.891, or about $\frac{1}{9}$th leſs than common water. Phil. Tranſ. Vol. L.

The ſcales made uſe of to determine the ſpecific gravities of bodies, are called the *hydroſtatic balance*.

BOOK III. PART II.

OF PNEUMATICS.

CHAP. I.

Of the WEIGHT *and* PRESSURE *of the* AIR.

DEF. The *Air*, or *Atmoſphere*, is that fluid which encompaſſes the earth.

PROP. XLVIII. The air has weight.

This appears from experiment.

EXP. 1. The air being exhauſted, by an air-pump, from a glaſs receiver, the veſſel will be held faſt by the preſſure of the external air.

2. If a ſmall receiver be placed under a larger, and both be exhauſted, the larger will be held faſt, whilſt the ſmaller will be eaſily moved.

3. If the hand be placed upon a ſmall open veſſel, in ſuch a manner as to cloſe its upper orifice, it will be held down with great force.

4. The upper orifice of an open receiver being cloſely covered with a piece of bladder, upon exhauſting the receiver, the bladder will burſt.

5. In the ſame ſituation a thin plate of glaſs will be broken.

6. Let a glaſs bulb, having a long neck, be put, with the neck downwards, into a veſſel of water; put the whole under a receiver, and exhauſt the air: on re-admitting the air, its preſſure upon the ſurface of the water in the veſſel will cauſe the water to riſe in the bulb.

7. Pour mercury into a wooden cup, cloſely placed upon the upper orifice of an open receiver: when the air beneath is exhauſted, the preſſure of the external air will force the mercury through the wood, and it will deſcend in a ſhower.

8. On a transferrer let the air be exhauſted from a long receiver; then let water be admitted through a pipe, by means of a cock; the water will riſe in a *jet d'eau.*

9. Condenſe the air within a globular veſſel, having a long neck by blowing through the neck, the preſſure of the air within the veſſel will force out the water.

10. The glaſs bulb and veſſel (as in Exp. 6.) being placed within a condenſing receiver; on increaſing the quantity of air in the receiver, the fluid will riſe into the bulb.

11. The quickſilver in the gauge of the condenſer will, by the increaſing preſſure of the air, be forced upwards in the tube.

12. Any quantity of air may be transferred from one veſſel to another.

13. A thin glaſs veſſel, of a cubical form and cloſed up, may be broken in a condenſer.

14. Fill a glaſs tube, about 3 feet long and cloſed at one end, with mercury; then inſert the open end in a veſſel of mercury; the mercury will remain ſuſpended in the tube, by the preſſure of the external air upon the ſurface of the mercury in the veſſel: when this preſſure is removed, by placing the tube and veſſel under a receiver, and exhauſting the air, the mercury will ſink in the tube, and on re-admitting the air, will riſe.

15. If the ſame immerſed tube be ſuſpended from the beam of a balance, the weight neceſſary to counterpoiſe it, excluſive of the weight of the tube, is equal to that of the mercury ſuſtained in the barometer by the preſſure of the atmoſphere: for, the weight of the column of air incumbent upon the tube not being counterbalanced by the contrary preſſure from below, which is employed in bearing up the mercury within the tube, muſt preſs upon the beam.

16. Let a barometer tube, inſtead of being hermetically ſealed at the top, be cloſely covered with a piece of bladder; the mercury will riſe to the ſame height as in a common barometer; and on piercing the bladder with a needle, to admit the air, it will fall.

Schol. Hence the preſſure of the atmoſphere on or near the ſurface of the earth is known: the weight of any column of air being equal to the weight of the column of mercury, of the ſame diameter, ſupported in the barometer. And, ſince the height of this column varies with the weight of the atmoſphere, the varieties in the weight of the atmoſphere are known by the barometer.

18. Let the air be exhauſted from a glaſs veſſel, and by means of a cock let the veſſel be kept exhauſted; weigh the veſſel whilſt it is exhauſted, and when the air is re-admitted; the difference is the weight of ſo much air as the veſſel contains; which difference will be about 324 grains for a thouſand cubic inches.

PROP. XLIX. The air preſſes equally in all directions.

Exp. 1. Let a bladder, filled with air, be placed within a condenſing receiver, the condenſed air will make the bladder flaccid.

2. In a tall phial let an orifice be made about 3 inches above the bottom; ſtop this orifice; through a cork in the neck of the phial inſert a long tube open at each end; and let its lower end be below the orifice in the ſide of the phial. The mouth of the phial being cloſed up about the tube, pour water into the tube till it is full. Upon opening the orifice, the water will be diſcharged till its ſurface in the tube is level with the orifice; after which it will ceaſe to flow, becauſe the external lateral preſſure of the air balances the perpendicular preſſure upon the water in the tube.

3. If a glaſs veſſel be filled with water, and covered with a looſe piece of paper, on inverting the glaſs, the water will be kept from falling by the upward preſſure of the air.

4. If a veſſel be perforated in ſmall holes at the bottom, but cloſed at the top, the upward preſſure of the air will keep the water within the veſſel; as will appear by ſucceſſively ſtopping and unſtopping a ſmall hole in the top of the veſſel.

5. Two braſs hemiſpherical cups put cloſe together, when the air between them is exhauſted, will be preſſed together with conſiderable force.

6. A ſyringe being faſtened to a plate of lead, and the piſton of the ſyringe being drawn upwards with one hand, whilſt the lead is held in the other, the air, by its upward

preſſure, will drive back the ſyringe upon the piſton : whereas, if the loaded ſyringe be hung in a receiver, and the air be exhauſted, the ſyringe and lead will deſcend ; but upon re-admitting the air, they will again be driven upwards.

7. If a thin glaſs veſſel, whoſe aperture is cloſed, be placed under the receiver of an air-pump, and the air exhauſted from the receiver ; the veſſel will be broken by the preſſure of air within.

PROP. L. The preſſure of the atmoſphere varies at different altitudes.

Exp. Put a glaſs tube, open at both ends, through a cork into a large phial containing a ſmall quantity of coloured water ; let the lower end of the tube be in the water ; and let the cork and tube be cloſely cemented to the neck of the bottle. Then, blow through the tube, till the quantity of air within the phial is ſo increaſed, that the water will riſe above the neck of the phial. Let this phial be placed in a veſſel of ſand, to keep the air within of the ſame temperature ; the water will ſtand at different heights in the tube, according to the elevation of the place where it is placed : from whence it appears, that the preſſure of the atmoſphere varies at different altitudes.

Cor. Hence the proportion of the ſpecific gravity of air to that of water may be determined. If the difference in height of the two places where the above experiment is made be 54 feet, and that difference cauſe a difference of $\frac{3}{4}$ of an inch in the height of the water ; it follows, that a column of water of $\frac{3}{4}$ of an inch, or $\frac{1}{16}$ of a foot, is equiponderant to a column of air of 54 feet having the ſame baſe : therefore the gravity of air to that of water, is as 54 to $\frac{1}{16}$, or 864 to 1. In aſcending the mountain of Snowden in Wales, which is 3720 feet perpendicular height, it was found that the barometer ſunk 3 inches and $\frac{8}{16}$. See Art. Barometer, Prop. LVII.

PROP. LI. The force with which the wind ſtrikes upon the ſail of a ſhip, the velocity of the air, and the dimenſion of the ſail being given, will be as the ſquare of the coſine of the angle of incidence.

Plate 5. Fig. 16. Let AD repreſent the ſail of a ſhip, with its edge towards the eye ; and let a circle be drawn upon the centre K ; whence K will be the middle of the ſail, and AD its length. If the wind blows perpendicularly againſt the ſail, all the air included within the ſpace FADG will ſtrike upon it. But if the ſail is inclined in the poſition BE, all the air, which ſtrikes upon it, is included within the ſpace HBEI.

If it were poſſible that the ſail ſhould be ſtruck with the ſame quantity of air in the perpendicular poſition AD as in the oblique poſition BE, yet the quantity of the oblique ſtroke would be to the quantity of the direct ſtroke (by Book II. Prop. XIX.) as the coſine of incidence to the radius, that is, ſince (ſuppoſing LK drawn parallel to BH, the direction of the wind, and BC perpendicular to KL) BKL is the ſupplement of the angle of incidence LKE, and BL its coſine, as BL to AK.

Again, if it were poſſible that the oblique ſtroke of the wind upon the ſail BE ſhould be equal to the direct ſtroke upon AD ; yet, the column of air which ſtrikes upon the ſail directly, having AD for its baſe, and the column which ſtrikes obliquely, having BC for its baſe, the quantity of air which ſtrikes obliquely, is to that which ſtrikes directly, as BC to AD, that is, as BL to AK; but the velocities in either caſe are ſuppoſed to be the ſame: therefore the momenta, or forces with which the ſails are ſtruck will be as the quantities of matter, that is, as BL the coſine of incidence to AK the radius.

Thus, the force with which the wind ſtrikes the ſail BE obliquely, is to the force with which it ſtrikes an equal ſail AD directly, as BL to AK on two accounts; firſt, becauſe an oblique ſtroke is to a direct ſtroke in this ratio; and ſecondly, becauſe the quantity of air which ſtrikes the oblique ſail is to that which ſtrikes the direct one in the ſame ratio. Conſequently, upon both accounts together, the oblique force is to the direct one as BL × BL to AK × AK, or as the ſquare of BL the coſine of incidence to the ſquare of AK the radius. But, the length of the ſail, or AD being given, AK the radius is a given quantity. Therefore the force of the wind in different obliquities of the ſail, will be as BL the ſquare of the coſine of incidence.

CHAP. II.

Of the ELASTICITY *of the* AIR.

PROP. LII. The air is an elaſtic fluid, or capable of compreſſion and expanſion.

Exp. 1. A blown bladder, preſſed with the hand, will return into the form which it had before the preſſure.

2. A flaccid bladder, put under a receiver, when the external air is exhauſted, becomes extended by the elaſticity of the internal air.

3. A bladder ſuſpended within the receiver, with a ſmall weight hanging from it which touches the bottom, when the external air is exhauſted, by the expanſion of the internal air, will raiſe the weight.

4. The bladder being put into a box, and a weight laid upon the lid, the weight, on exhauſting the air, will be lifted up.

5. If a tube, cloſed at one end, be inſerted at its open end in a veſſel of water, the fluid in the tube will not riſe to the level of the water in the veſſel, being reſiſted by the elaſtic force of the air within the tube. On this principle the diving bell is formed.

6. If a bladder be incloſed in a glaſs veſſel ſo cloſely that the air in the veſſel without the bladder cannot eſcape, but the air within the bladder communicates with the external air through the neck of the veſſel; the external air being exhauſted, the bladder will be cloſely preſſed by the air in the veſſel; and when the air is re-admitted, the bladder will be diſtended.

7. A ſhrivelled apple, under an exhauſted receiver, will have its coat diſtended by the internal air.

8. In the ſame ſituation, the air contained in a freſh egg will expel its contents from an orifice made in its ſmaller end.

9. On green vegetables, and other ſubſtances, placed in a veſſel of water under a receiver, whilſt the air is exhauſting, bubbles will be raiſed by the expanſion of the internal air.

10. Beer, a little warmed, will, from the ſame cauſe, whilſt the internal air is exhauſting, have the appearance of boiling.

11. Let a cylindrical piece of wood (made juſt ſpecifically heavier than water by faſtening to it a ſmall plate of lead) be placed in a veſſel of water under a receiver; upon exhauſting the air the wood will ſwim; ſome particles of air eſcaping from the wood, and hereby diminiſhing its ſpecific gravity.

12. The bulb inſerted in a veſſel of water (as in Prop. XLVIII. Exp. 6.) being nearly filled with water by exhauſting the air, on its readmiſſion, the air within the bulb, by its elaſticity, will expel the water from the bulb.

13. Place a double transferrer upon the air-pump, with two receivers, exhauſt one receiver; then open the pipe between the two receivers; and the air in the unexhauſted receiver will, by its elaſticity, be in part driven into the exhauſted receiver; and both receivers will have equal portions of air; but this air will be rarer in both than the external air; whence both the receivers will be held faſt by the external preſſure.

PROP. LIII. The elaſtic ſpring of the air is equivalent to the force which compreſſes it.

If the ſpring with which the air endeavours to expand itſelf when it is compreſſed were leſs than the compreſſing force, it would yield ſtill farther to that force; if it were greater, it would not have yielded ſo far. Therefore, when any force has compreſſed the air ſo that it remains at reſt, the ſpring of the air ariſing from its elaſticity can neither be greater nor leſs than this force, that is, muſt be equal to it.

Exp. Let the air be exhauſted from an open tube, whoſe lower part is inſerted in a veſſel containing a ſmall quantity of mercury, and let the air within the veſſel be prevented from eſcaping; this air, by its elaſticity, will force the mercury up the tube nearly to the height to which it would be raiſed by the preſſure of the atmoſphere.

PROP. LIV. The ſpace which any given quantity of air fills is inverſely, and its denſity directly, as the force which compreſſes it.

Plate 5. Fig. 20. Exp. Let there be a bent tube of the form *nkg*, open at *n* and cloſed at *g*. Let a ſmall portion of mercury be at the bottom *ki*. Then *gi* is filled with air compreſſed by the weight of the atmoſphere, equivalent to the weight of a column of mercury about 29½ inches in height. If more mercury be poured into the orifice *n*, the weight of this mercury is an additional compreſſing force acting upon the air *ig*. Since (by Prop. V.) the columns of equal heights *lk*, *hi*, balance each other, the air in the ſpace *gi*, is preſſed both by the weight of the atmoſphere and the column *ml*. If therefore *ml* be 29½ inches, the air in *gi* is preſſed with double the weight of the atmoſphere, or with two atmoſpheres; and it will be found, that it will be compreſſed into the ſpace *gh*, half the ſpace which the ſame quantity of air took up when it was preſſed only with the weight of the atmoſphere: therefore the ſpace is inverſely as the compreſſing force. And its denſity (Def. V.) is inverſely as its bulk, or the ſpace filled by it. Since therefore, both the compreſſing force and the denſity of the air are inverſely as the ſpace, the denſity muſt be directly as the compreſſing force.

PROP. LV. The air conſiſts of particles, which repel each other with forces which are inverſely as the diſtances between their centres.

Plate 5. Fig. 17.

An elaſtic fluid equally compreſſed in all directions muſt have all its particles at equal diſtances from each other: for if the diſtances are unequal, where it is the leaſt, the repelling force will be greateſt, and the particles will move towards the ſide where there is leſs repulſion, till the forces become equal, that is, till the particles are equally diſtant, or the fluid becomes every where of the ſame denſity. Suppoſe, then, two equal cubes of air, A and B; it is manifeſt, from the nature of the cube, that the number of particles in the whole maſs A is equal to the cube of the number of particles in the line *de*; and, in like manner, that the number of particles in the maſs B is equal to the cube of the number of particles in the line *hi*. And the denſity of theſe two equal cubes of air A and B will be as the number of particles contained in them. Therefore the denſity of the cube A is to the denſity of the cube B, as the cube of the number of particles in the line *de* to the cube of the number of particles in the line *hi*. But, ſince theſe lines *de*, *hi*, are of a given length, the number of particles in each, will be greater as the diſtances between their centres is leſs, that is, will be inverſely as thoſe diſtances. Whence, the cube of the number of particles in *de*, *hi*, will be inverſely as the cube of the diſtance between their centres. And it has been ſhewn, that the denſity of the maſs A is to the denſity of the maſs B, as the cube of the number of particles in *de* to the cube of the number of particles in *hi*. Therefore the denſity of A is to the denſity of B, inverſely as the cube of the diſtance between the centres of the particles.

Alſo, in compreſſing any maſs, A, every ſurface, as *defg*, is preſſed cloſer to the ſurface next beyond it. And the repulſion of the ſurface *defg* againſt the ſurface next beyond it will be (all other circumſtances being equal) as the number of repelling particles in that ſurface, that is, as the ſquare of the number of particles in the line *de*. But the number of particles in the line *de* is inverſely as the diſtance between their centres. Therefore the ſquare of the number of particles in *de*, that is, the number of repelling particles in the ſurface *defg*, that is, the repulſion of this ſurface againſt the next beyond it, is inverſely as the ſquare of the diſtance between the particles. Again, where the number of particles in each ſurface is given, if it be ſuppoſed that the particles repel each other with a force which is inverſely as the diſtance between their centres, ſince the ſurfaces are at the ſame diſtance from each other with the particles which compoſe them, the repulſion of the ſurfaces muſt be in the ſame ratio. Thus, the repulſion in the maſs A is to that in the maſs B, inverſely as the diſtances of the particles, if only their approach to each other be conſidered. And it has been ſhewn that the repulſion is inverſely as the ſquare of theſe diſtances, if only the number of particles be conſidered. Therefore on both accounts taken together, the repulſion is inverſely as the cube of the diſtance of the particles. And (by Prop. LIII.) the compreſſion is as the repulſion: therefore the compreſſion is inverſely as the cube of the diſtance of the particles.

Now it was ſhewn above, that the denſity of A is to the denſity of B inverſely as the cube of the diſtance of the particles. Therefore, when a fluid conſiſts of particles which repel each other with forces inverſely as the diſtances between the centres of the particles, the denſity of the fluid will be as the compreſſing force. But it was ſhewn (Prop. LIV.) that the denſity of the air is as the compreſſing force. Therefore the air conſiſts of particles which repel each other with forces which are inverſely as the diſtances between their centres.

SCHOL. From the doctrine of the elaſticity of the air, the phenomena of Sound may be explained.

When the parts of an elaſtic body are put into a tremulous motion, by percuſſion, or the like, as long as the tremors continue, ſo long is the air included in the pores of that body, and likewiſe that which preſſes upon its ſurface, affected with the like tremors and agitations. Now, the particles of air being ſo far compreſſed together by the weight of the incumbent atmoſphere as their repulſive forces permit, it follows, that thoſe which are immediately agitated by the reciprocal motions of the particles of the elaſtic body, will, in their approach towards thoſe which lie next them, impel theſe alſo towards each other, and hereby cauſe them to be more condenſed than they were by the weight of the incumbent atmoſphere, and in their return will ſuffer them to expand themſelves again : hence the like tremors and agitations will be propagated to them ; and ſo on, till having arrived at a certain diſtance from the body, the vibrations ceaſe, being gradually deſtroyed by a continual ſucceſſive propagation of motion to freſh particles of air throughout their progreſs.

Thus it is that ſound is communicated from a tremulous body to the organ of hearing. Each vibration of the particles of the ſounding body is ſucceſſively propagated to the particles of the air, till it reaches thoſe which are contiguous to the *tympanum* of the ear, (a fine membrane diſtended acroſs it) and theſe particles, in performing their vibrations, impinge upon the tympanum, which agitates the air included within it ; which being put into a like tremulous motion, affects the auditory nerve, and thus excites in the mind the ſenſation or idea of what we call *ſound.*

Now, ſince the repulſive force of each particle of air is equally diffuſed around it every way, it follows, that when any one approaches a number of others, it not only repels thoſe which lie before it in a right line, but the reſt laterally, according to their reſpective ſituations ; that is, it makes them recede every way from itſelf, as from a centre. And this being true of every particle, the tremors will be propagated from the ſounding body in all directions, as from a centre : and further, if they are confined for ſome time from ſpreading themſelves by paſſing through a tube, or the like, will, when they have paſſed through it, ſpread themſelves from the end in every direction. In like manner, thoſe which paſs through a hole in an obſtacle they meet with in their way, will afterwards ſpread themſelves from thence, as if that was the place where they began ; ſo that the ſound will be heard in any ſituation whatever, that is not at too great a diſtance.

The utmoſt diſtance at which ſound of any kind, has been heard is about 200 miles, which is ſaid to have been obſerved in the war between England and Holland, in the year 1672. The watch word, *All's well,* given at New Gibraltar, was heard at the Old, a diſtance of 12 miles. In both theſe caſes, the ſound paſſed over water, which with reſpect to conducting ſound, is of the greateſt conſequence. By an experiment made on the river Thames, a perſon was heard to read diſtinctly at the diſtance of 140 feet on the water, on land at that of 76 : in the latter caſe no noiſe intervened, but in the former there was ſome occaſioned by the flowing of the water againſt the boats. Watermen obſerve, that when the water is ſtill, the weather calm, and no noiſe intervene, a whiſper may be heard acroſs the river. After water, ſtone may be reckoned the beſt conductor of ſound. Brick has nearly the ſame properties as ſtone.

Since the repulſive force with which the particles of air act upon each other, is reciprocally as their diſtances (by Prop. LV.) it follows, that when any particle is removed out of its place by the tremors of a ſounding body, or the vibrations of thoſe which are contiguous to it, it will be driven back again by the repulſive force of thoſe towards which it is impelled, with a velocity proportional to the diſtance from its proper place, becauſe the velocity will be as the repelling force. The conſequence of this is, that, let the diſtance be great or ſmall, it will return to its place in the ſame time; for the time a body takes up in moving from place to place will always be the ſame, whilſt the velocity it moves with is proportional to the diſtance between the places. The time therefore in which each vibration of the air is performed, depends on the degree of repulſion in its particles, and ſo long as that is not altered, will be the ſame at all diſtances from the tremulous body: conſequently, as the motion of ſound is owing to the ſucceſſive propagation of the tremors of a ſounding body through the air, and as that propagation depends on the time each tremor is performed in, it follows, that the velocity of ſound varies as the elaſticity of the air, but continues the ſame at all diſtances from the ſounding body.

The velocity of ſound, according to Mr. Derham, is at the rate of 1142 feet in a ſecond of time. Hence, with a ſtop watch, may be eaſily eſtimated the diſtance of thunder, for by multiplying the number of ſeconds between the flaſh and clap of thunder by 1142, the diſtance is given in feet. Or thus, perſons in good health have about 75 pulſations at the wriſt in a minute, conſequently in 75 pulſations, ſound flies about 13 miles, that is, one mile in about ſix pulſations. Example. On ſeeing the flaſh of a gun at ſea, 54 pulſations at the wriſt were counted before the report was heard, conſequently the diſtance of the ſhip is $\frac{54}{6} = 9$ miles.

Moreover, ſince the undulatory motion of the air, which conſtitutes ſound, is propagated in all directions from the ſounding body; it will frequently happen, that the air, in performing its vibrations, will impinge againſt various objects, which will reflect it back, and ſo cauſe new vibrations the contrary way; now, if the objects are ſo ſituated, as to reflect a ſufficient number of vibrations back to the ſame place, the ſound will be there repeated and is called an *echo*. And, the greater the diſtance of the objects is, the longer will be the time before the repetition is heard. And when the ſound in its progreſs meet with objects, at different diſtances, ſufficient to produce an echo, the ſame ſound will be repeated ſeveral times ſucceſſively, according to the different diſtances of thoſe objects from the ſounding body; which makes what is called a *repeated* echo. Echoes repeat more by night than in the day.

If the vibrations of the tremulous body are propagated through a long tube, they will be continually reverberated from the ſides of the tube into its axis, and by that means prevented from ſpreading, till they get out of it; whereby they will be exceedingly increaſed, and the ſound rendered much louder than it would otherwiſe be; as in the *Speaking Trumpet*.

The difference of *muſical* tones depend on the different number of vibrations communicated to the air, in a given time, by the tremors of the ſounding body; and the quicker the ſucceſſion of the vibrations is, the acuter is the tone, and the reverſe.

PROP. LVI. The elaſticity of air is increaſed by heat.

Exp. To the bottom of a hollow glaſs ball let an open bended tube be affixed. Let the lower part of the bended tube and part of the ball be filled with mercury: the external ſurface will be preſſed by the weight of the atmoſphere; and the internal ſurface will be equally ly preſſed by the ſpring of the air incloſed within the veſſel. If the ball be immerſed in boiling water, the increaſed elaſticity of the included air will raiſe the mercury in the ſmall tube. The ſame may be ſhewn by immerſing in boiling water a tube, cloſed at one end, into which a ſmall quantity of mercury has been admitted, incloſing a portion of air within the tube.

Schol. 1. The Wind is no other than the motion of the air upon the ſurface of the globe. The principal cauſe of the wind is, that the atmoſphere is heated over one part of the earth more than over another. For, in this caſe, the warmer air being rarefied, becomes ſpecifically lighter than the reſt; it is therefore overpoiſed by it and raiſed upwards, the upper parts of it diffuſing themſelves every way over the top of the atmoſphere; while the neighbouring inferior air ruſhes in from all parts at the bottom; which it continues to do, till the equilibrium is reſtored. Upon this principle it is, that moſt of the winds may be accounted for.

Under the *Equator*, the wind is always obſerved to blow from the eaſt point. For, ſuppoſing the ſun to continue vertical over ſome one place, the air will be more rarefied there; and conſequently, the neighbouring air will ruſh in from every quarter with equal force. But, as the ſun is continually ſhifting to the weſtward, the part where the air is moſt rarefied, is carried the ſame way; and therefore the tendency of all the lower air, taken together, is greater that way, than any other. Thus the tendency of the air towards the weſt, becomes general, and its parts impelling one another, and continuing to move till the next return of the ſun, ſo much of its motion, as was loſt by his abſence, is again reſtored, and therefore the eaſterly wind becomes *perpetual.*

On each ſide of the *Equator*, to about the thirtieth degree of latitude, the wind is found to vary from the eaſt point, ſo as to become north-eaſt on the northern ſide, and ſouth-eaſt on the ſouthern. The reaſon of which is, that, as the *equatorial* parts are hotter than any other, both the northern and ſouthern air ought to have a tendency that way; the northern current, therefore, meeting in this paſſage with the eaſtern, produces a north-eaſt wind on that ſide; as the ſouthern current, joining with the ſame, on the other ſide the *Equator*, forms a ſouth-eaſt wind there.

This is to be underſtood of open ſeas, and of ſuch parts of them as are diſtant from the land; for near the ſhores, where the neighbouring air is much rarefied, by the reflection of the ſun's heat from the land, it frequently happens otherwiſe; particularly on the *Guinea* coaſt, the wind always ſets in upon the land, blowing weſterly inſtead of eaſterly. This is becauſe the deſarts of *Africa* lying near the *Equator*, and being a very ſandy ſoil, reflect a greater degree of heat into the air above them; which being thus rendered lighter than that which is over the ſea, the wind continually ruſhes in upon the land to reſtore the equilibrium.

That part of the ocean, which is called the *Rains*, is attended with perpetual calms, the wind ſcarcely blowing ſenſibly either one way or other. For this tract being placed between the weſterly wind blowing from the ocean towards the coaſt of *Guinea*, and the eaſterly wind blowing from the ſame coaſt to the weſtward thereof, the air ſtands in equilibrio between both, and its gravity is ſo much diminiſhed thereby, that it is not able to ſupport the vapour it contains, but lets it fall in continual rain, from whence this part of the ocean has its name.

There is a ſpecies of winds, obſervable in ſome places within the *Tropics*, called by the ſailors *Monſoons*, or Trade Winds, which, during ſix months of the year, blow one way; and the remaining ſix the contrary. The occaſion of them in general is this: when the ſun approaches the northern *Tropic*, there are ſeveral countries, as *Arabia*, *Perſia*, *India*, &c. which become hotter, and reflect more heat than the ſeas beyond the *Equator*, which the ſun has left; the winds therefore, inſtead of blowing from thence to the parts under the *Equator*, blow the contrary way: and when the ſun leaves thoſe countries, and draws near the other *Tropic*, the winds turn about, and blow on the oppoſite point of the compaſs.

From the ſolution of the general trade winds, we may ſee the reaſon, why in the *Atlantic* ocean, a little on this ſide the thirtieth degree of north latitude, there is generally a weſt, or ſouth-weſt wind. For, as the inferior air, within the limits of thoſe winds, is conſtantly ruſhing towards the *Equator*, from the north-eaſt point, or nearly ſo, the ſuperior air moves the contrary way; and therefore, after it has reached theſe limits, and meets with air, that has little or no tendency to any one point more than to another, it will determine it to move in the ſame direction with itſelf.

In our own climate, we frequently experience, in calm weather, gentle breezes blowing from the ſea to the land, in the heat of the day: which phenomenon is very agreeable to the principle laid down above: for the inferior air over the land being rarefied by the beams of the ſun, reflected from its ſurface, more than that which impends over the water, the latter is conſtantly moving on to the ſhore, in order to reſtore the equilibrium, when not diſturbed by ſtronger winds from another quarter.

From what has been obſerved, nothing is more eaſy than to ſee, why the northern and ſouthern parts of the world, beyond the limits of the trade winds, are ſubject to ſuch variety of winds. For the air, upon account of the leſſer influence of the ſun in thoſe parts, being undetermined to move towards any fixed point, is continually ſhifting from place to place, in order to reſtore the equilibrium, wherever it is deſtroyed, by the heat of the ſun, the riſing of vapours or exhalations, the melting of ſnow upon the mountains, or other circumſtances.

EXP. Fill a large diſh with cold water; into the middle of this put a water-plate, filled with warm water. The firſt will repreſent the ocean; and the other an iſland, rarefying the air above it. Blow out a wax candle, and if the place be ſtill, on applying it ſucceſſively to every ſide of the diſh, the ſmoke will be ſeen to move towards the plate. Again, if the ambient water be warmed, and the plate filled with cold water, let the ſmoking wick of the candle be held over the plate, and the contrary will happen.

SCHOL. 2. Heat expands all bodies, ſolid as well as fluid.

EXP. 1, 2. Water may be rarefied into ſteam, and will become exceedingly elaſtic, acting with great power, as in the eolipile, and in ſteam engines. See Art. X. Prop. LVII.

3. Metals expand by heat, and the degrees of their expanſion are meaſured by the PYROMETER, which is an inſtrument invented to render the ſmalleſt expanſions ſenſible. Various machines have been contrived for this purpoſe, by Ferguſon, Deſſaguliers, De Luc, Plate 12.
&c. but the general principle may be thus illuſtrated. Let *abc* be a lever, whoſe fulcrum is Fig. 13.
b, acting upon another lever *cde*, whoſe fulcrum is *d*, this again acts upon a third lever *efg*, whoſe

whoſe fulcrum is *f*, and let *x* be a metallic rod, one end of which reſts againſt an immoveable obſtacle P, and the other end againſt the lever *abc*, at *a*. If a lamp be put under this rod, the heat will increaſe its length, and put the levers in motion: Now by the principle of the lever,

Vel. of a : Vel. of c : : ab : bc
Vel. of c : Vel. of e : : cd : de
Vel. of e : Vel. of g : : ef : fg.

Therefore Vel. of a : Vel. of g : : $ab \times cd \times ef$: $bc \times de \times fg$.

Hence if *ab*, *cd*, *ef*, be ſmall in proportion to *bc*, *de*, *fg*, a trifling increaſe in the length *x* will produce a very conſiderable motion in the point *g*, which may be meaſured upon the graduated arc *yz*.

Exa. If *ab*, *cd*, *ef*, be each equal to 1, and *bc*, *de*, *fg*, each equal to 15, then, if the rod increaſe but the 3375th part of an inch, the point *g* will deſcribe 1 inch, conſequently, by dividing each inch in the graduated arc into 20 parts, an expanſion in the rod of leſs than a 60 thouſandth part of an inch eaſily becomes viſible.

Mr. Ferguſon found the expanſion of metals to be in the following proportion: iron and ſteel 3; copper $4\frac{1}{2}$; braſs 5; tin 6; and lead 7. An iron rod 3 feet long, is about $\frac{1}{70}$ of an inch longer in ſummer than in winter. See Ferguſon's firſt Lecture and Supplement; Deſſagulier's Exp. Phil. Chambers's Cyclopædia, by Dr. Rees.

4. Mercury, expanding or contracting by an increaſe or decreaſe of heat in the air, is made the meaſure of heat in thermometers. See Art. viii. Prop. lvii.

Schol. 3. It is found by experiment, that air is neceſſary to the exiſtence of ſound, of animal life, of fire, and of exploſion.

Exp. 1. Let a bell ring under an exhauſted receiver, and in a condenſer.
2. Let a lighted candle be extinguiſhed under a receiver.
3. Let gun-powder fall upon red hot iron placed within an exhauſted receiver.

Schol. 4. The elaſticity of the air affords a method of determining the depth of the ſea where a line cannot be uſed.

A wine glaſs immerſed in water with its mouth downwards will not become filled, becauſe the ſpring of air will prevent the water from entering beyond a certain point. The *diving bell* is conſtructed on this principle.

PROP. LVII. To explain the nature and uſe of ſundry Hydraulic and Pneumatic Inſtruments.

I. The Syphon.

Plate 5. Fig. 23.

Let DEC be a bended tube, having one leg longer than the other. This inſtrument, uſed for drawing off liquors, is called the ſyphon. If the ſhorter leg of the tube be inſerted in a veſſel of fluid, and if by ſucking with the mouth a vacuum be produced in the tube, or if the tube be filled with the fluid before it is uſed, the fluid will run off from the veſſel. The cauſe of which may be thus explained: the orifice C, of the longer leg, is expoſed to the preſſure of the atmoſphere: alſo, ſince the fluid within the ſhorter leg is ſupported by the ſurrounding fluid in the veſſel, the preſſure upon the orifice D is that of the atmoſphere. The two equal orifices are then acted upon by equal preſſures; the difference of the lengths of the columns of atmoſphere being too ſmall to cauſe any perceptible difference in their preſſure. But theſe equal preſſures are counteracted by the preſſures of two unequal columns of fluid ED, EC. If, therefore, the preſſures of the columns of atmoſphere be more than ſufficient to balance thoſe of the columns of fluid, that which acts with the leſſer force, that is, the leſſer column DE is more preſſed againſt the column CE, than the column CE is preſſed againſt DE at the vertex E. Conſequently, the column EC, muſt yield to the greater preſſure, and flow off through the orifice C.

Exp. 1. Draw off water by a ſyphon.

2. Whilſt mercury is paſſing off from a veſſel by a ſyphon, let the air be exhauſted from the veſſel, and the fluid will ceaſe to run.

3. Intermitting fountains are natural ſyphons.

II. The Syringe.

Let a hollow cylindrical tube have a ſmall orifice at one end: at the other end inſert a ſolid cylinder ſo exactly fitted to the tube, that no air can paſs along its ſides, and fix a handle to the ſolid cylinder. If that end of this inſtrument which has the ſmaller orifice, be inſerted in water, and the ſolid cylinder, or piſton, be drawn back, a vacuum will be produced within the ſyringe; and the preſſure of the atmoſphere on the ſurface of the water, meeting with no oppoſite preſſure, will force the water into the tube, from whence it may be forcibly expelled, by puſhing down the piſton.

III. The Common Pump.

Plate 5. Fig. 21.

In this uſeful inſtrument, a handle, acting upon a pin as a lever of the firſt kind, draws up a piſton AD, fitted to the ſhaft or barrel of the pump, as deſcribed in the ſyringe. This piſton has an hole, over which is a valve of leather, loaded with lead, opening upwards. Towards the lower part of the ſhaft is inſerted a plug C, which alſo has in it a hole, and a valve which opens upwards. When the piſton, or ſucker, is drawn up from the plug, a vacuum is produced in the ſhaft between D and C, into which the air contained in the lower part of the pipe expands itſelf. By repeated ſtrokes the air eſcapes through the upper valve, and the vacuum becomes ſo perfect, that the external air, preſſing without counteraction, upon the ſurface of the water, in the well or reſervoir in which the ſhaft is ſuppoſed to be inſerted, forces the water through the valves at C and D, into the ſpace AD; from whence it is prevented from returning downward, by the valves, which are cloſely preſſed down by the incumbent fluid. If therefore the handle be repeatedly lifted up, the column of water will increaſe upon every ſtroke, till it riſes to the level of the ſpout, and is diſcharged. But if the height be more than 34 feet, the water cannot be raiſed: for ſuch a column is equal to the weight of a column of the atmoſphere of the ſame diameter.

IV. The Forcing Pump.

Plate 5. Fig. 22. In this pump, the piſton is one entire cylinder, as in the ſyringe. The water is raiſed into the pipe between A and D, as in the common pump: from hence it is forced, by the downward preſſure of the piſton, or forcer, through a tube inſerted in the ſide of the main ſhaft. In this ſide-tube a valve is inſerted at E to prevent the water from returning, and when a ſufficient quantity is raiſed, it is diſcharged by the ſpout.

The common engine for *extinguiſhing fires* conſiſts of two ſuch forcing pumps, which convey the water into a reſervoir made air-tight, into which a pipe is inſerted. As this reſervoir fills with water, the air within it is proportionally condenſed, and therefore forces the water up a cylinder from which it is conveyed, at pleaſure, by leathern pipes.

V. The Condenser.

This inſtrument, which is uſed to force air into any veſſel, is a ſyringe, having a ſolid piſton, and a valve in the lower part of its barrel which opens downwards. By thruſting down the piſton, the air is forced through the valve, which is afterwards held cloſe by the elaſticity of the condenſed air. When the piſton is lifted up, a vacuum is produced, till it is raiſed above a ſmall hole in the barrel, when the air ruſhes in, and is again diſcharged through the valve.

Artificial Fountains are formed by the help of a condenſer, which throws any quantity of air into a veſſel in part filled with water; which, by its elaſticity, forces the water up into pipes from which it is conveyed at pleaſure.

The Air-Gun is an inſtrument, in the form of a gun, by which a quantity of condenſed air is ſuddenly ſet free, and drives a ball through the barrel with great force.

VI. The Air-Pump.

Plate 12. Fig. 12. This inſtrument, the uſe of which is to exhauſt the air from any veſſel, has two ſtrong barrels A, A, which communicate with a cavity in D; within each of which, near the bottom, is fixed a valve opening upwards, and two piſtons, one in each barrel, having a valve which likewiſe opens upwards. Theſe piſtons are moved by means of a cog-wheel in the piece TT, to the axis of which the handle B is fixed, and whoſe teeth catch in the racks of the piſtons CC, and move them upwards or downwards. PQR is a circular braſs plate, having at its centre the orifice K of a concealed pipe that communicates with the cavity. In the piece D at V is a ſcrew that cloſes the orifice of another pipe, for the purpoſe of admitting the external air when required. Upon W is placed the ſhort barometer gage for the purpoſe of ſhewing the degrees of exhauſtion. When the handle is turned, one of the piſtons is raiſed, and a vacuum produced in its barrel. By means of the pipe, which paſſes from the orifice K in the plate upon which the receiver LM, or veſſel to be exhauſted, ſtands, to the part of the barrel beneath the lower valve, the air contained in the receiver, communicating with the barrel, raiſes the lower valve by its elaſtic ſpring, and expends into the vacuum. Thus a part of the air in the receiver is extracted. By turning the handle the contrary way, the ſame effect is produced in the other barrel; whilſt the firſt piſton, being depreſſed, the air which had paſſed from the receiver is compreſſed, and eſcapes through the valve in the piſton. This operation is continued till the air is nearly exhauſted

from the receiver: for it can never be perfectly exhauſted, ſince at each ſtroke only ſuch a part of the air which remained is taken away, as is to the quantity before the ſtroke, as the capacity of the barrel, to that of the receiver, pipe and barrel taken together: which may be eaſily proved in the following manner.

Let $R =$ the content of the receiver and pipe, $B =$ the content of the barrel.

If $L =$ the quantity of air in R before the ſtroke, and $l =$ the quantity exhauſted by it; and ſince, the piſton being raiſed, the air is uniformly diffuſed through R and B, and that in B extracted by the ſtroke, conſequently $L : l :: R + B : B$ or $l : L :: B : R + B$, that is, the quantity of air extracted, is to the quantity before the ſtroke, as the capacity of the barrel, is to that of the receiver, pipe and barrel taken together.

Cor. 1. Let L, M, N, &c. be the quantities of air, before any ſucceſſive ſtrokes; l, m, n, &c. the quantities exhauſted by each ſtroke; and $L : l :: R + B : B :: M : m :: N : n$, &c. by diviſion $L : L - l\ (M) :: M : M - m\ (N) :: N : N - n\ (O)$ &c. or $L : M :: M : N :: N : O$, &c. therefore L, M, N, &c. are in decreaſing geometric progreſſion, whoſe common ratio is that of $R + B : R$. If $R = 2B$, then $R + B : R :: 3 : 2$, and $M = \frac{2\,L}{3}$ $N = \frac{2\,M}{3}$, &c. and the quantities of air are equal to L, $\frac{2}{3}L$, $\frac{4}{9}L$, $\frac{8}{27}L$, &c.

Cor. 2. Since $L : M :: l : m$; $M : N :: m : n$, &c. l, m, n, &c. are in a decreaſing geometric progreſſion, whoſe common ratio is that of $R + B : R$. If, as in the laſt Cor. $R + B : R :: 3 : 2$, then $m = \frac{2\,l}{3}$, $n = \frac{2\,m}{3}$, &c. and the quantities of air exhauſted by the ſucceſſive ſtrokes are l, $\frac{2}{3}l$, $\frac{4}{9}l$, $\frac{8}{27}l$, &c.

Cor. 3. If R be to B in any finite ratio as 3 : 2, the receiver can never be perfectly exhauſted by any finite number of turns, for let the number of turns be n, and Q the laſt remainder, then $Q = L \times \overline{\tfrac{2}{3}}\big)^{n}$, ſuppoſing L to be the quantity of air in the receiver at firſt; and this quantity $L \times \overline{\tfrac{2}{3}}\big)^{n}$ is finite, ſince n is finite.

VII. The Barometer.

(1) If a glaſs globe be exhauſted of air, and balanced at one end of a beam, upon admitting the air the globe preponderates. This experiment not only, in common with others beforementioned, ſhews that the air has weight, but alſo what that weight is. The denſity of air was found, by Mr. Haukſbee, to be 885 times leſs than that of water, when the barometer ſtood at $29\frac{1}{2}$ inches. Hence as a cubic inch of water, weighs 253.18 grains Troy, a cubic inch of air weighs 0.286 grains. And if mercury be 14 times heavier than water, the ſpecific gravity of air is to that of mercury 1 to $885 \times 14 = 12390$.

Plate 12. Fig. 4.

(2) If a glaſs tube AB, of about 32 or 33 inches long, hermetically ſealed at one end, be filled with mercury, and then inverted into a baſon D of the ſame fluid, the mercury in the tube will ſtand at an altitude above the ſurface of that in the baſon between 28 and 31 inches. A tube thus filled, and graduated from 28 to 31 inches, is called a barometer. The height of the mercury in the tube above the ſurface of the mercury in the baſon is called the ſtandard altitude, which, in this country, fluctuates between 28 and 31 inches; and the difference, between the greateſt and leaſt altitudes, is called the ſcale of variation.

Now the mercury in the barometer tube will ſubſide, till the column be equivalent to the weight of the external air upon the ſurface of the mercury in the baſon, and is therefore a true criterion to meaſure that weight, and chiefly directed to that purpoſe, in order to foretel the changes in the weather.

Plate 12. Fig. 5.

If each inch of the ſcale of variation AD, (fig. 5. made larger for ſake of perſpicuity) be divided into ten equal parts, marked 1, 2, 3, increaſing upwards, and a *vernier* LM, whoſe length is $\frac{11}{10}$ths of an inch, be likewiſe divided into ten equal parts, increaſing downwards, and ſo placed as to ſlide along the graduated ſcale of the barometer, the altitude of the mercury in the tube, above the ſurface of that in the baſon may be found, in inches and hundredth parts of an inch, by this proceſs. If the ſurface E of the mercury in the tube do not coincide with a diviſion in the ſcale of variation, place the index of the vernier M even with this ſurface, and obſerving where a diviſion of the *vernier* coincides with one of the ſcale, the figure in the *vernier* will ſhew what hundredth parts of an inch are to be added to the tenths immediately below the index. If the ſurface of the mercury be between 6 and 7 tenths above 30 inches, and the index of the *vernier* being placed even with it, and the figure 8 upon the *vernier* coincide with a diviſion upon the ſcale, the altitude of the barometer will be 30 inches $\frac{6}{10}$ and $\frac{8}{100}$ of an inch: For each diviſion of the *vernier* being greater than that of the ſcale by $\frac{1}{100}$ of an inch, (for the tenth part of a tenth of an inch is the hundredth part of an inch) and there being eight diviſions, the whole muſt be $\frac{8}{100}$ of an inch above the number 6 in the ſcale, and the height of the mercury is therefore 30.68 inches.

COR. 1. Hence, if the atmoſphere were homogeneous, its altitude would be eaſily found. For by the former part of this article, when the mercury ſtood at $29\frac{1}{2}$ inches, the denſity of the air was to that of mercury as 1 to 12390; conſequently, the altitude of an homogeneous atmoſphere would be equal to $12390 \times 29\frac{1}{2} = 5.77$ miles. The real height of the atmoſphere may be determined from the beginning and end of twilight. See Book VII. Prop. XXXIX.

COR. 2. The barometer has been applied to the meaſuring of the heights of towers, mountains, &c. Since 12390 inches of air, near the ſurface of the earth, is equal to one inch of mercury; 1239 inches, or about 103 feet of air, muſt be equal to $\frac{1}{10}$ of an inch of mercury. Therefore if a barometer be carried up any great eminence, the mercury will deſcend $\frac{1}{10}$ of an inch for every 103 feet that the barometer aſcends. This corollary ſuppoſes that the atmoſphere near the ſurface of the earth is every where of the ſame denſity, which is ſo far from being true, that the concluſions drawn from the ſuppoſition deviate from fact even in ſmall altitudes, as appears from the following obſervations made by Dr. Nettleton.

	Perpendi. Altitudes.	Loweſt Station.	Higheſt Station.	Diff.
Town of Halifax - -	102	29.78	29.66	0.12
Coal mine - -	236	29.50	29.23	0.27
Halifax-hill - -	507	30.00	29.45	0.55

See Abr. Phil. Tranſ. Vol. VI.

M. De Luc, Sir Geo. Shuckburgh, and General Roy, have conſidered this ſubject very attentively, and have laid down certain rules, which, with proper corrections, on account of the difference of the temperature of the air, will hold good for all altitudes within our reach. See De Luc on the Modifications of the Atmoſphere. Phil. Tranſ. Vol. LXXVII.

Cor. 3. When the mercury in the barometer ſtands at the altitude of 30 inches, the preſſure of the air upon every ſquare inch is rather more than 15lb. avoirdupoiſe. Now, ſuppoſing the ſurface of a middle ſized man to be 14½ ſquare feet, the preſſure upon him, when the air is lighteſt, will be 13.2 tons, and when heavieſt, it will be 14.3 tons, the difference of which is 2464lb. This difference of preſſure muſt affect us in regard to our health and animal ſpirits, eſpecially when the change takes place ſuddenly.

For a deſcription of the different kinds of barometers, ſee Parkinſon's Hydroſtatics, p. 97.

VIII. The Thermometer.

The thermometer is an inſtrument calculated for meaſuring the temperature of the air, and other bodies contiguous to it, as to heat and cold, being uſually a cylindrical glaſs tube, containing air, water, oil, ſpirits of wine, mercury, &c. which fluids are found to occupy different portions of the tube in different temperatures, and theſe portions being meaſured, exhibit the different expanſions of the included fluid.

AB repreſents a glaſs tube, whoſe end A is blown into a bulb: this bulb and part of the tube being filled with quickſilver, the leaſt change of the bulk of quickſilver, and conſequently of the temperature of contiguous bodies, is ſhewn by the riſe or fall of the ſurface in the tube, which is indicated by the ſcale *ab* affixed to the frame of the inſtrument. Plate 12. Fig. 2.

The thermometer chiefly uſed in Great Britain, is that conſtructed by Fahrenheit; in which there are 180 diviſions between the freezing and boiling water points, the freezing point being reckoned 32° above zero, or the commencement of the ſcale; conſequently the boiling water point is 212°.*

A good thermometer muſt poſſeſs the following properties: the capacity of the tube ſhould be very ſmall and regular, and its upper end muſt be hermetically ſealed. The empty ſpace muſt be as free as poſſible from air. The ſcale muſt be well adjuſted, and accurately divided according to the capacity of the tube. Thermometers with ſmall bulbs, and tubes in proportion, are the moſt to be depended upon, for a large volume of mercury is not ſufficiently ſenſible to the change of temperature.

Since the thermometers of Fahrenheit and Reaumeur are thoſe moſtly in uſe, it will be often found convenient to be able readily to convert the degrees on Fahrenheit's ſcale into thoſe of Reaumeur, and vice verſa: and as one degree on Reaumeur's ſcale is equal to 2.25°, or to $\frac{9}{4}$° of Fahrenheit; and as the former ſcale places the freezing point at zero, and the latter places it at 32; the following canons will reduce the degrees on the one to the correſponding ones on the other.

1. To convert the degrees of Fahrenheit into thoſe of Reaumeur; $\frac{\overline{F - 32}}{9} \times 4 = R$: thus the 167° of Fahrenheit anſwers to the 60° of Reaumeur.

2. To convert the degrees of Reaumeur to thoſe of Fahrenheit; $\frac{R \times 9}{4} + 32 = F$.

Thus the 40° of Reaumeur anſwers to the 122° of Fahrenheit. See N° 4, Appendix to Lavoiſier's Chemiſtry.

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* Note. The ſcale on Reaumeur's thermometer, which is principally uſed on the continent, begins at the freezing point, and proceeds both ways, from 0 or zero. From freezing to boiling water, are 80 degrees. For the conſtruction, uſes, &c. of this and ſeveral other thermometers, ſee Parkinſon's Hydroſtatics, p. 154—169.

It is evident, that the thermometers hitherto deſcribed, are limited in their extent. The mercurial thermometer extends no farther than the heat of boiling mercury, which anſwers to 600° of Fahrenheit's ſcale; but the heat of ſolid bodies in the ſtate of ignition exceeds that of boiling mercury. To remedy this defect, Mr. Wedgewood has contrived a thermometer for meaſuring the higher degrees of heat, by means of a diſtinguiſhing property of argillaceous bodies, viz. the diminution of their bulk by fire. This diminution commences in a dull red heat, and proceeds regularly as the heat increaſes, till the clay becomes vitrified. This thermometer, therefore, marks, with preciſion, the different degrees of ignition from the red heat viſible only in the dark, to the heat of an air furnace. Its conſtruction is extremely ſimple. It conſiſts of two rulers fixed upon a flat plane, a little farther aſunder at one end than at the other, leaving an open longitudinal ſpace between them. Small pieces of alum and clay, mixed together, are made of ſuch a ſize as juſt to enter at the wide end; they are then heated in the fire along with the body, whoſe heat we wiſh to determine. The fire, according to the degree of heat it contains, contracts the earthy body, ſo that applied to the wide end of the gage, it will ſlide on towards the narrow end, leſs or more, according to the degree of heat to which it has been expoſed. Each degree of Mr. Wedgewood's thermometer anſwers to 130 degrees of Fahrenheit; and the ſcale begins from 1077 of Fahrenheit. Hence the following

TABLE.

	Fahrenheit's Scale.	Wedgewood's Scale.
Extremity of Wedgewood's ſcale	32277°	240°
Caſt iron melts	21877	160
Leaſt welding heat of iron	12777	90
Fine gold melts	5237	32
Fine ſilver melts	4717	28
Braſs melts	3807	21
Red heat fully viſible in day light	1077	0
Red heat fully viſible in the dark	947	1
MERCURY BOILS, alſo expreſſed oils	600	
Lead melts	540	
Biſmuth melts	460	
Tin melts	408	
Nitrous acid boils	242	
Cow's milk boils	213	
WATER BOILS	212	
Heat of the human body	92 to 99	
Oil of olives begin to congeal	43	
WATER FREEZES and ſnow melts	32	
Milk freezes	30	
Urine and vinegar freeze	28	
Strong wine freezes	20	
A mixture of ſnow and ſalt freezes	0 to 4	
MERCURY FREEZES	— 39 or 40	
Cold produced at Hudſon's Bay, by a mixture of vitriolic acid and ſnow	— 69	

Note. If theſe three metals be mixed together by fuſion, in the proportion of 5, 8, and 3, the mixture will melt in a heat below boiling water.

IX. The

IX. The Hygrometer.

The hygrometer is an inſtrument for the meaſuring the degrees of moiſture in the air; of which there are various kinds; for whatever contracts, and expands by the moiſture, and dryneſs of the atmoſphere, is capable of being formed into an hygrometer. Such are moſt kinds of wood; catgut, twiſted cord; the beard of wild oat, &c. The following are very ſimple in their conſtruction, and will ſerve to explain the principle of the inſtrument.

1. Stretch a catgut, or a common cord, ABD, along a wall, paſſing it over a pulley B: fixing it at one end A, and to the other hanging a weight E, carrying a ſmall index F. Againſt the ſame wall, ſit a metal plate HI, divided into any number of equal parts, and the hygrometer is complete. Plate 12. Fig. 6.

For it is known, that moiſture ſenſibly ſhortens catgut, cord, &c. and that as the moiſture evaporates, they return to their former length. Hence the weight E, with the index, will aſcend when the air is moiſt, and deſcend when it becomes drier; and the diviſions on the ſcale will ſhew the degrees of moiſture or dryneſs. This hygrometer may be made more ſenſible and accurate by ſtraining the catgut over ſeveral pullies placed in a parallel or any other poſition.

2. The ſponge hygrometer is conſtructed as follows: BC is the beam of a balance; to the end B is hung a piece of ſponge, ſo cut as to contain as large a ſuperficies as poſſible, which muſt be exactly balanced on the other ſide by another thread of ſilk D, on which is ſtrung ſome very ſmall leaden ſhot at equal diſtances, ſo adjuſted as to cauſe an index E to point to G, the middle of the graduated arc FGH, (made large for diſtinction's ſake) when the air is in a middle ſtate between the greateſt moiſture and the greateſt dryneſs. Under this ſilk, ſtrung with ſhot, is placed a ſhelf I, for that part of the ſhot to reſt upon which is not ſuſpended. When the moiſture imbibed by the ſponge increaſes its weight, it will raiſe the index, and vice verſa when the air is dry. Plate 12. Fig. 7.

To prepare the ſponge, it may be proper firſt to waſh it in water very clean, and, when dry again, dip it in water or vinegar in which there has been diſſolved ſal amoniac, or ſalt of tartar; after which let it dry again. Salt of tartar, or any other ſalt, or pot-aſhes, may be put into the ſcale of a balance, and uſed inſtead of the ſponge.

X. Steam Engine.

The ſteam engine is a machine which derives its moving power from the elaſticity and condenſation of the ſteam of boiling water. The high importance of this machine to the mechanical arts of life, eſpecially where immenſe powers are required, has given birth to many conſiderable improvements both in its conſtruction and mode of operation.

The following is a deſcription of one of the earlieſt ſteam engines, which, as it exhibits the general principles of the machine, will be deemed ſufficient in a work only introductory to ſcience. A hiſtory of the ſteam engine, from its firſt conſtruction by Captain Savary, down to the preſent time, in which are included all the great improvements made by the ingenious Mr. Watt, of Birmingham, will be found in the Encyclopœdia Britannica, Vol. xvii. Part ii.

H repreſents the boiler on its furnace. E the cylindrical veſſel of iron, in which the piſton OO moves up and down, the cavity between the piſton and bottom of the cylinder is made air-tight. F is a cock to admit the ſteam into the cylinder. IK is a lever, attached to the piſton at I, and at K to the piſton of the pump which works on that ſide. PQ is a ſolid piſton moving in the pipe RM, and loaded with a heavy weight at P. ABC is the main Plate 12. Fig. 11.

main pipe that receives the water forced from RM through a valve at C, opening outwards. N is an air veſſel communicating with the main pipe. At D is a valve opening upwards, and at M is the water to be raiſed.

The engine is repreſented at the end of a forcing ſtroke, which is likewiſe its poſition when at reſt. Suppoſe the main pipe ABC to be filled with water, and the water in the boiler H to boil ſtrongly. The cock F being opened, the ſteam ruſhes into the cylinder, and being much lighter than air, riſes to the top, and expels the air through a valve in the bottom of the cylinder. F is then ſhut, and the cock G communicating with the main pipe is opened, which by ſpouting cold water againſt the bottom of the piſton, condenſes the ſteam. A vacuum being thus obtained, the preſſure of the atmoſphere forces the piſton down to the bottom of the cylinder; the lever IK is moved, and the piſton PQ with its weight is raiſed, and the water aſcends in the pipe MR upon the principle of the common pump. The cock G being now ſhut, and F opened, the ſteam enters the cylinder, and counteracts the preſſure of the atmoſphere on the piſton OO: conſequently, the weight P prevails, and drives down the piſton RQ, forcing the water through the valve C into the main pipe and its air veſſel. The uſe of the air veſſel is to prevent the main pipe from burſting by the ſudden entrance of the water; for the air at N being elaſtic, gives way to the ſtroke, and its re-action during the time of elevating the piſton PQ continues the motion of the water, ſo that its velocity is no more than half what it would have been if it had been impelled by ſtarts, and reſted during the raiſing of the piſton. By opening the cock G, and ſhutting F, (which is done by a ſingle operation) the ſteam is again condenſed, the preſſure of the atmoſphere again prevails, and thus the work may be continued at pleaſure.

The power of ſome of the ſteam engines, conſtructed by Meſſrs. Boulton and Watt, is thus deſcribed as taken from actual experiment. An engine, having a cylinder of 31 inches in diameter, and making 17 double ſtrokes per minute, performs the work of 40 horſes, working night and day, (for which three relays, or 120 horſes, muſt be kept) and burns 11.000 pounds of Staffordſhire coal per day. A cylinder of 19 inches, making 25 ſtrokes of 4 feet each per minute, performs the work of 12 horſes, working conſtantly, and burns 3700 pounds of coals per day. Theſe engines will raiſe more than 20.000 cubic feet of water, 24 feet high, for every hundred weight of good pit coal conſumed by them.

XI. The HYDROMETER.

Plate 12. Fig. 3. The hydrometer, an inſtrument uſually applied to find the ſpecific gravities of liquids, is thus conſtructed: AB is a hollow cylindrical tube of glaſs, ivory, copper, &c. joined to a hollow ball D, at the bottom of which is a ſmaller ball E, containing ſome quickſilver, or ſhot, by which the inſtrument is ſo poiſed, that it ſwims vertically in a liquid. The ſtem AB is graduated in ſuch a manner, that the figures exhibit the magnitudes of the parts below, and, conſequently, the ſpecific gravities of the different fluids in which it deſcends to thoſe figures. Thus if the parts immerſed in *water*, and *ſpirits of wine*, be as 10 to 11.1, then the ſpecific gravity of the water will be to that of the ſpirits of wine as 11.1 to 10.

To make this inſtrument of more ſervice, there has been added a little plate, or diſh, at the top of the tube, upon which may be placed weights, as convenience may require. For example; if the whole inſtrument float, immerſed in ſpirits to a certain point, it will require an additional weight to ſink it to the ſame depth in water. Suppoſe the inſtrument to weigh 10 dwts. and to be adjuſted to rectified ſpirits of wine, it will then require an additional weight of 1.6 dwt. to ſink it to the ſame point in water. Conſequently, the ſpecific gravity of water is to that of rectified ſpirits of wine as 11.6 to 10, or as 10 to 8.6.

BOOK IV.

OF MAGNETISM.

Def. I. THE earth contains a mineral ſubſtance which attracts iron, ſteel, and all ferruginous ſubſtances; this is called a *natural magnet.*

Def. II. The ſame ſubſtance has the power to communicate its properties to all ferruginous bodies: thoſe bodies, after having acquired the magnetical properties, are called *artificial magnets.*

Thoſe magnets are alſo made without the aſſiſtance of the natural magnet, as will hereafter be ſhewn.

Schol. The property of attraction in the magnet was that by which it was firſt diſcovered. Every ſubſtance that contains iron, is more or leſs attracted by the magnet; and ſo univerſally is this truly important metal diſſeminated, that there are very few ſubſtances which are not in ſome degree capable of being attracted by the magnet. In this way iron is found to enter into the compoſition of animals, vegetables, minerals, and even into that of the atmoſphere. On this ſubject, ſee Cavallo on Magnetiſm, Chap. vi. Part 1.

Def. III. Thoſe points in a magnet which ſeem to poſſeſs the greateſt power, are called the *poles of the magnet.*

Def. IV. The *magnetical meridian*, is a vertical circle in the heavens, which interſects the horizon in the points to which the magnetical needle, when at reſt, is directed.

Def. V. The *axis of a magnet*, is a right line which paſſes from one pole to the other.

Def. VI. The *equator of a magnet*, is a line perpendicular to the axis, and exactly between the two poles.

Schol. The diſtinguiſhing and characteriſtic properties of a magnet, are, (1.) Its attractive and repulſive powers. (2.) The force by which it places itſelf, when freely ſuſpended, in a certain direction towards the poles of the earth. (3.) Its dip, or inclination towards a point below the horizon. (4.) The property which it poſſeſſes of communicating the foregoing powers to iron and ſteel.

Def. VII. The direction of the *dipping* needle in any place, is called the *magnetical line.*

PROP. I. That mineral ſubſtance which is called the loadſtone, or magnet, has the property of attracting iron: but no other body whatever, unleſs it has a mixture of iron.

Exp. 1. The action of the magnet on iron, may be ſhewn on needles, ſteel filings, &c.

2. Let a needle be ſuſpended from a loadſtone, and a ſtring paſſing through its eye be faſtened to the beam of a balance placed under it; the degree of force with which it is attracted, may be meaſured.

Schol. Some philoſophers have ſuppoſed that iron is not the only ſubſtance attracted by the magnet. Mr. Kirwan ſays, that *nickel*, when purified from iron, becomes more, inſtead of leſs, magnetic, and acquires the properties of a magnet. Mr. Cavallo inſtituted a number of experiments, with a view of aſcertaining whether any other bodies than ferruginous ones, were attracted by the magnet. After all, he does not decide poſitively on the buſineſs.

PROP. II. The action, and re-action of the magnetic power, are mutual and equal.

A piece of iron, or ſteel, or other ferruginous ſubſtance, being brought within a certain diſtance of one of the poles of a magnet, is attracted by it, ſo as to adhere to the magnet, and not to ſuffer itſelf to be ſeparated without an evident effort. This attraction is alſo mutual, for the iron attracts the magnet, as much as the magnet attracts the iron; ſince if they be placed on pieces of wood or cork, ſo as to float upon the ſurface of water, it will be found that the iron advances towards the magnet, as well as the magnet advances towards the iron; or, if the iron be kept ſteady, the magnet will move towards it.

Schol. The ſtrength of magnetic attraction varies according to different circumſtances; ſuch as, the ſtrength of the magnet;—the weight and ſhape of the body preſented to it;—the magnetic, or unmagnetic ſtate of that body; the diſtance between it and the magnet, &c.

The attraction is ſtrongeſt near the ſurface of the magnet, and diminiſhes as it recedes from it; the law of this diminution has not yet been aſcertained.

The four following experiments, accurately made by Profeſſor Muſchenbroek, will exhibit ſome of the irregularities reſpecting magnetic attraction. In theſe experiments, the magnet was ſuſpended to one ſcale of an accurate balance, and under it there was ſucceſſively placed on a table at different diſtances, another magnet, or piece of iron, and at each diſtance, the degree of attraction between the iron and the magnet was aſcertained by weights put into the other ſcale. The reſults were as follow:

Exp. i. In this experiment a cylindrical magnet, weighing 16 drams, was ſuſpended to the ſcale; and on the table a piece of iron of the ſame ſhape and weight.

Diſt. in inches.		Attract. in grains.
6	—	3
5	—	$3\frac{1}{2}$
4	—	$4\frac{1}{2}$
3	—	6
2	—	9
1	—	18
0	—	57

Exp. ii. A ſpherical magnet, of the ſame diameter as the laſt, but of greater ſtrength, was affixed to the ſcale, and the cylindrical magnet uſed in the preceding experiment, was placed on the table.

Diſt. in inches.		Attract. in grains.
6	—	21
5	—	27
4	—	34
3	—	44
2	—	64
1	—	100
0	—	260

Exp. iii. Inſtead of the cylindrical magnet, the cylinder of iron was placed on the table, and under the globular magnet.

Diſt. in inches.		Attract. in grains.
6	—	7
5	—	$9\frac{1}{2}$
4	—	15
3	—	25
2	—	45
1	—	92
0	—	340

Exp. iv. A globe of iron of the ſame diameter as the magnet, was now placed on the table.

Diſt. in inches.		Attract. in grains.
8	—	1
7	—	2
6	—	$3\frac{1}{4}$
5	—	6
4	—	9
3	—	16
2	—	30
1	—	64
0	—	290

Cor. It appears from the ſecond and third experiments, that, when in contact, a magnet attracts another magnet with leſs force than it does a piece of iron. This has been confirmed by many other experiments. But the attraction between the two magnets begins from a greater diſtance than between the magnet and iron; hence it muſt follow a different law of decrement.

PROP. III. The attraction and repulſion of magnetiſm is not ſenſibly affected by the interpoſition of bodies of any ſort except thoſe which are ferruginous.

Exp. 1. Suppoſe a magnet placed at an inch diſtance from a piece of iron requires an ounce of force to remove it, or, which is the ſame thing, ſuppoſe that the attraction towards each other is equal to one ounce; it will be found that the ſame degree of attraction remains conſtantly unaltered, though a plate of other metal, glaſs, paper, &c. be interpoſed between the magnet and the iron, or though they be incloſed in ſeparate boxes of glaſs or other matter.

2. Move ſteel filings placed on a braſs plate, in water, &c. by holding a magnet under the veſſel.

3. Sprinkle ſteel duſt on a ſheet of paper, under which is placed a magnet, or two magnets, having their poles oppoſite to each other, and at the diſtance of about an inch.

4. A needle under an exhauſted receiver, will be attracted at the ſame diſtance as in the open air.

Schol. 1. Heat weakens the power of a magnet; and a white heat deſtroys it entirely. Hence it appears, from this cauſe alone, beſides others which may concur, the power of a magnet muſt be continually varying.

Schol. 2. The attractive power of a magnet may be increaſed conſiderably by gradually adding more weight to it; for it is found that a magnet will keep ſuſpended on one day a little more weight than it did the preceding; which, additional weight being added to it on the following day, it will be found that the magnet can keep ſuſpended a weight ſtill greater, and ſo on, as far as to a certain limit.

On the contrary, by putting a very ſmall weight of iron to it, the magnet may gradually loſe much of its ſtrength.

Schol. 3. Among natural magnets, the ſmalleſt generally poſſeſs a greater attractive power, in proportion to their ſize, than thoſe which are larger. There have been natural magnets not exceeding 20 or 30 grains, which would lift a piece of iron that weighed 40 or 50 times more than themſelves. A ſmall magnet, worn by Sir I. Newton in a ring, weighing but about 3 grains, is ſaid to have taken up 746 grains, or nearly 250 times its own weight. And Mr. Cavallo has ſeen one of 6 or 7 grains weight, which was capable of lifting a weight of 300 grains. But magnets of two pounds and upwards, ſeldom lift up ten times their own weight of iron.

PROP. IV. The magnetic power may be communicated from the loadſtone; and from one piece of iron to another, which then becomes an artificial magnet; and this communication of power is without apparent loſs of power in the loadſtone.

Exp. 1. Take a bar of ſoft iron, about three feet long and one inch thick, (ſome kitchen pokers will anſwer for this experiment) and place it upright, or rather in the magnetical line. Then preſent a magnetic needle to the various parts of the bar from top to bottom, and the lower half of the bar will be poſſeſſed of the north polarity, capable of repelling the north, and of attracting the ſouth pole of the needle, and the upper half is poſſeſſed of the ſouth polarity. The attraction is ſtrongeſt at the very extremities of the bar, it diminiſhes as it recedes from them, and vaniſhes about its middle point.

If the bar is turned upſide down, the ſouth pole will become north, and the north will become the ſouth pole. In the ſouthern parts of the globe, the lower part is a ſouth pole; or more generally, the extremities of the bar will acquire the polarities correſponding to the neareſt poles of the earth.

If an iron bar be left a long time in the direction of the magnetic line, or even in a perpendicular poſture, it will ſometimes acquire a great magnetic power. Tongs, pokers, &c. by being often heated, and ſet to cool again in an erect poſture, frequently acquire a conſiderable magnetic virtue. Magnetiſm is often communicated to iron and ſteel by repeated blows of the hammer: by ſome experiments of Mr. Cavallo, it appears that this effect is often produced on braſs, hence it is neceſſary carefully to examine the braſs before it is uſed in the conſtruction of theodolites, &c.

Plate 13. Fig. 1.

2. Place two magnets A and B in a right line, ſo that the north end of the one is oppoſed to the ſouth end of the other, and at ſuch a diſtance that the bar to be touched may reſt upon them. Take now two other bars, D and E, and apply the north end of D,* and the ſouth end of E to the middle of the untouched bar C, elevating their other ends ſo as to make an acute angle with the ſaid bar.

Separate the bars D and E, drawing them different ways along the ſurface of C, but preſerving the ſame elevation: then removing the bars D and E to the diſtance of a foot or more from the bar C, and bringing the north and ſouth ends into contact, and then apply them again to the middle of C. This proceſs being repeated ſeveral times to each ſurface of the bar C, it will be found to have acquired a ſtrong and permanent magnetiſm.

Schol. 1. The magnetic virtue may be readily communicated by the horſe-ſhoe magnet, much in the ſame way as in the preceding experiment.

Schol. 2. A ſmall compaſs needle may be touched by being put between the oppoſite poles of two magnetic bars: Whilſt it is receiving the magnetiſm, it will be violently agitated, moving backwards and forwards as if it were animated; and when it has received as much magnetiſm as it can acquire in this way, it becomes quieſcent.

Another method of communicating magnetiſm to a compaſs needle, is by means of the combined horſe-ſhoe magnet, from the centre of which draw that half of the needle which is to have the contrary pole; from a conſiderable diſtance draw the needle over it again. This repeated twenty times or more, and the ſame for the other half, will ſufficiently communicate the power.

3. Take twelve bars, ſix of ſoft ſteel, and ſix of hard, the former to be each three inches long, $\frac{1}{4}$ of an inch broad, $\frac{1}{20}$ of an inch thick; with two pieces of iron, each half the length of one of the bars, but of the ſame breadth and thickneſs. The 6 hard bars to be each $5\frac{1}{2}$ inches long, $\frac{1}{2}$ an inch broad, and $\frac{1}{20}$ of an inch thick, with two pieces of iron of half the length but of the ſame breadth and thickneſs of one of the hard bars; and let all the bars be marked with a line quite round them at one end: then take an iron poker and tongs, or two bars of iron, the larger they are, and the longer they have been uſed, the better; and fixing the poker upright, or rather in the magnetical line, between the knees, hold to it, near the top, one of the ſoft bars, having its marked end downwards, by a piece of ſewing ſilk, which muſt be pulled tight by the left hand that the bar may not ſlide: then graſping the tongs with the right hand, a little below the middle, and holding them nearly in a vertical poſition, let the bar be ſtroked by the lower end from the bottom to the top about 10 times on each ſide, which will give it a magnetic power ſufficient to lift a ſmall key at the marked end: which end, if the bar were ſuſpended on a point, would turn towards the north, and is therefore called the north pole; and the unmarked end, for the ſame reaſon, is called the ſouth pole. Four of the ſoft bars being impregnated after this manner, lay the other two parallel to each other, at a quarter of an inch diſtance, between the two pieces of iron belonging to them, a north and a ſouth pole againſt each piece of iron; then take two of the four bars already made magnetical, and place them together ſo as to make a

Plate 13. Fig. 2.

a double

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* Note. The north ends of magnetic bars are generally marked with a croſs, or ſtraight line, as are alſo the north ends of the horſe-ſhoe, or any other ſhaped magnets.

Plate 13. Fig. 3. a double bar in thickneſs, the north pole of one even with the ſouth pole of the other; and the remaining two being put to theſe, one on each ſide, ſo as to have two north, and two ſouth poles together, ſeparate the north from the ſouth poles at one end by the interpoſition of ſome hard ſubſtance I, and place them perpendicularly with that end downward on the middle of one of the parallel bars AC, the two north poles towards its ſouth end, and the two ſouth poles towards its north end. Slide them three or four times backward and forward the whole length of the bar; then removing them from the middle of this bar, place them on the middle of the other bar BD as before directed, and go over that in the ſame manner; then turn both the bars the other ſide upwards, and repeat the former operation: this being done, take the two from between the pieces of iron; and placing the two outer-moſt of the touching bars in their ſtead, let the other two be the outer-moſt of the four to touch theſe with; and this proceſs being repeated till each pair of bars have been touched three or four times over, which will give them a conſiderable magnetic power.

When the ſmall bars have been thus rendered magnetic, in order to communicate the magnetiſm to the large bars, lay two of them on the table, between their iron conductors, as before: then form a compound magnet with the ſix ſmall bars, placing three of them with the north poles downwards, and the three others with the ſouth poles downwards. Place theſe two parcels at an angle, as was done with four of them, the north extremity of the one parcel being put contiguous to the ſouth extremity of the other, and, with this compound magnet, ſtroke four of the large bars, one after another about twenty times on each ſide, by which means they will acquire ſome magnetic power.

When the four large bars have been ſo far rendered magnetic, the ſmall bars are laid aſide, and the large ones are ſtrengthened by themſelves, in the manner as was done with the ſmall bars.

To expedite the operation, the bars ought to be fixed in a groove, or between braſs pins, otherwiſe the attraction and friction between the bars will be continually deranging them, when placed between the conductors.

This whole proceſs may be gone through in about ½ an hour, and each of the large bars, if well hardened, will lift about 28 ounces Troy, and they are fitted for all the purpoſes of magnetiſm, in navigation and experimental philoſoſophy. The half dozen being put into a caſe in ſuch a manner as that no two poles of the ſame name may be together, and their irons with them as one bar, they will retain the virtue they have received: but if their power ſhould, by making experiments, be ever ſo much impaired, it may be reſtored without any foreign aſſiſtance in a few minutes.

This method of communicating magnetiſm was ſent to the Royal Society by Mr. Canton, in the year 1751.

PROP. V. Two magnets having a free motion, will attract when different poles are directed towards each other, and repel when the adjacent poles are of the ſame name.

Exp. A needle turning on its centre will be attracted or repelled by another, as different, or the ſame poles are brought near to each other.

Schol. 1. If the magnetic powers are very unequal, or the two bodies are forcibly brought together, they will attract with the ſame poles.

Exp. 1. Suſpend a magnet by a thread, and let a ſmall needle be brought near it at the ſame poles.

2. Bring two very unequal needles into contact at the ſame poles, ſuſpended in the ſame manner, they will cohere.

Schol. 2. The following experiments will ſhew the attraction of the magnet on ferruginous bodies which are not magnetic.

Properly ſpeaking, however, the magnet has no action upon unmagnetic bodies, for any ferruginous body becomes magnetic, on being preſented to the magnet, and then is attracted by it.

Exp. 1. Place a magnetic needle upon a pin ſtuck on a table, and when it ſtands ſteady, place an iron bar 8 inches long, and ½ an inch thick upon the table, ſo that one end of it may be on one ſide of the north pole of the needle, and near enough to draw it a little out of its natural direction. In this ſituation approach gradually the north pole of a magnet to the other extremity of the bar, and you will ſee that the needle's north end will recede from the bar, in proportion as the magnet is brought nearer to the bar.

The reaſon of this phenomenon is, that, by the approach of the north pole of the magnet, in the firſt caſe, the extremity of the iron bar next to it, acquires a ſouth polarity, and conſequently, the oppoſite extremity acquires a north polarity, by which the needle is repelled: but in the ſecond caſe, when the north pole of the magnet is brought near the bar, the end of the bar next to it acquires a ſouth polarity, and the oppoſite end acquiring the north polarity, cauſes the north end of the needle to recede.

2. Tie two pieces of ſoft iron wire, AB, AB, each to a ſeparate thread AC, which join at top, and ſuſpend them on a pin ſo that the wires may hang at ſome diſtance from the wall. Then bring the marked end D of a magnetic bar juſt under them, and it will be ſeen that the wires repel each other more or leſs in proportion to the diſtance of the magnet. The ſame may be ſhewn by means of the ſouth pole of the magnet. Plate 13. Fig. 5.

If the wires be of ſoft iron, they will, on removing the magnet, ſoon collapſe; but if ſteel wires, or two ſewing needles be uſed, they will retain their magnetic virtue, and continue to repel each other.

3. Take four pieces of ſteel wire, or four common ſewing needles, tie threads to them, and join them two and two as in the laſt experiment; then bring the ſame pole of the magnet under both pairs, by which means they will acquire a permanent magnetiſm, and the wires of each pair will repel each other. After putting the magnet aſide, bring one pair of the wires near the other pair, ſo that their lower extremities may be level, and the four wires will repel each other, and form a kind of ſquare.

4. Strew ſome iron filings upon a ſheet of paper laid on a table, and place a ſmall artificial magnet among them, then give a few gentle knocks to the table with the hand, ſo as to ſhake the filings, and they will diſpoſe themſelves round the bar in the manner repreſented

Plate 13. Fig. 6. ſented by the figure; many particles clinging to one another, and forming themſelves into lines, which at the very poles, are in the ſame direction with the axis of the magnet; a little ſideways of the poles they begin to bend, and then they form complete arches, reaching from a point in the north half of the magnet to a point in the other half, which is poſſeſſed of the ſouth polarity.

Plate 13. Fig. 7. 5. Tie a thread to one end of a bit of ſoft iron wire AB, about four inches long, and ſuſpend it freely; let a bar of ſoft iron CD be ſo ſupported, as to have one of its extremities C about $\frac{1}{4}$ of an inch diſtant from the lower extremity B of the wire. Bring now either pole of a ſtrong magnet EF under it, and the end B of the wire will recede from C, becauſe they are both poſſeſſed of the ſame polarity: but if the magnet be applied to the upper part of the wire, in the ſituation GH, then the end B of the wire will be attracted by the extremity C of the iron bar, becauſe, ſuppoſing G to be north pole of the magnet, C acquires a ſouth polarity, and attracts the end B; becauſe B being fartheſt from the north pole G, acquires alſo the north polarity.

Schol. 3. Hence methods are eaſily deviſed to aſcertain whether a body poſſeſſes any magnetiſm, and in caſe it does, to find out the poles.

Exp. 1. *To aſcertain whether a body has any attraction towards the magnet.*

If the body contain an evident quantity of iron, it will be perceived as ſoon as it is brought in contact with the magnet, as a certain force will be required to ſeparate them.

If the body be not ſenſibly attracted by the magnet in this way; let it be placed, by means of a piece of cork or wood, upon ſome water, or mercury, in a common ſoup plate, in which ſituation let a magnet be brought ſideways to it, and the attraction will be manifeſt by the body coming towards the magnet.

2. *To aſcertain whether a given body has any magnetiſm.*

The only difference in this experiment from the laſt is, that inſtead of a magnet, muſt be uſed a piece of ſoft clean iron, about one inch long, and of half an ounce weight.

3. *A magnetic body being given to find out its poles.*

Preſent the various parts of the ſurface of the magnetic body ſucceſſively to one of the poles of a magnetical needle, and the parts poſſeſſed of a contrary polarity will be diſcovered by the needle's ſtanding perpendicularly towards them. Then preſent the various parts of the ſurface of the ſame body to the other pole of the needle, &c.

Def. VIII. There is a point between the two poles where the magnet has no attraction nor repulſion, this point is called the *magnetic centre*, though it is not always exactly between the poles.

PROP. VI. If a magnet be cut through the middle, or any way broken in two, each piece will become a complete magnet, and the parts which were contiguous will become oppoſite poles.

Exp. 1. Take a magnetic bar AB, ſix or eight inches long, and $\frac{1}{4}$ of an inch thick, having only two poles A and B. The magnetic centre of this bar will be in, or very near, its middle C. Now if, by a ſmart ſtroke of a hammer, part of the magnet be broken off as FB, it will be found that the part of the fragment contiguous to the fracture has acquired the contrary polarity, and a magnetical centre E will be generated. Plate 13. Fig. 8.

At firſt the magnetic centre of this fragment is nearer to the fracture F, but in time it advances towards the middle of the fragment. The original centre C of AF, after the fracture, will likewiſe advance nearer to the middle of it.

2. A ſteel bar, of the ſame ſize as that mentioned in the laſt experiment, being made quite hard, may be broken into two parts, and ſo preſſed together as to appear whole. In this ſituation it may be rendered magnetic by the application of very powerful magnets to its extremities; and the whole bar will be found to have two poles at its extremities, and one magnetic centre in its middle; but if the parts be ſeparated, each will be found to have two poles and a magnetic centre.

Cor. Hence it is ſeen that the magnetic centre may be removed;—it may be removed alſo, by ſtriking a magnetic bar, by heating it, by hard rubbing, &c.

PROP. VII. Magnetiſm requires ſome time to penetrate through iron.

Exp. Place a bulky piece of iron, weighing 40 or 50 pounds, ſo near a magnetic needle as to draw it a little out of its direction, apply one of the poles of a ſtrong magnet to the other extremity of the iron, and ſeveral ſeconds will be required before the needle can be affected by it. The interval is greater or leſs according to the ſize of the iron and the ſtrength of the magnet.

Def. IX. A magnet is ſaid to be *armed* when its poles are ſurrounded with plates of ſteel or iron.

PROP. VIII. A magnet will take up much more iron when *armed*, than it can alone.

As both magnetic poles together attract a much greater weight than a ſingle one, and as the two poles of a magnet are generally in oppoſite parts of its ſurface, in which ſituation, the ſame piece of iron cannot be adapted to them both at the ſame time; therefore it has been common, to place two broad pieces of ſoft iron to the poles of a magnet, and projecting on one ſide, becauſe in that caſe, the pieces of iron being rendered magnetic, another piece of iron could be conveniently adapted to their projections, ſo that both poles may act at the ſame time. Thoſe pieces of iron called the *armature*, are generally held faſt upon the magnet by means of a ſilver or braſs box. Thus AB repreſents the magnet, CD, CD repreſents the *armature* or pieces of iron, the projections of which are DD, and to which the piece of iron F is made to adhere. The dotted lines repreſent the braſs box having a ring E at top by which the armed magnet may be ſuſpended. In this manner the two poles of the magnet, which are at A and B, are made to act at DD. Plate 13. Fig. 4.

For this purpoſe, and to avoid armature, artificial magnets have been conſtructed in the ſhape of a horſe-ſhoe, having their poles in the truncated extremities.

PROP. IX. A magnetical needle, accurately balanced on a pivot or centre, will ſettle in a certain direction, either duly, or nearly north and ſouth, called the magnetical meridian.

This is known by long experience.

The directive power of the magnet is the moſt wonderful and uſeful part of the ſubject. By it mariners are enabled to conduct their veſſels through vaſt oceans in any given direction; by it miners are guided in their works below the ſurface of the earth; and travellers conducted through deſerts, otherwiſe impaſſable.

The uſual method is to have an artificial magnet ſuſpended, ſo as to move freely, which will always place itſelf in or near the plane of the meridian north and ſouth: then by looking upon the direction of this magnet the courſe is to be directed ſo as to make any required angle with it. Thus, ſuppoſe that a veſſel ſetting off from any place in order to go to another which is due weſt of the former; in that caſe, the veſſel muſt be ſo directed that its courſe may be always at right angles with the ſituation of the magnetic needle, the north end of which muſt be to the right hand. A little reflection will ſhew how the veſſel may be ſteered in any other direction. An artificial ſteel magnet fitted for this purpoſe in a proper box, is called the *mariner's compaſs*, or *ſea compaſs*, or, ſimply, the *compaſs*; which inſtrument is too well known to need any particular deſcription. The mariner's compaſs, with the addition of ſights, divided circles, &c. for obſerving azimuths, and amplitudes of the heavenly bodies, is called the *azimuth compaſs*.

DEF. X. The deviation of the horizontal needle from the meridian; or the angle which it makes with the meridian, when freely ſuſpended in an horizontal plane, is called the *declination or variation of the needle*.

PROP. X. There is generally a ſmall variation in the direction of the magnetic needle, which differs in degree at different places and times.

This is known by obſerving the different points of the compaſs at which the ſun riſes or ſets, and comparing them with the true points of the ſun's riſing or ſetting, according to aſtronomical tables. Thus, if the magnetic *amplitude* is 80°. eaſtward of the north, and the true *amplitude* is 82°. towards the ſame ſide, then the variation of the needle is 2°. weſt. The variation may be eſtimated from the azimuths in the ſame way.

SCHOL. 1. A needle is continually changing the line of its direction, traverſing ſlowly to certain limits towards the eaſt and weſt. The firſt good obſervations, on the variations, were made by Mr. Burrowes about the year 1580, when the variation, at London, was 11°. 15′. eaſt, and ſince that time the needle has been moving to the weſtward at that place; alſo by the obſervations of different perſons it has been found to point, at different times, as in the following table.

Years,

Years.	*Obſervers.*	*Variation E. or W.* ° ′
1580	Burrowes.	11 15 Eaſt.
1622	Gunter.	5 56
1634	Gellibrand.	4 3
1640	Bond.	3 7
1657	Bond.	0 0
1665	Bond.	1 23 Weſt.
1666	Bond.	1 36
1672	——	2 30
1683	——	4 30
1692	——	6 00
1723	Graham.	14 17 Weſt.
1747	————	17 40
1774	Royal Soc.	21 16
1775	Royal Soc.	21 43
1776	Royal Soc.	21 47
1777	Royal Soc.	22 12
1778	Royal Soc.	22 20
1779	Royal Soc.	22 28
1780	Royal Soc.	22 41

By this table it appears, that from the firſt obſervation in 1580 till 1657, the change in the variation at London was 11°. 15′. in 77 years, which, at a mean rate, is nearly 9′. a year. And from 1657 to 1780, it changed 22°. 41′. which is at the rate of 11′. a year nearly.

			°	′
At Paris the Variation of the Needle was	in	1550	8	0 Eaſt.
		1640	3	0
		1660	0	0
		1681	2	2 Weſt.
		1759	18	10
		1760	18	20

			°	′
At St. Helena the Variation of the Needle was	in	1600	8	0 Eaſt.
		1623	6	0
		1677	0	40
		1692	1	0 Weſt.

Near the equator, in long. 40°. eaſt, the higheſt variation from the year 1700 to 1756, was 17°. 15′. weſt; and the leaſt 16°. 30′. W. In lat. 15°. N. and long. 60°. W. the variation was conſtantly 5°. E. In lat. 10°. S. and long. 60°. E. the variation decreaſed from 17°. W. to 7°. 15′. W. In lat. 10°. S. and long. 5°. W. it increaſed from 2°. 15′. to 12°. 45′. W. In lat. 15°. N. and long. 20°. W. it increaſed from 1°. W. to 9°. W. In the Indian ſeas the irregularities were greater, for in 1700, the weſt variations ſeem to have decreaſed regularly from long. 50°. E. to long. 100°. E.; but in 1756, the variation decreaſed ſo faſt, that there was eaſt variation in long. 80°. 85°. and 90°. E. and yet, in long. 95°. and 100°. E. there was weſt variation.

In the year 1775, in lat. 58°. 17′. S. and long. 348°. 16′. E. it was 0°. 16′. W. In lat. 2°. 24′. N. and long. 32°. 12′. W. it was 0°. 14′. 45″. W. In lat. 50°. 6′. 30″. N. and long. 4°. 0′. W. it was 19°. 28′. W.

Schol. 2. The variation of the needle is affected by heat and cold. The following is the reſult of obſervations made by Mr. Canton at different hours of the day, and alſo the mean variation for each month in the year.

The Declination obſerved at different Hours of the ſame day.

June 27, 1759.

	Hours.	Min.	Declin.	Weſt.	Degrees of the Thermometer.
Morning	0	18	12°	2′	62°
	6	4	18	58	62
	8	30	18	55	65
	9	2	18	54	67
	10	20	18	57	69
	11	40	19	4	$68\frac{1}{2}$
Afternoon	0	50	19	9	70
	1	38	19	8	70
	3	10	19	8	68
	7	20	18	59	61
	9	12	19	6	59
	11	40	18	51	$57\frac{1}{2}$

The mean Variation for each Month in the Year.

	′	″
January - -	7	8
February - -	8	58
March - -	11	17
April - -	12	26
May - -	13	0
June - -	13	21
July - -	13	14
Auguſt - -	12	19
September - -	11	43
October - -	10	36
November - -	8	9
December - -	6	58

PROP. XI. A needle which, before it receives the magnetic power, reſts on its centre parallel to the horizon, on becoming magnetical will incline towards the earth: this is called the inclination or dip of the needle.

Exp. Let a ſmall dipping needle be carried from one end of a magnetic bar to the other; when it ſtands over the ſouth pole, the north end of the needle will be directed perpendicularly to it; as the needle is moved, the dip will grow leſs, and when it comes to the magnetic centre it will be parallel to the bar; afterwards the ſouth end will dip, and the needle will ſtand perpendicular to the bar when it is directly over the north pole.

Schol. 1. This property of the magnetic needle was diſcovered accidentally by Robert Norman, a compaſs maker at Radcliffe, about the year 1576. He relates, that it being his cuſtom to finiſh and hang up the needles of his compaſſes, before he touched them, he found that immediately after the touch, the north point would always dip or incline downward, pointing in a direction under the horizon; ſo that, to balance the needle again he was forced to put a piece of wax on the ſouth end as a counterpoiſe. The conſtancy of the effect led him to meaſure the angle which the needle would make with the horizon, and he found it at London to be 71°. 50′.

It is not yet abſolutely aſcertained whether the dip varies at the ſame place; it is now, and has been ſince the year 1772, about 72°. according to ſeveral obſervations made by Mr. Nairne and the Royal Society. The trifling difference between the firſt obſervations of Mr. Norman, and theſe laſt of Mr. Nairne, &c. leads us to ſuppoſe that the dip is unalterable at the ſame place, It is certain, however, that the dip is different in different latitudes, and that it increaſes in going northward. It appears from a table of obſervations made with a marine dipping needle of Mr. Nairne's, in a voyage towards the north pole in 1773, that

° ′ ° ′
In latitude 60 18, the dip was 75 0.
In latitude 70 45, the dip was 77 52.
In latitude 80 12, the dip was 81 52.
In latitude 80 27, the dip was 82 2½.

See Phil. Tranſ. Vol. LXV.

Schol. 2. The phenomena of the compaſs, and the dipping needle, and of the magnetiſm acquired by an iron bar in a vertical poſition, leave no room to doubt but that the *cauſe* exiſts in the earth. Dr. Halley ſuppoſed that the earth has within it a large magnetic globe, not fixed within to the external parts, having four magnetic poles, two fixed and two moveable, which will account for all the phenomena of the compaſs and dipping needle. This would make the variation ſubject to a conſtant law, whereas we find caſual changes which cannot be accounted for upon this hypotheſis. This the doctor ſuppoſes may ariſe from an unequal and irregular diſtribution of the magnetical matter. The irregular diſtribution alſo of ferruginous matter in the ſhell may likewiſe cauſe ſome irregularities.

Mr. Cavallo's opinion is, that the magnetiſm of the earth ariſes from the magnetic ſubſtances therein contained, and that the magnetic poles may be conſidered as the centres of the polarities of all the particular aggregates of the magnetic ſubſtances; and as theſe ſubſtances are ſubject to change, the poles will change. Perhaps it may not be eaſy to conceive how theſe ſubſtances can have changed ſo materially, as to have cauſed ſo great a variation in the poles, the poſition of the compaſs having changed from the eaſt towards the weſt about 33°. in 200 years. Alſo the gradual, though not exactly regular change of variation ſhews, that it cannot depend upon the accidental changes which may take place in the matter of the earth.

Mr. Churchman of America, ſays, there are two magnetic poles of the earth, one to the north, and the other to the ſouth, at different diſtances from the poles of the earth, and revolving in different times; and from the combined influence of theſe two poles, he deduces rules for the poſition of the needle in all places of the earth, and at all times, paſt, preſent, or to come.

The north magnetic pole, he ſays, makes a complete revolution in 426 years, 77 days, 9 hours, and the ſouth pole in about 5459 years. In the beginning of the year 1777, the north magnetic pole was in 76°. 4′. north latitude; and in longitude from Greenwich 140°. eaſt; and the ſouth was in 72°. ſouth latitude, 140°. eaſt from Greenwich.

BOOK V.

OF ELECTRICITY.

Def. I. THE earth, and all bodies with which we are acquainted, are ſuppoſed to contain a certain quantity of an exceedingly elaſtic fluid, which is called the *electric fluid.*

Schol. This certain quantity belonging to all bodies, may be called their natural ſhare; and ſo long as each body contains neither more nor leſs than this quantity, it ſeems to lie dormant, and to produce no effect.

Def. II. When any body becomes poſſeſſed of more, or leſs than its natural quantity, it is ſaid to be *electrified*, and is capable of exhibiting appearances which are aſcribed to the power of *electricity.*

Schol. This equilibrium could never be diſturbed, or, if it was diſturbed, would be immediately reſtored, and therefore be inſenſible; but that ſome bodies do not admit the paſſage of the electric fluid through their pores, or along their ſurfaces, though others do.

Def. III. When a body has acquired an additional quantity of electric matter; or loſt a part of what naturally belonged to it, and it is at the ſame time ſurrounded by bodies through which it cannot paſs, it muſt remain in that ſtate, and is ſaid to be *inſulated.*

PROPOSITION I.

The Electric Fluid, being excited, becomes perceptible to the ſenſes.

Exp. 1. Let a long glaſs tube be rubbed with the hand, or with a leathern cuſhion; the electric fluid, being thus excited, will attract light ſubſtances, and give a lucid ſpark to the finger, or any metallic ſubſtance, brought near it.

The glaſs tube is called the *electric*, and all thoſe bodies which are capable, by any means, to produce ſuch effects, are called *electrics*. The hand, or any other body that rubs an electric, is called the *rubber*.

2. As the exciting a tube is very laborious for the operator, and the electricity procured by that means is ſmall in quantity; globes and cylinders are uſed for this purpoſe. Theſe,

by a proper apparatus, are made to revolve on their axes, and a rubber of leather is applied to the equatorial parts of the revolving glaſs, which become electrical by the friction. The electricity of the globe, or cylinder, is received by a metallic conductor inſulated on a glaſs ſupporter.

A cylinder or globe thus fitted up is called an electrical machine. C repreſents a glaſs cylinder about one foot in diameter and 20 inches long, which is turned by means of a wheel; the rubber or cuſhion is ſupported behind the cylinder by two upright ſprings that appear beneath, and are faſtened to two croſs bars of glaſs. B is a metallic conductor, ſupported on two pillars of glaſs: from the end neareſt to the cylinder iſſue ſeveral points, and at the other end the ball E projects by means of a wire. Sparks given by the conductor of a machine of this conſtruction and magnitude are from 12 to 14 inches long. A chain D muſt connect the rubber with the earth. Plate 13. Fig. 9.

Schol. 1. In all experiments in electricity the greateſt care ſhould be taken to keep every part of the apparatus clean, and as free as poſſible from duſt and moiſture. When the weather is clear, and the air dry, eſpecially in clear froſty weather, the electrical machine will always work well. But in very hot, or damp weather, the machine is not ſo powerful.

Before the machine is uſed the cylinder ſhould be wiped firſt very clean with a ſoft linen cloth; and afterwards with a clean hot flannel, or old ſilk handkerchief.

Sometimes it will be neceſſary to apply to the rubber a very ſmall quantity of amalgam made with one part of zinc, and four or five of mercury.

Schol. 2. Reſpecting the theory of electricity, there are two different hypotheſes, one that there is only one fluid, and the other that there are two. Dr. Franklin's hypotheſis is the former, and it depends on the following principles. (1) That all terreſtrial bodies are full of the electric fluid. (2) That the electric fluid violently repels itſelf, and attracts all other matter. (3) By exciting an electric the equilibrium of the electric fluid contained in it is deſtroyed, and one part contains more than its natural quantity, and the other leſs. (4) Conducting bodies, connected with that part which contains more electric fluid than its natural quantity, receive it, and are charged with more than their natural quantity; this is called *poſitive* electricity; if they be connected with that part which has leſs than its natural quantity, they part with ſome of their own, and contain leſs than their natural quantity; this is called negative electricity. (5) When one body poſitively and another negatively electrified are connected by any conducting ſubſtance, the fluid in the body which is poſitively electrified ruſhes to that which is negatively electrified, and the equilibrium is reſtored. Theſe are the principles of *poſitive* and *negative* electricity. The other hypotheſis is, that there are two diſtinct fluids, which was ſuggeſted by M. Du Faye, upon his diſcovery of the different properties of excited glaſs, and excited reſins, ſealing-wax, &c. The following are the principles of this theory. (1) That the two powers ariſe from two different fluids which exiſt together in all bodies. (2) That theſe fluids are ſeparated in non-electrics, by the excitation of electrics, and from thence they become evident to the ſenſes, they deſtroying each other's effects when united. (3) When ſeparated they ruſh together again with great violence, in conſequence of their ſtrong mutual attraction as ſoon as they are connected by any conducting ſubſtance. Theſe are the principles of *vitreous* and *reſinous* electricity.

PROP. II. The electric fluid paſſes eaſily along the ſurfaces of ſome bodies; whilſt other bodies do not convey it: the former are called *Conductors*, the latter *Non-conductors*, or *Electrics.*

Exp. The metallic cylinder being fixed upon glaſs ſupporters, and placed near the electric machine, will, by means of pointed wires, receive the electric fluid from the glaſs cylinder, and the fluid will be diffuſed over the whole ſurface of the metallic cylinder, from whence it cannot paſs through the glaſs ſupporters which are electric, but may be conveyed away by any metallic or other conducting ſubſtances, brought near, or into contact with it. This metallic cylinder is called the *Prime Conductor*, or the *Conductor.*

PROP. III. Some conductors are more perfect than others; and the electric fluid paſſes through that which is moſt perfect.

Exp. The fluid will paſs through a wire held in the hand.

Schol. 1. The following bodies are conductors and electrics, diſpoſed in the order of their degrees of perfection: Conductors; gold, ſilver, copper, braſs, iron, tin, quickſilver, lead, the ſemi-metals, ores, charcoals, water, ice, ſnow, ſalts, ſoft ſtones, ſmoke, ſteam: Non-Conductors, or Electrics; glaſs, and all vitrifications, even thoſe of metals; precious ſtones, reſins, gums, amber, ſulphur, baked wood, bituminous ſubſtances, wax, ſilk, cotton, feathers, wool, hair, paper, air, oil, hard ſtones. Many electrics become conductors, when heated, and all when moiſtened.

Schol. 2. Glaſs veſſels, made for electrical purpoſes, are often rendered very good electrics, by uſe and time, though they might be very bad ones when new. And ſome glaſs veſſels, which had been long uſed for excitation, have ſometimes loſt their power almoſt entirely. Dr. Prieſtley mentions ſeveral inſtances of very long tubes which, when firſt made, anſwered the purpoſes of electricity admirably, but after a few months they have become almoſt uſeleſs.

Schol. 3. An exhauſted glaſs veſſel on being rubbed ſhews no ſigns of electricity upon its external ſurface. But the electric power of a glaſs cylinder is the ſtrongeſt when the air within is a little rarefied. If the air be condenſed, or the cylinder be filled with ſome conducting ſubſtance, it is incapable of being excited. Nevertheleſs a ſolid ſtick of glaſs, ſealing-wax, ſulphur, &c. may be excited.

Schol. 4. The ſame ſubſtance, by different preparations, is ſometimes a conductor, and at others an electric. A piece of wood juſt cut from a tree is a good conductor;—let it be baked, and it becomes an electric;—burn it to charcoal, and it is a good conductor again;—laſtly, let this coal be reduced to aſhes, and theſe will be impervious to electricity. Such changes are alſo obſervable in many other bodies; and very likely in all ſubſtances there is a gradation from the beſt conductors to the beſt non-conductors of electricity.

PROP. IV. Non-conductors retain the fluid on a ſmall part of their ſurface where the friction has acted; conductors diffuſe it over all their ſurface, and therefore cannot confine it, unleſs they be ſurrounded entirely by non-conductors, or be *inſulated.*

EXP. Obſerve the partial diſtribution of the fluid on an excited electric, and its univerſal diffuſion over a conductor. If a finger, or any other conductor, be preſented to an excited glaſs, cylinder, tube, &c. it will receive a ſpark, and in that ſpark, a ſmall part only of the electricity of the electric: becauſe the excited electric being a non-conductor, cannot convey the electricity of all its ſurface to that point to which the conductor has been preſented. But if any conducting ſubſtance be brought to a charged metallic conductor, it will receive in one ſpark nearly the whole of the electricity accumulated upon it. The ſmall part which remains is very trifling in compariſon of the firſt ſpark, and is called the *reſiduum.*

DEF. IV. A body is ſaid to be *poſitively electrified*, when it has thrown upon it a greater quantity of the electric fluid than its natural ſhare.

DEF. V. A body is ſaid to be *negatively electrified* when it has a leſs quantity of the electric matter than is natural to it.

PROP. V. The electric fluid may be excited by rubbing, by pouring a melted electric into another ſubſtance, by heating and cooling, and by evaporation.

EXP. 1. In working the electrical machine, the fluid is excited by friction. Rubbing is the general mean by which all electric ſubſtances that are at all excitable may be excited. Whether they be rubbed with electrics of a different ſort, or conductors, they always ſhew ſigns of electricity, and in general ſtronger when rubbed with conductors, and weaker when rubbed with electrics.

2. When ſulphur is melted into an earthen veſſel, if the veſſel be ſupported by a conducting ſubſtance, the ſulphur, when cold and ſeparated from the veſſel, is ſtrongly electrical, and will attract light bodies.

3. If ſulphur be melted into glaſs veſſels, when cold, the glaſs, whether ſupported by electrics or not, will be poſitively electrical, and the ſulphur negative.

4. Melted ſealing-wax, when poured into ſulphur, becomes poſitively electrified, and the ſulphur negative.

5. Melted ſealing-wax poured into glaſs cups acquires a negative electricity: upon being ſeparated the glaſs is poſitive.

6. Sulphur melted into cups, ſhews no ſigns of electricity till it is ſeparated from the cup, when the cup is negative and the ſulphur is poſitive.

7. If a ſtick of ſealing-wax be broken into two pieces, the extremities that were contiguous will be found electrified, one poſitively, and the other negatively.

8. The tourmalin, a ſtone which is generally of a deep red, or purple colour, about the ſize of a walnut, and found in the Eaſt Indies, while kept in the ſame degree of heat, ſhews no ſigns of electricity, but will become electrical by increaſing or diminiſhing its heat, and ſtronger in the latter than in the former caſe. (1) Its electricity does not appear all over its ſurface, but only on two oppoſite ſides, which may be called its poles, and they are always in one right line with the centre of the ſtone, and in the direction of the ſtrata; in which direction the ſtone is abſolutely opaque, though on the other ſide it is ſemitranſparent. (2) Whilſt the tourmalin is heating, one of its ſides (called A) is electrified plus; the other (called B) minus. But when it is cooling A is minus, and B is plus. (3) If this ſtone be excited by friction, then both its ſides at once may be made poſitive. (4) If a tourmalin be cut into ſeveral parts, each piece will have its poſitive and negative poles, correſponding to the poſitive and negative ſides of the ſtone from which it was cut.

Schol. Theſe properties are now found to belong to ſeveral hard and precious ſtones as well as to the tourmalin.

9. Electricity may be produced by the evaporation of water in this manner:—Upon an inſulating ſtand, as a wine glaſs, place an earthen veſſel, as a crucible, a baſin, &c. and put into it three or four lighted coals. Let a wire be put with one end among the coals, and with the other let it touch a very ſenſible electromer. Then pour in a ſpoonful of water at once upon the coals, which will occaſion a quick evaporation; and at the ſame time the electromer will diverge. For a deſcription of the electromer, ſee Prop. XII. Schol.

PROP. VI. The electric fluid may be lodged in electrics, or in inſulated conductors, in a greater quantity than naturally belongs to them, or they may be *poſitively* electrified.

Exp. In working the machine, the cylinder acquires more than its natural quantity of fluid by excitation, the conductor, by communication: for, while there is a free conveyance of fluid from the earth to the rubber, by means of a conducting ſupporter, the conductor will be highly electrified.

Schol. The electric matter with which the prime conductor is loaded, is not *produced* by the friction of the cylinder againſt the rubber. It is only *collected* by that operation from the rubber, and all the bodies that are contiguous to it. If, therefore, the rubber be well inſulated, the friction of the cylinder will produce but little electricity: for in that caſe the rubber can only part with its own ſhare, which is very inconſiderable. In this ſituation, if the finger be preſented to the rubber, ſparks will be ſeen to dart from it to the rubber, to ſupply the place of that electric matter which had paſſed from it to the cylinder: if the conductor be alſo inſulated, theſe ſparks will ceaſe as ſoon as it is fully loaded.

PROP. VII. The electric fluid being accumulated on any body, will paſs to any conductor brought near to the body: if it paſs from, or be received by, pointed wires, it will be conveyed in a continued ſtream; if it paſs from, or be received by, a ſurface which has no ſharp points, it will be diſcharged with an inſtantaneous exploſion or ſpark.

Exp. 1. Receive the fluid from the conductor upon a pointed wire, and upon a braſs ball.

2. The fluid will be diffuſed through the ſurrounding atmoſphere, by wires placed upon the conductor.

Cor. Hence ariſes the neceſſity of keeping the whole ſurface of the conductor free from points.

Schol. When a conductor is electrified by communication, its whole electric power is diſcharged at once, on the near approach of a conductor communicating with the earth; whereas an excited electric, in the ſame circumſtances, loſes its electric power only in the parts near to the conductor.

PROP. VIII. If conductors be inſulated, they will retain a greater or leſs quantity of the electric fluid (the power of the machine being given) proportional to the extent of ſurface in the conductor.

Exp. Obſerve the difference in the magnitude and diſtance of ſparks taken from a ſmall conductor, and of thoſe taken from a large one.

There is a limit, beyond which this Propoſition will not hold true, but which experiment has not yet aſcertained. For it is certain, that if the conductor be very long, it will diſcharge itſelf over the cylinder back to the rubber long before it is fully charged. The late Mr. G. C. Morgan, whoſe memory will be ever dear to the editor of this work, aſſerts, that by the moſt powerful excitation of a cylinder 14 inches in diameter, the ſpark afforded by a conductor 8 inches in diameter, and 12 feet long, did not equal half the length of that procured from the ſame cylinder with a conductor of equal diameter but ſhortened to 6 feet. And he thinks that a conductor of half that length even, and about 16 inches in diameter, would have yielded a longer ſpark than either of the preceding. See Morgan's Lect. on Elect. Vol. I. p. 54, &c.

PROP. IX. A body may be deprived of part of its natural portion of electric fluid, or be *negatively* electrified.

Exp. If the rubber which communicates the fluid to the glaſs cylinder, and from thence to the conductor be inſulated, becauſe by working the machine a quantity of its fluid is conveyed away, and it cannot receive a freſh ſupply through its ſupporter, it will be in an exhauſted or negative ſtate.

Schol. If negative electricity be required, then the chain which connects the rubber with ſurrounding objects, and conſequently with the earth, the great reſervoir of the electric fluid, muſt be removed from the inſulated rubber, and hung to the prime conductor; for in this caſe the electricity of the conductor will be communicated to the ground, and the rubber will appear ſtrongly negative. Another conductor may be connected with the inſulated rubber, and then as ſtrong negative electricity may be obtained from this as poſitive can be in the caſe before mentioned.

The patent machine of Mr. Nairne is admirably adapted for the purpoſes both of poſitive and negative electricity.

PROP. X. When bodies are negatively electrified they receive the fluid from other bodies brought near them.

Exp. 1. Let two inſulated conductors, one of which is connected with the glaſs cylinder, the other with the rubber, be electrified; whilſt they are in this ſtate let them be brought near each other; a ſpark will paſs from that which (by Prop. VI.) is poſitively, to that which (by Prop. IX.) is negatively, electrified.

2. Let two perſons ſtanding on glaſs feet be electrified, firſt both poſitively, or both negatively, they will not, on contact, communicate the fluid to each other; but let them be electrified, the one poſitively and the other negatively, by making a communication from one to the conductor, and from the other to the rubber, on contact, the former will give, and the latter receive a ſpark.

PROP. XI. From a pointed body poſitively electrified the fluid will be ſeen to ſtream out, towards any electrified body brought near it, in a conical pencil of rays; whereas in paſſing from the unelectrified body to a pointed body negatively electrified, it will form a globular flame, or ſtar, about its point.

Exp. 1. Obſerve, in a dark room, the different appearances of the electric fluid at the extremity of a pointed wire, when the point is preſented to an inſulated conductor poſitively, and when it is preſented to one negatively, electrified; or when ſuch a wire is fixed upon a conductor poſitively or negatively electrified.

2. Within a *luminous conductor* electrified poſitively, (viewed in a dark room) the fluid will be ſeen paſſing in the form of a pencil from one wire, and received in the form of a ſtar upon the other; and the reverſe if it be electrified negatively.

PROP. XII. If two bodies be electrified, both poſitively, or both negatively, they repel each other; but if one be electrified poſitively, and the other be negatively or not at all electrified, they attract each other.

Exp. 1. Light feathers, or hair, connected with the conductor, appear repellent, but are attracted by bringing any non-electrified body near them.

2. The hair of a perſon electrified becomes repellent.

3. In the graduated electrometer the ball is repelled according to the degree in which the conductor is electrified.

4. Downy feathers, paper figures, threads of flax, thiſtle down, gold leaf, braſs duſt, or other light bodies, brought near to the conductor, are alternately attracted and repelled. This will not take place if the bodies be laid on a plate of glaſs.

5. Two bells being ſuſpended by wires from a braſs rod connected with the conductor, and a third by a ſilk cord, and two ſmall bells of braſs ſuſpended by a ſilken thread between the bells, the fluid will be communicated from the conductor to the outer bells, and by the balls to the middle bell, and from thence conveyed by a chain to the earth: the balls in receiving and communicating the fluid are attracted and repelled ſucceſſively, and produce ringing.

6. Let water flow from a capillary tube, from which, before it is electrified, it paſſes in drops; upon being electrified, the particles of fluid will be ſeparated, and their motion accelerated.

Theſe appearances will be preſented, whether the conductor be poſitively or negatively electrified.

7. Mr. Symmer, in the year 1759, preſented to the Royal Society ſome papers upon the electricity of ſilk ſtockings. He had been accuſtomed to wear two pair of ſilk ſtockings, a white pair under black ones. When theſe were pulled off together, no ſigns of electricity appeared, but on pulling off the black ones from the white, he heard a ſnapping noiſe, and in the dark perceived ſparks of fire. On this ſubject he has related a number of very curious experiments on the attraction and repulſion of the ſtockings, and upon their different ſtates of electricity.

Cor. Since it is found that rubbed glaſs electrifies any inſulated conductor poſitively, it may be determined whether any body is electrified poſitively or negatively, by bringing it near to a pith-ball, or down-feather, poſitively electrified, and obſerving whether the ball or feather be attracted or repelled by the body.

Exp. Bring a pith-ball or down-feather, ſuſpended by a ſilken thread and poſitively electrified by any rubbed glaſs ſurface, near to another pith-ball or feather ſuſpended by a flaxen thread from a conductor connected with the cylinder; then bring the ſame near to a conductor connected with the rubber.

Schol. The electrometer is an inſtrument invented to meaſure the degree of electrification of any body. Small degrees of electricity are ſhewn by the divergence of two very ſmall pith-balls, *a*, *b*, ſuſpended upon parallel threads, ſtraws, &c. Theſe balls preſented to a body in its natural ſtate will not be affected; but if the body be electrified, they will be attracted by it, and diverge. Plate 3. Fig. 10.

Another very uſeful and common electrometer conſiſts of an upright ſtick, AB, to which is affixed a graduated ſemicircle: D is a pith-ball ſtuck upon the end of a fine ſtraw, which by means of an axis at C, is moveable in a plane parallel to that of the ſemicircle. This electrometer is fixed upright on a prime conductor; and when it is not electrified, the radius will hang down, and according to the intenſity of the electric ſtate given to the conductor, the repulſion muſt cauſe the ball to aſcend. The aſcent will be marked by the graduations. Plate 13. Fig. 11.

Plate 13. Fig. 12. Mr. Cavallo has invented a very ſenſible electrometer, well adapted for the obſervation of the preſence and quality of natural and artificial electricity. ABC is the braſs caſe containing the inſtrument. When the part AB is unſcrewed, and the electrometer taken out, it appears as repreſented in ABDC. A glaſs tube, CDNM, is cemented into the piece AB. The upper part of the tube is ſhaped tapering to a ſmall extremity, which is entirely covered with ſealing-wax. Into this tapering part, a ſmall tube of glaſs is cemented, the lower extremity being alſo covered with ſealing-wax, projects a ſmall way within the tube CDNM. Into this ſmaller tube, a wire is cemented, which, with its under extremity, touches a flat piece of ivory H, faſtened to the tube by means of a cork. The upper extremity of the wire projects about a quarter of an inch above the tube, and ſcrews into the braſs cap EF, which cap is open at the bottom, and ſerves to defend the waxed part of the inſtrument from the rain. From H are hung two fine ſilver wires, having very ſmall corks at the lower ends, which, by their repulſion, ſhew the electricity. IM, and KN, are two ſlips of tin-foil ſtuck to the inſide of the glaſs, and communicating with the braſs bottom AB. They ſerve to convey away that electricity, which, when the corks touch the glaſs, is communicated to it, and might diſturb their free motion.

When this inſtrument is uſed to obſerve artificial electricity, it is ſet on a table, and electrified by touching the braſs cap EF with an electrified body; in this ſtate, if any electrified ſubſtance is brought near the cap, the corks of the electrometer, by their converging, or diverging more, will ſhew the ſpecies of electricity.

When it is to be uſed to try the electricity of fogs, &c. it muſt be unſcrewed from its caſe, and held a little above the head by the bottom AB, ſo that the obſerver may conveniently ſee the corks, which will immediately diverge if there is any ſufficient quantity of electricity in the air, the nature of which may be aſcertained by bringing an excited piece of ſealing-wax towards the braſs cap EF.

PROP. XIII. From the ſharp points of electrified bodies there proceeds a current of air.

Exp. 1. A wire, with ſharp points bended in oppoſite directions, and ſuſpended on the point of a perpendicular wire inſerted in the conductor, will be carried round by the current proceeding from the points.

2. Let ſeveral pieces of gilt paper be ſtuck like vanes into the ſide of a cork, through the centre of which a needle paſſes; ſuſpend the whole by a magnet, and preſent one of the vanes to the point of a wire inſerted in the conductor; they will be put into motion.

PROP. XIV. Some bodies upon being rubbed, are electrified poſitively, and others negatively; and the ſame bodies are capable of being electrified poſitively, or negatively, as they are rubbed with different ſubſtances.

Exp. Smooth glaſs becomes poſitively electrified by being rubbed with any ſubſtance hitherto tried, except the back of a living cat; rough glaſs becomes poſitively electrified by being rubbed with dry oiled ſilk, ſulphur, and metals; negatively, with woollen cloth, ſealing-wax, paper, the human hand. White ſilk becomes poſitively electrified by being rubbed with black ſilk, metals, black cloth; negatively, with paper, hairs, the hand. Black ſilk will be poſitively electrified with red ſealing-wax; negatively, with hare's ſkin, metals, the hand. Sealing-wax will be poſitively electrified with the hand, leather, woollen cloth, paper, hare's ſkin. Baked wood will be poſitively electrified with ſilk; negatively, with flannel. If theſe and other ſubſtances, being electrified, be brought near to a pith-ball or down-feather, as deſcribed Prop. XII. Cor. Exp. it will appear whether they are electrified poſitively or negatively.

PROP. XV. Bodies inſulated, if placed within the influence of an electrified body, will be electrified, at the part adjacent to that body, in the manner contrary to that of the electrified body.

Exp. 1. Bring a conductor (without pointed wires) near to the glaſs cylinder, whilſt the machine is working; if the conductor be not inſulated, it will be negatively electrified till it is brought ſo near as to receive ſparks from the cylinder; if the conductor be inſulated, it will, in the ſame ſituation, be electrified negatively, in the parts neareſt the cylinder, and poſitively in the parts more remote; as may be ſeen by bringing an excited glaſs tube (which is poſitively electrified) near to a ball ſuſpended from the conductor. Compare Prop. XII. Cor.

2. Let two pith-balls be ſo ſuſpended by flaxen threads as to be in contact when unelectrified; on being brought near to a body electrified poſitively, they will repel each other, being electrified negatively: if the balls be ſuſpended in the ſame manner by ſilken threads, they will, in the ſame ſituation, be poſitively electrified.

Plate 13. Fig. 13.

3. Let PC be an electrified prime conductor, and AB a metallic body placed within its atmoſphere, but beyond the ſtriking diſtance. Now from the principles already explained, it is evident that the electrical atmoſphere of the prime conductor muſt be *poſitive* or *negative*. (1.) If it be *poſitive*, then the adjacent part A of the metallic body AB, will be found to be electrified negatively; the remote part B, will be electrified poſitively; and there will be a certain point D, in its natural ſtate, or not electrified at all. (2.) If the prime conductor be charged with negative electricity, then A will be poſitive, B negative, and ſtill ſome point, as D, will be found unelectrified; which is called the *neutral* point.

Earl Stanhope has demonſtrated, by a conſiderable number of experiments, that the neutral point D, is the fourth point of an harmonical diviſion of the line CAB. Conſequently, the points C, A, and B, being given, the neutral point D, may be always found. For by the proportion aſſumed by his lordſhip, as the whole line BC is to the part CA, ſo is the remote end BD to the middle term DA; therefore, by compoſition, BC + CA (BA + 2AC) : CA : : BD + DA (BA) : AD. Thus, if BA be 40 inches, and CA 36, then AD is equal to 12$\frac{6}{7}$ inches.

Cor. 1. From the nature of this propoſition, it is evident, that the neutral point D can never be farther from A than half the diſtance between A and B, ſuppoſing the electrified conductor PC to be removed to an infinite diſtance.

Cor. 2. It is likewiſe evident, that the evaneſcent poſition of the neutral point D muſt be A, when the end A of the metallic body AB comes into contact with the charged body PC.

Schol. From the above conſiderations, Lord Stanhope has, with great ingenuity, proved by an elaborate mathematical demonſtration, illuſtrated and confirmed by a great variety of experiments, that the denſity of an electrical atmoſphere ſuperinduced upon any body muſt be inverſely as the ſquare of the diſtance from the charged body.

4. Let a circular plate compoſed of roſin and ſulphur, or of ſealing-wax, be negatively electrified by rubbing it with flannel; whilſt it is in this ſtate, let a metallic plate of the ſame form and ſize, having a glaſs handle faſtened to its centre, be placed, by means of the handle, on the electrified plate; then receive a ſpark from the metallic plate with the finger: after which the metallic plate, being removed by the glaſs handle, will be found to be poſitively electrified. This inſtrument is called an electrophorus.

5. Let one ſide of a plate of glaſs be electrified poſitively, the other ſide will attract light bodies, being negatively electrified.

6. Let a plate of glaſs be placed between two metallic plates about two inches in diameter ſmaller than the plate of glaſs, and let the plates be ſupported by a conductor; upon poſitively electrifying the upper metallic plate, by means of a wire connected with the prime conductor, the fluid not being able to paſs along the glaſs, will be accumulated upon the part contiguous to the upper metallic plate; whilſt the lower metallic plate, being within the electric influence of the upper, will be negatively electrified.

PROP. XVI. When any electric ſubſtance is electrified, it will continue in that ſtate till ſome conductor conveys away the accumulated or reſtores the deficient fluid; which will be done more or leſs rapidly, according to the degree of conducting power in the conductor, and the number of points in which it touches the electric.

Exp. 1. When the metallic plate in the electrophorus is electrified (as deſcribed Prop. XV. Exp. 4.) by ſetting it upon the electric plate, touching it with the finger, and ſeparating it ſucceſſively, many ſparks may be obtained, without again exciting the electric plate; for this plate being negatively electrified, the metallic plate, on being touched with the hand, becomes poſitively electrified (by Prop. XV.) and the electric plate remains long in its negative ſtate, becauſe, not being a conductor, its deficiency will be ſlowly ſupplied from the air where its ſurface is not covered.

2. If a glaſs veſſel, a common drinking-glaſs, for inſtance, held in the hand, receive the electric fluid on the inſide from a wire, or chain, fixed on the conductor, pith-balls, placed under the veſſel upon a conducting ſupporter, will continue long in motion.

3. Let a plate of glaſs be electrified in the manner deſcribed in Prop. XV. Exp. 6. Becauſe one ſide of the plate is poſitively electrified, and the other negatively, if a communication is made from one metallic plate to the other by means of ſome conductor, part of the accumulated fluid will ſuddenly paſs to the ſide which is deficient; upon a ſecond application of the plates of metal to the glaſs, there will be a ſecond exploſion.

Schol. AB is an electric jar, coated with tin-foil on the inſide and outſide, within three inches of the top, having a wire with a round braſs knob K, at its extremity. This wire paſſes through the cork D, that ſtops the mouth of the jar, and, at its lower end, is bended or branched ſo as to touch the inſide coating in ſeveral places. Coated jars may be made of any form and ſize, and are called *Leyden Phials*, or *Leyden Jars*. Plate 13. Fig. 14.

A number of jars combined, make what is termed an electrical *battery*; they all ſtand in a box, the bottom of which is covered with tin, thus all their outſides are connected; and by means of wires and braſs rods, their inſides are alſo connected.

The diſcharging rod conſiſts of a glaſs handle A, and two curved wires BB, which move by a joint C, fixed to the braſs cap of the glaſs handle A. The wires BB are pointed, and the points enter the knobs DD, to which they are ſcrewed, and may be unſcrewed from them at pleaſure. By this conſtruction, the balls or points may be uſed as occaſion requires. The wires being moveable at the joint C, may be adapted to ſmaller or larger jars at pleaſure. Plate 13. Fig. 15.

PROP. XVII. If a glaſs plane, or cylindrical veſſel, *coated* on both ſides with tin-foil, or any other conducting ſubſtance, be *charged*, that is, poſitively electrified on one ſide, and conſequently negatively electrified on the other; a communication being made from one ſide to the other by ſome conductor, the plane, or veſſel, will be ſuddenly *diſcharged*, with an exploſion.

There is a ſtrong attraction (compare Prop. XII. and XV.) between the fluids on oppoſite ſides of the glaſs, or the fluid which is accumulated on one ſide makes a powerful effort towards the other ſide where the fluid is deficient; but the ſubſtance of the glaſs itſelf being impervious to the electric fluid, the accumulated fluid cannot paſs to the deficient ſide till a communication is made between them by ſome conducting ſubſtance. When ſuch a communication is made, becauſe the metallic coating touches the whole ſurface of the electrified glaſs, the whole quantity of redundant fluid eaſily paſſes from the ſide which was poſitively electrified to the other.

Exp. 1. Let a plate of glaſs, coated with tin-foil, (except about 1½ inch from the edge) be charged, as deſcribed Prop. XV. Exp. 6. Upon making a communication from one ſide to the other by the diſcharging rod, there will be a ſudden diſcharge.

2. Let the ſame be done with the *Leyden Phial*.

3. Charge a jar coated on the inſide with water, ſhot, or braſs duſt, and held on the outſide by the hand, then diſcharge it in a dark room.

4. If two equal circular braſs plates, one of which is ſuſpended by a long metallic rod from the conductor parallel to the horizon, and the other, ſupported by a conductor, is placed parallel and oppoſite to the firſt, be electriſied; the plate of air between them will be charged by the braſs plates.

5. Let one coated jar be ſuſpended by a wire under another; let the upper jar be charged by taking ſparks from the conductor; the lower uninſulated jar will be charged with the fluid which paſſes from the ſide negatively electriſied of the upper jar.

6. Diſcharge, in a dark room, a jar imperfectly coated.

Cor. 1. A coated jar cannot be charged unleſs its outer ſurface be connected with ſome conductor. For without ſuch a conductor, the fluid cannot paſs from or to the outer ſurface, which is neceſſary in order to charge the jar.

Cor. 2. When a coated glaſs veſſel is charged, the charge of electric fluid is in the glaſs, and not in the coating.

Exp. Lay a plate of glaſs between two metallic plates, as deſcribed Prop. XV. Exp. 6. Having charged the plate of glaſs, remove the upper plate of metal by a glaſs handle, with ſome non-conducting ſubſtance, as ſilk; remove the electrified glaſs plate, and place it between two other plates of metal unelectrified and inſulated; the plate of glaſs thus coated afreſh will ſtill be charged.

Schol. The diſcharge of a plate of glaſs, Leyden Phial, &c. is made by reſtoring the equilibrium which was deſtroyed by the charging; and it is effected by forming a communication between the overloaded and the exhauſted ſide; and if the communication be made by metal, or other good conductors, the equilibrium will be reſtored with violence, the redundant electricity on one ſide will ruſh with great rapidity through the metallic communication to the exhauſted ſide, and a large exploſion will be made, that is, the flaſh of electric light will be very viſible, and the report will be very loud.

PROP. XVIII. If the conductor be electrified poſitively, that ſide of the jar with which it has a communication will be electrified poſitively, the other negatively.

Exp. 1. Charge one jar on the inſide poſitively, and another negatively, and obſerve, in a dark room, the different appearances of the fluid, upon the point of a wire brought near to the ball which is connected with the inner ſide of each jar: when the point is preſented to the jar poſitively electrified on the inner ſide, it will exhibit the appearance of a ſtar; when preſented to the other, that of a pencil.

2. Obſerve the different appearances, in a dark room, when with the ſame charged jar the point is preſented towards the ſide poſitively, and towards the ſide negatively, electrified.

3. Between two jars, charged one negatively and the other poſitively, ſuſpend by a ſilken ſtring a cork ball, from which ſhort threads hang freely; the ball will paſs with a rapid motion from one to the other, and, being firſt attracted towards the jar poſitively electrified, then towards the other, it will receive the fluid from the former, and communicate it to the latter, till both are diſcharged. If both be charged in the ſame manner, the cork will remain at reſt.

4. If, after a jar is charged, the uncoated part of the jar be moiſtened by the breath, or by ſteam, the jar placed upon a conductor will be gradually diſcharged, and the fluid will be ſeen, in a dark room, to flaſh ſtrongly from one ſide to the other: if the jar be inſulated, the flaſhes will be greateſt on the ſide poſitively electrified.

5. Let a diſcharging rod be applied without its balls to a charged jar, in ſuch manner as to diſcharge the jar gradually: the point which approaches towards the ſide poſitively electrified, will, in a dark room, exhibit a ſtar; the other point, a pencil.

6. Within the receiver of an air-pump place two well poliſhed braſs balls, the lower ſupported on a braſs ſtem by the plate of the pump, the other fixed on a ſtem which is moveable in the neck of the receiver: let the balls be brought within the diſtance of four or five inches from one another; then let the upper ball be connected with the conductor, and electrified poſitively: a lucid atmoſphere will, in a dark room, appear on the lower ſurface of the upper ball: whereas if the upper ball be negatively electrified, the lucid atmoſphere will be ſeen on the lower ball.

Schol. The laſt experiment eſtabliſhes the theory of a ſingle electric fluid: for if there were two contrary fluids, there muſt in this experiment be an atmoſphere about each ball, attracting each other.

PROP. XIX. The electric fluid can be conveyed through an inſulated conductor of any length, and its paſſage from one ſide of a charged jar to the other, is apparently inſtantaneous, through whatever length of a metallic, or other good conductor, it is conveyed.

Exp. 1. Let a long wire, paſſing round a room, ſuſpended by ſilk cords, be a part of the circuit of communication from one ſide of a charged jar to the other, the diſcharge will be apparently at the ſame inſtant in which the communication from one ſide to the other is completed.

2. Let any number of perſons make a part of the circuit of communication; the fluid will paſs inſtantaneouſly through the whole circuit.

Schol. The ſhock of the Leyden jar has been tranſmitted through wires of ſeveral miles in length, without taking any ſenſible ſpace of time. Dr. Prieſtley relates ſeveral curious experiments made with a view of aſcertaining this point ſoon after the diſcovery of the Leyden Phial. See Prieſtley's Hiſt. of Elect.

PROP. XX. The ſudden diſcharge of a charged jar gives a painful ſenſation to any animal placed in the circuit of communication, called the *electric ſhock.*

The diſcovery of the effects of electricity, as exhibited by the Leyden jar, immediately raiſed the attention of all the philoſophers in Europe. The account which ſome of them gave of the experiments to their friends, border very much on the ludicrous. M. Muſchenbrock, who tried the experiment with a glaſs bowl, told M. Reaumur, in a letter written ſoon after the experiment, that he felt himſelf ſtruck in his arms, ſhoulder, and breaſt, ſo that he loſt his breath; and was two days before he recovered from the effects of the blow and the terror. He added, that he would not take a ſecond ſhock for the whole kingdom of France.

M. Allamand, who made the experiment with a common beer glaſs, ſaid, that he loſt his breath for ſome moments, and then felt ſuch an intenſe pain all along his right arm, that he was apprehenſive of bad conſequences; but it ſoon went off without any inconvenience.

Notwithſtanding the parade made by theſe philoſophers, the ſhock was probably, not, by any means, ſtronger than what many children of 6 or 7 years old would bear without the ſmalleſt heſitation. Their deſcriptions muſt have ariſen through terror, or the love of the marvellous.

Cor. The force of the electric ſhock may be increaſed, by increaſing the ſurface of coated glaſs.

Exp. 1. A battery being charged, a fine metallic wire brought into the circuit will be melted.

2. If a plain piece of metal be placed upon one of the rods of the diſcharger, and upon the other a needle with the point oppoſite to the ſurface of the metal, upon diſcharging the battery, the ſurface of the piece of metal will be marked with coloured circles, occaſioned by thin laminæ of metal raiſed in the exploſion.

3. If a piece of leaf gold be put between two pieces of glaſs, and the whole faſt-bound together, the metal will be melted, and a metallic ſtain will be ſeen in both glaſſes.

4. If a ſhock be ſent through a needle, it will give it magnetic polarity.

5. An animal, or plant, may be killed by being placed in the circuit of a battery.

Schol. Perſons not thoroughly converſant in electricity, ſhould be very cautious in uſing large batteries; they ſhould be ſure that they are perfect maſters of a ſmall force, before they meddle with a greater. Such a force of electricity as may be accumulated in batteries is not to be trifled with, ſince the conſequences, if not fatal, may be great and laſting. A large ſhock, taken through the arms and breaſt, which an operator is moſt in danger of receiving, might poſſibly injure the lungs, or ſome other vital part: and if the ſhock were taken through the head, which may eaſily happen when a perſon is ſtooping over the apparatus in order to adjuſt it, it might affect his intellects for the remainder of life.

PROP. XXI. If the circuit be interrupted, the fluid will become viſible, and where it paſſes, it will leave an impreſſion upon any intermediate body.

Exp. 1. Let the fluid paſs through a chain, or through any metallic bodies placed at ſmall diſtances from each other; the fluid, in a dark room, will be viſible between the links of the chain, or between the metallic bodies.

2. If the circuit be interrupted by ſeveral folds of paper, a perforation will be made through it, and each of the leaves will be protruded by the ſtroke from the middle towards the outward leaves.

3. Let a card be placed under wires which form the circuit, where the circuit is interrupted for the ſpace of an inch: the card will be diſcoloured. If one of the wires be placed under the card, and the other above it, the direction of the fluid may be ſeen.

4. Spirits of wine, or gun-powder, being made part of the circuit, may be fired.

5. Inflammable air may be fired by an electric gun.

PROP. XXII. The atmoſphere is electrified, ſometimes poſitively, and ſometimes negatively.

Exp. Let a kite be ſent up into the air with cord (conſiſting of copper thread twiſted with twine;) let the lower end of the cord be inſulated by a ſilk line: a metallic conductor ſuſpended from the lower end of the cord will be poſitively or negatively electrified. The air at ſome diſtance from houſes, trees, maſts of ſhips, &c. is generally electrified poſitively; particularly in froſty, clear, or foggy weather. For the particular conſtruction of the electrical kite, and other inſtruments uſed with it, ſee Cavallo's Elect. Vol. II. chap. I.

Schol. The following general laws have been deduced by Mr. Cavallo, from a great number of experiments made during two years in almoſt every degree of the atmoſphere, from 15°. to 80°. of Fahrenheit's thermometer.

I. The air appears to be electrified at all times; its electricity is conſtantly poſitive, and much ſtronger in froſty than in warm weather; but it is by no means leſs in the night than in the day-time.

II. The preſence of the clouds generally leſſens the electricity of the kite.

III. When it rains, the electricity of the kite is generally negative, and very ſeldom poſitive.

IV. The aurora borealis ſeems not to affect the electricity of the kite.

V. The electrical ſpark, taken from the ſtring of the kite, or from any inſulated conductor connected with it, eſpecially if it does not rain, is very ſeldom longer than $\frac{1}{4}$ of an inch, but it is exceedingly pungent. When the index of the electrometer is not higher than 20°. the perſon that takes the ſpark will feel the effect of it in his legs; it appearing more like the diſcharge of an electric jar, than the ſpark taken from a prime conductor.

VI. The electricity of the kite is in general ſtronger or weaker, according as the ſtring is longer or ſhorter; but it does not keep any exact proportion to it. The electricity, for inſtance, brought down by a ſtring of a hundred yards, may raiſe the index of the electrometer to 20°. when with double that length of ſtring, the index of the electrometer will not go higher than 25°.

VII. When the weather is damp, and the electricity is pretty ſtrong, the index of the electrometer, after taking a ſpark from the ſtring, or preſenting the knob of a coated phial to it, riſes ſurpriſingly quick to its uſual place, but in dry and warm weather it riſes exceedingly ſlow.

PROP. XXIII. The electric fluid and lightning are the ſame ſubſtance.

Their properties and effects are the ſame. Flaſhes of lightning are generally ſeen to form irregular lines in the air; the electric ſpark when ſtrong, has the ſame appearance. Lightning ſtrikes the higheſt and moſt pointed objects; takes in its courſe the beſt conductors; ſets fire to bodies; ſometimes diſſolves metals; rends to pieces ſome bodies; deſtroys animal life; in all which it agrees (as has been ſhewn) with the phenomena of electric fluid;—both cauſes have the ſame power of making iron magnetic. Lightning has been known to ſtrike men with blindneſs. Dr. Franklin produced a ſimilar effect on a pigeon by the electrical fluid. Laſtly, the lightning being brought from the clouds to an electrical apparatus, by a kite or wire, will exhibit all the appearances of the electric fluid.

Exp. Take a Leyden phial, 5 inches in diameter, and 13 inches in height; on the inſide let the coating riſe till its upper edge be 2½ inches from the rim of the veſſel; on the outſide let the coating riſe no higher than 1 inch from the bottom. When the phial is thus coated, let it be charged, and a ſpark will paſs from the tin-foil on the outſide to that on the inſide; but its form will reſemble that of a tree, whoſe trunk will increaſe in magnitude and brilliancy, and conſequently in power, as it approaches the edge, owing to ramifications which it collects from all parts of the glaſs. Within two inches of the edge, it becomes one body, or ſtream, and along that interval its greateſt force acts.

When two clouds, or the two correſpondent parts of a cloud, have their equilibrium reſtored by a diſcharge, the appearances are exactly ſimilar to thoſe of the preceding experiment. Each extremity of the flaſh is formed by a multitude of little ſtreams, which gather into one body, whoſe power is undivided in that interval only which ſeparates the poſitive from the negative.

PROP. XXIV. Buildings may be ſecured from the effects of lightning, by fixing a pointed iron rod higher than any part of the building and continuing it, without interruption, to the ground, or the neareſt water.

The electric fluid will, by means of the pointed rod, be gradually conveyed from the cloud to the earth by a continued ſtream, and thus prevent the effects of a ſudden and violent exploſion.

Exp. Let a board, ſhaped like the gable end of a houſe, be fixed perpendicularly upon an horizontal board: in the perpendicular board let a hole be made, about an inch ſquare and ½ inch deep; in this hole let a piece of wood nearly of the ſame dimenſions be ſo inſerted as to fall eaſily out of its place, and let a wire be faſtened diagonally to this ſquare piece of wood; let another wire, terminated by a braſs ball, be faſtened to the perpendicular board, with its ball above the board, and its lower end in contact with the diagonal wire in the ſquare piece of wood; let the communication be continued by a wire to the bottom of the perpendicular board. If the wires in this ſtate be made part of a circuit of communication, on diſcharging the jar the ſquare piece of wood will not be diſplaced; but if the communication be interrupted by changing the direction of the diagonal wire, the ſquare piece of wood will, upon the diſcharge, be driven out of its place.

If inſtead of the upper braſs ball, a pointed wire be placed above the perpendicular board, the diſcharge may be drawn off without an exploſion.

Schol. The following directions are given by Earl Stanhope, to perſons erecting conductors for lightning:

(1.) The rods muſt be made of ſuch ſubſtances as are, in their nature, the beſt conductors of electricity.

(2.) The rods muſt be uninterrupted, and perfectly continuous.

(3.) They muſt be of ſufficient thickneſs.

(4.) They muſt be perfectly connected with the common ſtock, that is, the earth, or neareſt water.

(5.) The upper extremity of the rods muſt be finely tapered, and as acutely pointed as poſſible.

(6.) The rods muſt be very prominent, and ſeveral feet above the chimneys.

(7.) Each rod muſt be carried in the ſhorteſt convenient direction from its upper end to the common ſtock.

(8.) There ſhould be no prominent bodies of metal on the top of the building propoſed to be ſecured, but ſuch as are connected with the conductor by ſome proper metallic communication.

(9.) There ſhould be a ſufficient number of ſubſtantially erected high and pointed rods. See "Principles of Electricity," by Charles Viſcount Mahon, now Earl Stanhope. To the ſame work, the reader muſt be referred for an account of a diſcovery made by his lordſhip in the ſcience of electricity, which he denominated the "*returning ſtroke*," by which, he aſſerts, that perſons may be killed, and other vaſt miſchief enſue by lightnings at the diſtance of ſeveral miles from the flaſh. It is proper alſo to obſerve, that ſeveral reſpectable electricians, though, willing to admit the fact as diſcovered by Earl Stanhope, yet do not ſeem to think that the danger attending the returning ſtroke can ever be great or formidable. See Cavallo's Electr. Vol. II. and III. Morgan's Lectures on Elect. Vol. II. Dr. Hutton's Dict. Art. *Returning Stroke*.

PROP. XXV. The electric fluid paſſes eaſily through a *vacuum*.

The air being a non-conductor, in proportion as it is removed, the effort of the electric fluid on the ſurface of the body poſitively electrified to paſs to the next conductor, meets with leſs reſiſtance, and therefore is diffuſed over a greater ſpace.

Exp. 1. Let a jar be charged *in vacuo*.

2. Let a *luminous conductor* be placed in the circuit, and obſerve the fluid paſſing through it.

3. Let a *vacuum* be made a part of the circuit in diſcharging a phial.

4. Make a *vacuum* in a double barometer, and let the fluid paſs from one leg to the other by connecting one of the veſſels of mercury with the conductor.

5. The electric fluid may be made to paſs through a large tube three feet in length, and four or five inches in diameter, if, being well exhauſted, one end of it be connected with a large conductor.—The preceding experiments are to be performed in a dark room.

Schol. 1. From the reſemblance between theſe electrical appearances, and the atmoſpherical phenomena of the *Aurora Borealis*, meteors, &c. it is inferred, that theſe phenomena are produced by the electric fluid.

Schol. 2. The ſucceſs of the foregoing experiments depends, it is highly probable, upon the air in the jar, tube, &c. being rarefied in a high degree; for Mr. W. Morgan, a gentleman deeply ſkilled in calculations and political arithmetic, has ſhewn, that a *perfect vacuum* is abſolutely impermeable to the electric fluid. See Phil. Tranſ. vol. LXXV.

PROP. XXVI. Some fiſhes have the property of giving ſhocks analogous to thoſe of artificial electricity; namely, the *Torpedo*, the *Gymnotus electricus*; and the *Silurus electricus*.

If the torpedo, whilſt ſtanding in water, or out of water, but not inſulated, be touched with one hand, it generally communicates a trembling motion or ſlight ſhock to the hand. If the torpedo be touched with both hands, at the ſame time, one hand being applied to its under, and the other to its upper ſurface, a ſhock will be received exactly like that occaſioned by the Leyden Phial. When the hands touch the fiſh on the oppoſite ſurfaces, and juſt over the electric organs, then the ſhock is the ſtrongeſt; but no ſhock is felt, if both hands are placed upon the electric organs of the ſame ſurface; which ſhews that the upper and lower ſurfaces of the electric organs are in oppoſite ſtates of electricity, anſwering to the *plus* and *minus* ſides of a Leyden Phial.

The ſhock given by the torpedo when in air, is about four times as ſtrong as when in water; and when the animal is touched on both ſurfaces by the ſame hand, the thumb being applied to one ſurface, and the middle finger to the oppoſite, the ſhock is felt much ſtronger than when the circuit is formed by both hands.

This power of the torpedo is conducted by the ſame ſubſtances which conduct electricity, and is interrupted by thoſe ſubſtances which are non-conductors of electricity. A circuit may be made of ſeveral perſons joining hands, and the ſhock will be felt by them all at the ſame time; but the ſhock will not paſs through the leaſt interruption of continuity, not even the diſtance of the two hundredth part of an inch.

No electric attraction or repulſion could be ever obſerved to be produced by the torpedo, nor, indeed, by any of the electric fiſhes. The ſhocks of the torpedo ſeem to depend on the will of the animal.

The gymnotus electricus, or electrical eel, poſſeſſes all the electric properties of the torpedo, but in a ſuperior degree. When ſmall fiſh are put into the water wherein the gymnotus is kept, they are generally ſtunned or killed by the ſhock, and then they are ſwallowed, if the animal be hungry.

The ſtrongeſt ſhock of the gymnotus will paſs a very ſhort interruption of continuity in the circuit. When the interruption is formed by the inciſion made by a pen-knife on a ſlip of tin-foil that is paſted on glaſs, and that ſlip is put into the circuit, the ſhock, in paſſing through that interruption, will ſhew a ſmall but vivid ſpark, plainly to be ſeen in a dark room.

The gymnotus ſeems alſo to be poſſeſſed of a ſort of *new ſenſe*, by which he knows whether the bodies preſented to him are conductors or not. This fact was aſcertained by a great number of experiments made by Mr. Walſh.

The ſilurus electricus is known to have the power of giving the ſhock, but we have a very imperfect account of its properties.

A fourth electrical fiſh was found on the coaſt of Johanna, one of the Comora iſlands, in lat. 12°. 13′. ſouth, by William Paterson; and an account of it was publiſhed in the 76th vol. of the Phil. Tranſ.

Schol. When electricity is ſtrongly communicated to inſulated animal bodies, the pulſe is quickened, and perſpiration increaſed; and if they receive, or part with their electricity on a ſudden, a painful ſenſation is felt at the place of communication. But what is more extraordinary is, that the influence of the brain and nerves upon the muſcles ſeems to be of an electric nature.

We are indebted for this diſcovery to M. Galvini, a learned Italian, who has denominated that part of ſcience, ANIMAL ELECTRICITY. We ſhall, without pretending to enter at large on the ſubject, give the reſult of the principal obſervations hitherto made, together with three or four illuſtrative experiments.

1. The nerve of the limb of an animal being laid bare, and ſurrounded with a piece of tinfoil, if a communication be formed between the nerve thus armed, and any of the neighbouring muſcles, by means of a piece of zinc, ſtrong contractions will be produced in the limb.

2. If a portion of the nerve which has been laid bare be armed as above, contractions will be produced as powerfully, by forming the communication between the armed and bare part of the nerve, as between the armed part and muſcle.

3. A ſimilar effect is produced by arming a nerve, and ſimply touching the armed part of the nerve with the metallic conductor.

4. Contractions will take place if a muſcle be armed, and a communication be formed by means of the conductor between it and a neighbouring nerve. The ſame effect will be produced if the communication be formed between the armed muſcle and another muſcle which is contiguous to it.

5. Contractions may be produced in the limb of an animal by bringing the pieces of metal into contact with each other at ſome diſtance from the limb, provided the latter make part of a line of communication between the two metallic conductors.

6. Contractions can be produced in the amputated leg of a frog, by putting it into water, and bringing the two metals into contact with each other at a ſmall diſtance from the limb.

7. The influence which has paſſed through, and excited contractions in, one limb, may be made to paſs through, and excite contractions in, another limb.

8. The heart is the only involuntary muſcle in which contractions can be excited by theſe experiments.

9. Contractions are produced more ſtrongly the farther the coating is placed from the origin of the nerve.

10. Animals which were almoſt dead have been found to be conſiderably revived by exciting this influence.

11. When theſe experiments are repeated upon an animal that has been killed by opium, or by the electric ſhock, very ſlight contractions are produced; and no contractions whatever will take place in an animal that has been killed by corroſive ſublimate, or that has been ſtarved to death.

12. Zinc appears to be the beſt exciter when applied to gold, ſilver, molybdena, ſteel, or copper. The latter metals, however, excite but feeble contractions when applied to each other. Next to zinc, in contact with theſe metals, tin and lead, and ſilver and lead, appear to be the moſt powerful exciters.

EXP. 1. Place the limb of an animal, a frog, for inſtance, upon a table; hold with one hand the principal nerve previouſly laid bare, and in the other hold a piece of zinc; let a ſmall plate of lead or ſilver be then laid upon the table, at ſome diſtance from the limb, and a communication be formed, by means of water, between the limb and the part of the table where the metal is lying. If now, the ſilver be touched with the zinc, contractions will be produced in the limb the moment that the metals come into contact with each other. The ſame effect will be produced, if the two pieces of metal be previouſly placed in contact, and the operator touch one of them with his finger.

2. Let two amputated limbs of a frog be taken; let one of them be laid upon a table, and its foot be folded in a piece of ſilver; let a perſon lift up the nerve of this limb with a ſilver probe, and another perſon hold in his hand a piece of zinc, with which he is to touch the ſilver including the foot; let the perſon holding the zinc in one hand, catch with the other the nerve of the ſecond limb, and he who touches the nerve of the firſt limb is to hold in the other hand the foot of the ſecond; let the zinc now be applied to the ſilver, including the foot of the firſt, and contractions will be immediately excited in both limbs.

3. Take a living flounder, lay it flat in a pewter-plate, or upon a ſheet of tin-foil, and put a piece of ſilver, as a ſhilling, or a half crown, upon the fiſh. Then by means of a piece of metal, complete the communication between the pewter-plate, or tin-foil, and the ſilver piece, on doing which the animal will give evident tokens of being affected.

4. Let a perſon lay a piece of zinc upon his tongue, and a half crown, or other ſilver, under it; on forming a communication between thoſe two metals, by bringing their two edges into contact, he will perceive a peculiar ſenſation, a kind of cool ſub-acid taſte, not exactly like, and yet not much different from that produced by artificial electricity. See Cavallo's Elect. Vol. III.

SCHOL. 2. Electricity has been adminiſtered for various diſeaſes. Mr. Cavallo has taken great pains in aſcertaining the caſes in which electricity has been ſucceſsfully applied. We are informed by that gentleman, that *rheumatic diſorders*, even of long ſtanding, are relieved, and generally quite cured. Deafneſs, the tooth-ach, ſwellings in general, inflammations of every ſort, palſies, ulcers, cutaneous eruptions, the St. Vitus' dance, ſcrophulous tumours, cancers, abſceſſes, nervous head-achs, the dropſy, gout, agues, and obſtructions, have all been conſiderably relieved, and in many inſtances perfectly cured, by the application of electricity. A full account of the method of adminiſtering electricity in the caſes above mentioned, with an accurate deſcription of the inſtruments uſed, may be ſeen in the 2d Vol. of Cavallo's Complete Treatiſe of Electricity.

PROP. XXVII. There is a conſiderable analogy and difference between magnetiſm and electricity.

The power of electricity is of two ſorts, poſitive and negative; bodies poſſeſſed of the ſame ſort of electricity, repel each other, and thoſe poſſeſſed of different ſorts attract each other. In magnetiſm, every magnet has two poles; poles of the ſame name repel each other, and the contrary poles attract each other.

In electricity, when a body in its natural ſtate is brought near to one electrified, it acquires a contrary electricity, and becomes attracted by it. In magnetiſm, when a ferruginous ſubſtance is brought near to one pole of a magnet, it acquires a contrary polarity, and becomes attracted by it.

One ſort of electricity cannot be produced by itſelf. In like manner, no body can have only one magnetic pole.

The electric virtue may be retained by electrics, but it eaſily pervades non-electrics. The magnetic virtue is retained by ferruginous bodies, but it eaſily pervades other bodies.

On the contrary, the magnetic power differs from the electric, in that it does not affect the ſenſes with light, ſmell, taſte, or noiſe, as the electric does.

Magnets attract only iron, whereas the electric power attracts bodies of every ſort.

The electric virtue reſides on the ſurface of electrified bodies, but the magnetic is internal.

A magnet loſes nothing of its power by magnifying other bodies, but an electrified body loſes part of its electricity by electrifying other bodies. See Cavallo's Magnetiſm, Part II. Chap. II.

BOOK VI.

OF OPTICS; OR, THE LAWS OF LIGHT AND VISION.

CHAP. I.

Of LIGHT.

DEF. I. LIGHT is that which, proceeding from any body to the eye, produces the perception of *ſeeing*.

DEF. II. A *Ray of Light*, is any exceedingly ſmall portion of light, as it comes from a luminous body.

DEF. III. A body which is tranſparent, or affords a paſſage for the rays of light, is called a *Medium*.

DEF. IV. Rays of light which, coming from a point, continually ſeparate as they proceed, are called *Diverging Rays*.

DEF. V. Rays which tend to a common point are called *Converging Rays*. The divergency, or convergency, of rays, is meaſured by the angle contained between the lines which the rays deſcribe.

DEF. VI. Rays of light are *parallel*, when the lines which they deſcribe are parallel.

DEF. VII. A *Beam* of light, is a body of parallel rays; a *Pencil* of rays, is a body of diverging or converging rays.

DEF. VIII. The point from which diverging rays proceed, is called the *radiant point*; that to which converging rays are directed, is called the *focus*.

Plate 6. Fig. 1. If the rays proceed from B; BD, BA, BC, BE, are diverging rays, and B is the radiant; if the rays tend towards B, DB, AB, &c. are converging rays, and B is the focus.

Fig. 2. If the rays AC, BC, converge to the focus C, paſſing on from thence in a right line, they become diverging, and C becomes a radiant.

Def. IX. A ray of light bent from a ſtraight courſe in the ſame medium, is ſaid to be *inflected*.

PROPOSITION I.

Rays of light conſiſt of particles of matter.

For, like all matter with which we are acquainted, they are capable of being inflected out of their courſe by attraction.

Exp. 1. If a beam of light be admitted into a dark room through a ſmall hole, and the edge of a knife be brought near the beam, the rays, which would otherwiſe have been in a ſtraight line, will be inflected towards the knife. The edge of any other thin plate of metal, &c. produces the ſame effect.

2. The ſhadow of a ſmall body, as a hair, a thread, &c. placed in a beam of the ſun's light, will be much broader than it ought to be if the rays of light paſſed by theſe bodies in right lines.

3. A beam of light paſſing through an exceedingly narrow ſlit, not above $\frac{1}{400}$ part of an inch broad, will be ſplit into two, and leave a dark ſpace in the middle.

PROP. II. Every viſible body emits particles of light from its ſurface in all directions, which, paſſing without obſtruction, move in right lines.

Wherever a ſpectator is placed with reſpect to a luminous body, every point of that part of the ſurface which is turned towards him is viſible to him: the particles of light are, therefore, emitted in all directions, and thoſe rays only are intercepted in their paſſage by an interpoſed object, which would be intercepted upon the ſuppoſition that the rays move in right lines.

Exp. 1. Let a portion of a beam of light be intercepted by any body: the ſhadow of that body will be bounded by right lines paſſing from the luminous body, and meeting the lines which terminate the opaque body.

2. A ray of light, paſſing through a ſmall orifice into a dark room, proceeds in a ſtraight line.

3. Rays will not paſs through a bended tube.

Schol. Rays of light are properly repreſented by right lines.

PROP. III. The rays of light move with great velocity.

The velocity of light is much greater than that of ſound; for the flaſh of a gun, fired at a conſiderable diſtance, is ſeen ſome time before the report is heard. The clap of thunder is not heard till ſome time after the lightning has been ſeen.

This propoſition is proved by obſervations made on the ſatellites of the planet Jupiter, and on the aberration of the rays of light from the fixed ſtars, as will be ſhewn in treating upon Aſtronomy: from whence it will be ſeen, that the velocity is at the rate of 200.000 miles in one ſecond of time.

PROP. IV. The particles of light are exceedingly ſmall.

Otherwiſe their velocity would render their momentum too great to be endured by the eye without pain.

Exp. 1. If a candle be lighted, and there be no obſtacle to obſtruct the progreſs of its rays, it will fill all the ſpace within two miles every way before it has loſt the leaſt ſenſible part of its ſubſtance.

2. Rays of light will paſs without confuſion through a ſmall puncture in a piece of paper, from ſeveral candles in a line parallel to the paper, and form diſtinct images on a ſheet of paſteboard placed behind the paper.

PROP. V. The quantities of light, received from a luminous body upon a given ſurface, are inverſely as the ſquares of the diſtances of the ſurface from the luminous body.

Plate 6. Fig. 3. Let ABD, EFG, be two concentric ſpherical ſurfaces; of which let ELFI, AHBK, be two ſimilar portions. Let the rays CE and CF, with the reſt proceeding from the centre C, fall upon the portion ELFI, and cover it: it is evident from inſpection, that the ſame rays at the diſtance CH will cover the portion AHBK only; now theſe rays being the ſame in number at each place, will be as much thinner in the former, than they are in the latter, as ELFI is larger than AHBK; but theſe ſpaces being ſimilar portions of the ſurfaces of ſpheres, have the ſame ratio to each other, that the ſurfaces themſelves have, that is, they are to each other as the ſquares of their radii CL, CH: the denſity of the rays is therefore inverſely as the ſquares of theſe radii, or of their diſtances from the luminous point C.

Plate 12. Fig. 8. Exp. The light, paſſing from a candle through a ſquare orifice, will diverge as it proceeds, and will illuminate ſurfaces which will be to each other as the ſquares of their diſtances from the candle. Thus at the diſtance AF the candle will illuminate the ſquare BF, at the diſtance AO it will illuminate the ſurface CO equal to four times BF, and at the diſtance AS it will illuminate the ſurface DS equal to nine times BF, but AF, AO, and AS, are as 1, 2, and 3, conſequently, the illuminated ſurfaces are as the ſquares of the diſtances.

PROP. VI. If the diſtance between rays diverging from different radiant points be the ſame, the diſtances of the radiant points are inverſely as the divergency of the rays.

Plate 6. Fig. 4.

Let D and E be two different radiants; and let the rays diverging from D deſcribe the lines DA, DB, and the rays diverging from E deſcribe the lines EA, EB; ſo that, at the points A and B, the diſtance between the former rays ſhall be the ſame with the diſtance between the latter, and let EC, DC, be the perpendicular diſtances of the radiants E, D. At the point E make the angle ZEC equal to ADC, which is half ADB: whence ZEC and ADC (El. V. 7.) have the ſame ratio to AEC. But if theſe angles are ſmall, they are very nearly in the proportion of their ſines ZC, AC. And becauſe the angle ADC is equal to the angle ZEC (El. I. 28.) AD is parallel to ZE: and becauſe theſe lines are parallel, (El. I. 29.) the angles CAD, CZE, are equal: whence the two triangles ZEC, ADC, are equiangular, and (El. VI. 4.) EC is to DC, as ZC to AC, or (from what was ſhewn above) as ADC to AEC: that is, the diſtance of the radiant E is to the diſtance of the radiant D, as half the angle of divergency of the rays which proceed from D is to half the divergency of the rays which proceed from E, or as the whole angle of divergency ADB to the whole angle of divergency AEB; that is, the diſtances of the radiants are inverſely as the divergency of the rays.

PROP. VII. If the diſtance between converging rays tending to different foci be the ſame, the diſtances of the foci are inverſely as the convergency of the rays.

Plate 6. Fig. 4.

Let AD, BD, be lines deſcribed by rays converging to the focus D, and AE, BE, lines deſcribed by other rays converging to E, and let the diſtance AB, at the points A and B, be the ſame between the former and the latter rays. The angles ADB, AEB, are in this caſe the angles of convergency; and EC, DC, are diſtances of the foci to which they reſpectively tend. Now it was proved in the laſt Prop. that EC is to DC as ADB is to AEB. Therefore the diſtances of the foci are inverſely as the convergency of the rays.

PROP. VIII. If rays proceed from a radiant at an infinite diſtance, their divergency is nothing, and the rays are conſidered as parallel.

Since (by Prop. VI.) the divergency of rays is inverſely as the diſtance of the radiant, when the diſtance of the radiant is infinitely great, the angle of divergency is infinitely ſmall, and the rays may be conſidered as parallel.

Cor. Hence all the rays which come from the centre, or any other given point, of the ſun's ſurface, are conſidered as parallel.

PROP. IX. If rays tend to a focus at an infinite diſtance, their convergency is nothing, and the rays are conſidered as parallel.

Since (by Prop. VII.) the convergency is inverſely as the diſtance of the focus, when that diſtance is infinitely great, the angle of convergency is infinitely ſmall.

CHAP.

CHAP. II.

Of Refraction.

SECT. I.—*Of the* Laws *of* Refraction.

Def. X. A ray of light bent from a ſtraight courſe by paſſing out of one medium into another, is ſaid to be *refracted.*

Def. XI. The *Angle of Incidence*, is that which is contained between the line deſcribed by the incident ray, and a line perpendicular to the ſurface on which the ray ſtrikes, raiſed from the point of incidence.

Def. XII. The *Angle of Refraction*, is that which is contained between the line deſcribed by the refracted ray, and a line perpendicular to the refracting ſurface at the point in which the ray paſſes through that ſurface.

Def. XIII. The *Angle of Deviation*, is that which is contained between the line of direction of an incident ray, and the direction of the ſame ray after it is refracted.

Plate 6. Fig. 6. AC is a ray of light; HK the ſurface of the refracting medium; CF the refracted ray; OP the perpendicular; ACO the angle of incidence; PCF the angle of refraction, and FCL the angle of deviation.

Plate 6. Fig. 5. Schol. The radiant point and focus may be either real or imaginary. If the rays rn, ro, diverging from the radiant r, paſs into a refracting medium, and move on in the directions of the lines nA, oB, which produced in the contrary direction would meet in R, this radiant point is imaginary.

If the rays Ip, Lq, tending towards the point F, be refracted at p and q, and acquire a direction towards f, the focus f is imaginary.

PROP. X. The attracting force of any medium, acting upon a ray of light, is every where perpendicular to the refracting ſurface.

Plate 6. Fig. 8. If the medium be uniform in all its parts, its immediate power upon the ray of light will be equally ſtrong in every point of a plane drawn parallel to the refracting ſurface; though its ſtrength may be different in the next parallel plane, and ſo onwards as far as that power is extended on each ſide of the ſurface of the medium. The extent of this power will therefore be terminated by two planes, parallel to one another and to the refracting ſurface. Let R be a particle of light, acted upon by the refractive power of the medium whoſe refracting ſurface is DC. It is evident that the refractive power at O will move the particle R in the direction RO: and taking any two points D, C, at equal diſtances on each ſide of O, the powers at D and C being equal, and acting at equal diſtances, RD, RC, equally inclined to RO, cannot move R in any direction but that of RO. The ſame may be ſhewn of the powers at every point of the line DC, and in every line parallel to DC, that is, of the whole power of the medium.

PROP. XI. A ray of light, in paſſing out of a rarer into a denſer medium, is refracted towards a perpendicular to the ſurface of the denſer, raiſed from the point in which the ray meets the medium; in paſſing out of a denſer into a rarer medium, it is refracted from the ſame perpendicular.

Let a ray of light, AC, paſs obliquely out of a rarer medium X, into a denſer medium Z; let HK be the plane ſurface of the denſer medium; from the point C, in which the ray AC paſſes into the denſer medium, raiſe the perpendicular OCP: the ray will be refracted out of the direction ACL, towards the perpendicular OCP. Plate 6. Fig. 6.

Becauſe the ray is more attracted by the denſer medium than by the rarer, it will be accelerated on entering the medium Z: for whilſt the ray is ſo near the ſurface of the medium Z as to be within its attraction, and more attracted downwards than upwards, this attraction conſpires with the motion of the ray, and, conſequently, increaſes its velocity. And, ſince the action of the attracting force of the medium Z, muſt (by prop. X.) be in the direction of a line OCP perpendicular to its ſurface, if the oblique motion of the ray in the direction AC be reſolved into two others, AD parallel to the ſurface HK, and AB, or DC, perpendicular to it, the parallel motion AD cannot be accelerated or retarded by the attraction which acts in the direction OC: the change of velocity, therefore, which the ray receives from the attracting force, muſt be made in the perpendicular part of its motion DC. Take CG greater than DC repreſenting the perpendicular motion of the ray after paſſing into the denſer medium; and take CE equal to AD repreſenting the parallel part of the motion of the ray, which, becauſe it is parallel to AB, remains the ſame when the ray enters the denſer medium. The ray, therefore, at its entering the medium Z, may be conſidered as acted upon by two forces CE, CG, and conſequently (Book II. Prop. XIV.) will deſcribe the diagonal CF of a parallelogram, the ſides of which are CE, CG. Now, of theſe ſides CE remaining the ſame, whilſt CG becomes greater than CD, the angle GCF (from the nature of the parallelogram) will be leſs than the angle NCL, equal (El. I. 15.) to ACD. Therefore the ray, after it has paſſed into the denſer medium, makes a leſs angle with the perpendicular OCP than AC, the ray before it paſſes into the denſer medium; that is, the ray, in paſſing out of the rarer into the denſer medium, is refracted towards the perpendicular. On the contrary, whilſt the ray of light FC is paſſing out of the denſer medium Z into the rarer medium X, it is more attracted by the denſer than by the rarer medium, and is therefore more drawn downwards than upwards; whence the attraction oppoſes the motion of the ray, and will retard it as much as in paſſing out of the rarer into the denſer medium it was accelerated; and, conſequently, the effect will be the reverſe of that which was ſhewn in the former caſe.

SCHOL. The principles of optics are demonſtrated upon the ſuppoſition that light is an homogeneal ſubſtance; and though light will appear to be compounded of ſeveral kinds of rays, yet the principles of refraction and reflection, &c. are mathematically true when applied to rays of any one ſort.

Plate 6. Fig. 7. EXP. 1. Let a perpendicular cylindrical veſſel be ſo placed that the ſun, ſhining upon its ſide NA, may caſt the ſhadow of the ſide to a point L in the bottom of the veſſel. This ſhadow is terminated by SNL, a ray which paſſes, in a right line, by the edge of the veſſel. If the veſſel be filled with water, the ſhadow will recede, as the water is poured into the veſſel, from the point L which terminated it when the veſſel was empty, towards the ſide NA, on which the ſun ſhines, and will be terminated by the ray ONC: that is, the ray SNL, which firſt terminated the ſhadow, by paſſing out of the air into the water, is refracted towards AN, a line drawn perpendicular to the ſurface of the water, at the point in which the ray enters the water: or the angle of refraction is leſs than the angle of incidence.

2. Let a ſmall bright object be laid upon the bottom of a cylindrical veſſel NBAL at C. Let the ſpectator's eye be ſo placed at S, as juſt to loſe ſight of the object at C, that is, ſo that a ray paſſing in a right line from the remote edge of the object towards the eye at S will be intercepted by the edge of the veſſel, or that the firſt ray which is not intercepted paſſes in the direction ONC above the eye. Whilſt the eye continues in the ſame ſituation, if the veſſel be filled with water, the object will become viſible; that is, the ray which paſſed from the remote edge of the object, in a right line CNO, by the veſſel, in entering the air is refracted into the direction NS, towards the eye, or from the perpendicular PNA.

PROP. XII. All refraction is reciprocal.

Plate 6. Fig. 6. The ray AC, in paſſing out of the medium X into Z, is refracted into CF, becauſe it is accelerated at its entrance into Z by the greater attraction of the denſer medium: and the ray FC in paſſing out of Z into X is refracted into CA, becauſe it is retarded by the ſame attraction. Since, then, the acceleration and retardation are produced by the ſame degree of attraction in oppoſite directions, they will be equal to one another, and the refractions produced by them will be equal, but in oppoſite directions; that is, if the refracted ray becomes the incident ray, the incident ray will become the refracted one: or, the refractions are reciprocal.

PROP. XIII. In any two given mediums, the ſine of any one angle of incidence has the ſame ratio to the ſine of the correſponding angle of refraction, as the ſine of any other angle of incidence has to the ſine of its correſponding angle of refraction.

Plate 6. Fig. 6. The oblique motion of the ray AC, in paſſing out of the rarer medium X into the denſer medium Z, may be reſolved into AD parallel to the ſurface HK, and AB or DC perpendicular to it. Of theſe (as was ſhewn in Prop. XI.) only the perpendicular motion DC is accelerated. Take CN equal to CD, and continue the refracted ray CS to F, ſo that if FG is drawn perpendicular to OP, FG may be equal to AD; and draw NS the ſine of the angle

of refraction NCS. The perpendicular velocity of the ray in the medium Z (by Prop. XI.) is CG, that is, the ray is accelerated in the ratio of CN to CG. But, becauſe CGF, CNS, are equiangular triangles, (El. VI. 4.) GF the ſine of the angle of incidence is to NS the ſine of the angle of refraction, as CG the velocity of the ray in the denſer medium Z, is to CN the velocity of the ray in the rarer medium X; that is, the ſines of the angles of incidence are inverſely as the velocities of the ray. But where the two mediums are given, as X and Z, their denſities, and conſequently, the differences of their attractions are given: and, ſince the ratio of velocities in different mediums depends upon the difference of their attracting force, where this difference is given, the ratio of the velocities cannot be altered by changing the obliquity of the ray, ſo that, if the velocity in one medium is to that in another, as CG to CN, and the ſine of the angle of incidence be to that of refraction in any one degree of obliquity; as GF to NS, this ratio will continue the ſame in every degree of obliquity; for GF will always be to NS in the ratio of CG to CN.

Cor. 1. Hence, when the angle of incidence is increaſed, the correſponding angle of refraction will alſo be increaſed: becauſe the ratio of their ſines cannot continue the ſame, unleſs they be both increaſed: and if two angles of incidence be equal, the angles of refraction will alſo be equal.

Cor. 2. Hence the angle of deviation varies with the angle of incidence.

Schol. 1. If a ray of light, AC, paſs obliquely out of air into glaſs, AD the ſine of the angle of incidence ACD, is to NS the ſine of the angle of refraction NCF, nearly as 3 to 2: therefore, ſuppoſing the ſines proportional to the angles, the ſine of FCL the angle of deviation, is as the difference between AD and NS, that is, as 3—2, or 1, whence the ſine of incidence is to the ſine of the angle of deviation as 3 to 1. In like manner it may be ſhewn, that, when the ray paſſes obliquely out of glaſs into air, the ſine of the angle of incidence will be to that of deviation, as NS to AD—NS, that is, as 2 to 1. In paſſing out of air into water, the ſine of the angle of incidence is to that of refraction, as 4 to 3, and to that of deviation, as 4 to 4—3, or 1: and in paſſing out of water into air, the ſine of the angle of incidence is to that of refraction, as 3 to 4, and to that of deviation, as 3 to 1. Plate 6. Fig. 6.

Cor. 3. Hence a ray of light cannot paſs out of water into air at a greater angle of incidence than 48° 36′, the ſine of which is to radius as 3 to 4. Out of glaſs into air the angle muſt not exceed 40° 11′, becauſe the ſine of 40° 11′ is to radius as 2 to 3 nearly: conſequently, when the ſine has a greater proportion to the radius than that mentioned, the ray will not be refracted.

Schol. 2. It muſt be obſerved, that when the angle is within the limit, for light to be refracted, ſome of the rays will be reflected. For the ſurfaces of all bodies are for the moſt part uneven, which occaſions the diſſipation of much light by the moſt tranſparent bodies; ſome being reflected, and ſome refracted, by the inequalities on the ſurfaces. Hence a perſon can ſee through water, and his image reflected by it at the ſame time. Hence alſo, in the duſk, the furniture in a room may be ſeen by the reflection of a window, while objects that are without are ſeen through it.

Plate 6. Fig. 6. Exp. Upon a ſmooth board, about the centre C, deſcribe a circle HOKP; draw two diameters of the circle OP, HK, perpendicular to each other; draw ADM perpendicular to OP; cut off DT and CI equal to ¾ DA; through TI, draw TIS, cutting the circumference in S; NS drawn from S perpendicularly upon OP, will be equal to DT, or ¾ of DA. Then if pins be ſtuck perpendicularly at A, C and S, and the board be dipped in the water as far as the line HK, the pin at S will appear in the ſame line with the pins at A and C. This ſhews, that the ray which comes from the pin S is ſo refracted at C, as to come to the eye along the line CA: whence the ſine of incidence AD is to the ſine of refraction NS, as 4 to 3. If other pins were fixed along CS, they would all appear in AC produced; which ſhews that the ray is bent at the ſurface only. The ſame may be ſhewn, at different inclinations of the incident ray, by means of a moveable rod turning upon the centre C, which always keep the ratio of the ſines AD, NS, as 4 to 3. Alſo the ſun's ſhadow, coinciding with AC, may be ſhewn to be refracted in the ſame manner.

Cor. 4. The image R of a ſmall object G, placed under water, is one fourth nearer the ſurface than the object. And hence the bottom of a pond, river, &c. is one third deeper than it appears to a ſpectator. If a river appears to be only 4½ feet deep, it will be 6 feet; a perſon not apprized of this, might venture into water at the hazard of his life. And hence alſo we have the reaſon of the common phenomenon of a ſhilling, or other object, placed in an empty veſſel, appearing to be elevated higher and higher as the veſſel is filled with water. Suppoſe the veſſel empty, CQ its ſide opake, (Fig. 16.) G the object; if the eye be at H, the object will be hidden by the ſide CQ; but by filling the veſſel with water up to DB, it will become viſible, and ſeen at R, the ray EG being refracted into HE. And if the eye be ſo placed as to ſee the object at G when the veſſel was empty, while it is filling the object will appear to riſe gradually. Hence appears the reaſon why a ſtraight ſtick HER, (Fig. 16.) immerſed obliquely into water, appears bent at the ſurface DB; the image of the part within lying above the object.

PROP. XIV. Rays of light which paſs perpendicularly out of one medium into another, ſuffer no refraction.

Plate 6. Fig. 6. When AC, the incident ray, coincides with OC, the perpendicular, the action of the medium Z or X to accelerate or retard the motion of the ray, being perpendicular to its ſurface, cannot turn the ray out of its perpendicular path.

PROP. XV. When parallel rays paſs obliquely out of one medium into another through a plane ſurface, they will continue parallel after refraction.

Plate 6. Fig. 9. Let AB, CD, be parallel rays, falling on the plane ſurface RBD of a medium of different denſity: becauſe they make equal angles of incidence with their reſpective perpendiculars OP, ST, they will ſuffer an equal degree of refraction; that is, the angles of refraction, EBP, FDT, will be equal; whence the refracted rays BE, DF, will be parallel.

PROP. XVI. Through a plane ſurface, if diverging rays paſs out of a rarer into a denſer medium, they are made to diverge leſs; and if they paſs out of a denſer into a rarer medium, they are made to diverge more: if converging rays paſs out of a rarer into a denſer medium, they will be made to converge leſs; if out of a denſer into a rarer, to converge more.

Plate 6. Fig. 10.

Let the diverging rays AB, AE, AF, paſs out of a rarer into a denſer medium, through the plane ſurface GH, and let the ray AB be perpendicular to that ſurface; the reſt being refracted towards their reſpective perpendiculars IK, LM, and that the leaſt which falls the fartheſt from B, they will proceed in the directions EN and FO, diverging in a leſs degree from the ray AP, than they did before refraction: whereas had they proceeded out of a denſer into a rarer medium, they would have been refracted from their perpendiculars EK, FM, and thoſe the moſt which were the moſt oblique, and therefore would have diverged more than before. Again, let the converging rays AB, CD, EF, paſs out of a rarer into a denſer medium, through the plane ſurface GH, and let the ray AB be perpendicular to that ſurface; the other rays being refracted towards their reſpective perpendiculars IK, LM, and EF being refracted more than CD, they will proceed in the directions DN, FN, converging in a leſs degree towards the ray AN, than they did before: whereas, had the firſt medium been the denſer, they would have been refracted the other way, and therefore have converged more.

Fig. 11.

DEF. XIV. A *Lens* is a round piece of poliſhed glaſs, which has both its ſides ſpherical, or one ſpherical, and the other plane.

A lens may either be convex on both ſides, plano-convex, concave, plano-concave, or convex on one ſide, and concave on the other; which laſt is called a *meniſcus*.

In plate 6. fig. 12. Sections of theſe, formed by a plane paſſing perpendicularly through their centres, are repreſented.

DEF. XV. The *Axis of a Lens*, is a right line, paſſing through its centre, perpendicular to both its ſurfaces, and the extremities of the axes are its poles.

Each kind of lens is generated by the revolution of a ſection of the lens about this line. Thus, in the firſt lens, if *acb*, *adb*, revolve about *cd*, the convex lens will be formed.

DEF. XVI. In every beam of light, the middle ray is called the *Axis*.

DEF. XVII. Rays are ſaid to fall *directly* upon a lens, if their axis coincides with the axis of the lens; otherwiſe, they are ſaid to fall *obliquely*.

DEF. XVIII. The point in which parallel rays are collected by paſſing through a lens, is called the *Focus of parallel rays* of that lens.

PROP. XVII. Through a convex ſurface of the denſer medium, parallel rays, paſſing out of a rarer into a denſer medium, will become converging;—diverging rays will be made to diverge leſs, to become parallel, or to converge, according to the degree of divergency before refraction, or of the convexity of the ſurface;—rays converging towards the centre of convexity will ſuffer no refraction;—rays converging to a point beyond the centre of convexity, will be made more converging;—and rays converging towards a point nearer the ſurface than the centre of convexity, will be made leſs converging by refraction:—and when the rays proceed out of a denſer into a rarer medium, the contrary occurs in each caſe.

Plate 6. Fig. 13. Let AB, ID, be parallel rays entering a denſer medium through the convex ſurface CDE, whoſe centre of convexity is L; and let one of theſe, ID, be perpendicular to the ſurface. This will paſs on through the centre without ſuffering any refraction, but the other being oblique to the ſurface, will be refracted towards the perpendicular LB, and will therefore be made to proceed in ſome line, as PK, converging towards the other ray, and meeting it in K, the focus. Had one ray diverged from the other, ſuppoſe in the line MP, it would, by being refracted towards its perpendicular LB, have been made either to diverge leſs, be parallel, or to converge. Let the line ID be produced to K; and if the ray had converged, ſo as to have deſcribed the line NBL, it would then have been coincident with its perpendicular, and have ſuffered no refraction. If it had proceeded with leſs convergency towards any point beyond L in the line IK, it would have been made to converge more by being refracted towards the perpendicular LB, which converges more than it; and had it proceeded with more convergency than BL, that is, towards any point between D and L, being refracted towards the perpendicular, it would have been made to converge leſs.

And the contrary happens, when rays proceed out of a denſer into a rarer medium, through a concave ſurface of the denſer. For being now refracted from their reſpective perpendiculars, as they were before towards them, if they are parallel before refraction, they diverge afterwards: if they diverge, their divergency is increaſed; if they converge in the direction of their perpendiculars, they ſuffer no refraction; if they converge leſs than their reſpective perpendiculars, they are made to converge ſtill leſs, to be parallel, or to diverge; if they converge more, their convergency is increaſed. All which may clearly be ſeen by the figure, imagining the rays AB, ID, &c. bent the contrary way in their refractions to what they were in the former caſes.

Exp. Let parallel, diverging, and converging rays, paſs through a convex lens; the ſeveral caſes of this propoſition will be confirmed.

If CDEH be a convex lens, whoſe axis is IK, let L be the centre of the firſt convexity CDE, and M that of the other CHE; and let the ray AB be parallel to the axis; through B draw the line LN, which will be perpendicular to the ſurface CDE at that point. The ray AB in entering the denſer ſubſtance of the lens will be refracted towards the perpendicular, and therefore proceed after it has entered the ſurface at B in ſome direction inclined towards the axis, as BP. Through M the centre of convexity of this ſurface and the point P draw the line MR, which paſſing through the centre will be perpendicular to the ſurface at P, and the ray now entering a rarer medium will be refracted from the perpendicular into ſome direction as PK. In like manner, and for the ſame reaſons, the parallel ray ST on the other ſide the axis, and alſo all the intermediate ones, as XZ, &c. will meet it in the ſame point, unleſs the rays AB and ST enter the ſurface of the lens at too great a diſtance from the axis IK, the reaſon of which will be afterwards explained.

The point K where the parallel rays AB, ST, &c. are ſuppoſed to be collected by paſſing through the lens CE, is called the focus of parallel rays of that lens.

If the rays come diverging from a point equally diſtant from the ſurface as the focus of parallel rays, they will be rendered parallel; if from a point farther from the ſurface than L, they will be brought to a point beyond L; if from a point nearer than L, they will diverge leſs; as may be inferred from Prop. XII.

If the rays come converging towards L, they will ſuffer no refraction; if towards a point beyond L, they will become more converging; if towards a point nearer the ſurface than L, they will become leſs converging: as is ſufficiently explained in the proof of this propoſition.

Plate 6. Fig. 14.

Schol. If the rays AB, CD, EF, be parallel to each other, but oblique to GH the axis of the lens IK, or if the diverging rays CB, CF, proceed as from ſome point C which is not ſituated in the axis of the lens, they will be collected into ſome point as L, not directly oppoſite to the radiant C, but nearly ſo: for the ray CD, which paſſes through the middle of the lens, and falls upon the ſurface of it with ſome obliquity, will itſelf ſuffer a refraction at D and N; but it will be refracted the contrary way in one place from that in the other; and theſe refractions will be equal in degree if the lens has an equal convexity on each ſide, as we may eaſily perceive if we imagine ND to be a ray paſſing out of the lens both at N and D, for it is evident the line ND has an equal inclination to each ſurface at both its extremities. Upon which account the difference between the ſituation of the point L, and one directly oppoſite to C, is ſo ſmall, that it is generally neglected; and the focus is ſuppoſed to be in that line, in which a ray, that would paſs through the middle point of the lens, were it to ſuffer no refraction, would proceed.

PROP. XVIII. When rays paſs out of a rarer into a denſer medium, through a concave ſurface of the denſer, if the rays are parallel before refraction, they are made to diverge :—if they are divergent, they are made to diverge more, to ſuffer no refraction, or to diverge leſs, according as they proceed from ſome point beyond the centre, from the centre, or from ſome point between the centre and the ſurface :—if they are convergent, they are either made leſs converging, parallel, or diverging, according to their degree of convergency before refraction :—and the reverſe, in paſſing out of a denſer into a rarer medium.

Let

Plate 6. Fig. 15. Let MF, OI, be two parallel rays entering a concave and denſer medium, the centre of whoſe convexity is H, and the perpendicular to the refracting ſurface at the point F is LH; the ray OI, if we ſuppoſe it perpendicular to the ſurface, will proceed on directly without refraction, but the oblique ray MF, being refracted towards the perpendicular HL, will recede from the other ray OI. If the ray MF had proceeded from a point in OI farther from the ſurface than H, it would have been bent nearer to the perpendicular, and therefore have diverged more: if it had diverged from the centre H, it would have fallen in with the perpendicular HL, and not have been refracted at all: and had it proceeded from a point nearer the ſurface than the centre H, it would by being refracted towards the perpendicular HL, have proceeded in ſome line nearer it than it otherwiſe would have done, and ſo would diverge leſs than before refraction. Laſtly, if it had converged, it would have been rendered leſs converging, parallel, or diverging, according to the degree of convergency, which it had before it entered into the refracting ſurface.

If the ſame rays proceed out of a denſer into a rarer medium through a convex ſurface of the denſer, the contrary happens in each ſuppoſition: the parallel rays are made to converge; thoſe which diverge leſs than their reſpective perpendiculars, that is, thoſe which proceed from a point beyond the centre, are made leſs diverging, parallel, or converging, according to the degree in which they diverge before refraction; thoſe which diverge more than their reſpective perpendiculars, that is, thoſe which proceed from a point between the centre and the refracting ſurface, are made to diverge ſtill more. And thoſe which converge, are made to converge more. All which may eaſily be ſeen by conſidering the ſituation of the rays with reſpect to the perpendicular HL.

Plate 6. Fig. 15. Exp. Let parallel, diverging, and converging rays, paſs through a concave lens: the ſeveral caſes of this propoſition will be confirmed: thus, let ABCD repreſent a concave lens, EO its axis, FH the radius of the firſt concavity, IK that of the ſecond; produce HF to L, and let MF be a ray of light entering the lens at the point F. This ray being refracted towards the perpendicular FL, will paſs on to ſome point, as K in the other ſurface, more diſtant from the axis than F, and being there refracted from the perpendicular IK, will be diverted farther ſtill from the axis, and proceed on the direction KN, as from ſome point O, on the firſt ſide of the lens. In like manner other rays, as PQ parallel to the former, will proceed after refraction at both ſurfaces as from the ſame point O; which upon that account will be the imaginary focus of parallel rays of this lens.

If the rays diverge before they enter the lens, their imaginary focus is then nearer the lens than that of the parallel rays. If they converge before they enter the lens proceeding towards ſome diſtant point in the axis, as E, they are then rendered leſs converging: if they converge to a point at the ſame diſtance from the lens with the focus of parallel rays, they then go out parallel; if to a point at a leſs diſtance, they remain converging, but in a leſs degree than before they entered the lens.

Schol. If the lens is plane on one ſide, and convex or concave on the other, the refraction is ſimilar, but in a leſs degree. In a meniſcus, if the convexity on one ſide is equal to the concavity on the other, the two ſides will produce equal and contrary effects, and the inclination of the rays to each other will be the ſame after refraction as before. If the convexity is greater than the concavity, the meniſcus will have the effect of a lens which has its convexity equal to the exceſs of the convexity of the meniſcus above its concavity; and the reverſe, if its concavity exceeds its convexity.

PROP. XIX. When diverging rays are made to converge by paſſing through a convex lens, as the radiant approaches towards the lens on one ſide, the focus departs from it on the other; and the reverſe.

For, the nearer the radiant point is to the lens, the more the rays which fall upon the lens diverge before refraction; whence (the power of the refracting medium being given) they will converge the leſs after refraction, and have their focal point at the greater diſtance from the ſurface: on the contrary, the more remote the radiant point is from the lens, the leſs the incident rays will diverge, and conſequently the more will the refracted rays converge, and the nearer will the focus be to the ſurface, till, at an infinite diſtance of the radiant, the rays are collected in the focus of parallel rays.

PROP. XX. When the radiant point is at that diſtance from the ſurface, at which parallel rays coming through it from the other ſide would be collected, rays flowing from that point become parallel on the other ſide.

It manifeſtly follows from Prop. XII. that if the parallel rays AB, ID, ST, in paſſing through CDE, are brought to a focus in K, rays from K as a radiant point will, after refraction, proceed in the parallel lines BA, DI, TS. Plate 6. Fig. 13.

PROP. XXI. When rays paſs out of one medium into another of different denſity through a plane ſurface, if they diverge, the focal diſtance will be to that of the radiant point;—if they converge, it will be to that of the imaginary focus of the incident rays, as the ſine of the angle of incidence is to that of the angle of refraction.

This propoſition admits of four caſes.

Caſe 1. Of diverging rays paſſing out of a rarer into a denſer medium.

Let X repreſent a rarer, and Z a denſer medium, ſeparated from each other by the plane ſurface AB; ſuppoſe CD and CE to be two diverging rays proceeding from the point C, the one perpendicular to the ſurface, the other oblique; through E draw the perpendicular PK. The ray CD being perpendicular to the ſurface, will proceed on in the right line CQ, but the other falling upon it obliquely at E, and there entering a denſer medium, will ſuffer a refraction towards the perpendicular EK. Let then EG be the refracted ray, and produce it back till it interſects DC produced alſo in F; this will the focal point. On the centre E with the radius EF, deſcribe the circle AFBQ, and produce EC to H; draw HI the ſine of the angle of incidence, and GK that of refraction; equal to this is FP or CM, which let be drawn. Now if we ſuppoſe the points D and E contiguous, or nearly ſo, then will the line HE be almoſt coincident with FD, and therefore FD will be to CD, as HE to CE, or Plate 6. Fig. 16. Plate 6. Fig. 16.

(El. VI. 4.) as HI to CM; that is, the focal diſtance of the ray CE is to the diſtance of the radiant point, as the ſine of the angle of incidence is to that of the angle of refraction.[*]

Caſe 2. Of diverging rays proceeding out of a denſer into a rarer medium.

Let X be the denſer, Z the rarer medium, FD and FE two diverging rays proceeding from the point F; and ſuppoſing the perpendicular PK drawn as before, FP will be the ſine of the angle of incidence of the oblique ray FE, which in this caſe being refracted from the perpendicular, will paſs on in ſome line as ER, which being produced back to the circumference of the circle will cut the ray FD ſomewhere, ſuppoſe in C, this therefore will be the imaginary focus of the refracted ray ER; draw RO the ſine of the angle of refraction, to which HI will be equal: but here alſo FP or its equal CM, is to HI, as EC to EH, or (if the point D and E be conſidered as contiguous) as DC to DF; that is, the ſine of the angle of incidence is to the ſine of the angle of refraction, as the focal diſtance to that of the real radiant point.

Caſe 3. Of converging rays paſſing out of a denſer medium into a rarer.

Plate 6. Fig. 16. Next; let Z be the denſer, X the rarer medium, and GE the incident ray; this will be refracted from the perpendicular into a line as EH: then all things remaining as before, GK, or its equal FP, or CM will be the ſine of the angle of incidence, and HI that of refraction: but theſe lines, as before, are to each other, as DC to DF; that is, the focal diſtance is to the diſtance of the imaginary focus, as the ſine of the angle of incidence to that of the angle of refraction.

Caſe 4. Of converging rays paſſing out of a rarer into a denſer medium.

Let Z be the rarer, X the denſer medium, and RE the incident ray; this will be refracted towards the perpendicular into a line, as EF; C will be the imaginary focus, and F the real one, HI which (El. I. 26.) is equal to RO, will be the ſine of the angle of incidence, and FP that of the angle of refraction: but theſe are to each other (El. VI. 2.) as DF to DC; and therefore the focal diſtance is to that of the imaginary focus, as the ſine of the angle of incidence is to that of the angle of refraction.

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[*] Whereas IE is to ME, or ND to CD, as HI to CM, that is, as the ratio of the ſine of the angle of incidence to that of the angle of refraction, which (Prop XIII.) is always the ſame, the line IN is in all inclinations of the ray CE, at the ſame diſtance from CM. Conſequently, had CE been coincident with CD, the point H had fallen upon N; and becauſe the circle paſſes through both H and F, F would alſo have fallen upon N; upon which account the focus of the ray CE would have been there. But the ray CE being oblique to the ſurface DB, the point H is at ſome diſtance from N; and therefore the point F is neceſſarily ſo too, and the more ſo, the greater that diſtance is. Hence it is manifeſt, that no two rays flowing from the radiant point C, and falling with different obliquities on the ſurface BD, will, after refraction there, proceed as from the ſame point; therefore, ſtrictly ſpeaking, there is no one point in the line D produced, that can more properly be called the focus of rays flowing from C, than another: for thoſe which enter the refracting ſurface near D, will, after refraction, proceed, as has been obſerved, from the parts about N; thoſe which enter near E, will flow as from the parts about F; thoſe which enter about T, as from ſome points in the line DF produced, &c. And it is farther to be obſerved, that when the angle DCE becomes large, the line NF increaſes apace; whence thoſe rays which fall near T, proceed, after refraction, as from a more diffuſed ſpace, than thoſe which fall at the ſame diſtance from each other near the point D. Upon which account it is uſual, with optical writers, to ſuppoſe the diſtance between the points where the rays enter the plane ſurface of a refracting medium, to be inconſiderable with regard to the diſtance of the radiant point, if they diverge; or to that of their imaginary focus, if they converge: and unleſs there be ſome particular reaſon to the contrary, they conſider them as entering the refracting medium in a direction as nearly perpendicular to its ſurfaces as may be.

PROP. XXII. When parallel rays fall upon a ſpherical ſurface of different denſity, the focal diſtance will be to the diſtance of the centre of convexity, as the ſine of the angle of incidence is to the difference between that ſine and the ſine of the angle of refraction.

Caſe 1. Of parallel rays paſſing out of a rarer into a denſer medium through a convex ſurface of the denſer.

Let AB repreſent a convex ſurface; C its centre of convexity; HA and DB two parallel rays, paſſing out of a rarer medium into a denſer, the one perpendicular to the refracting ſurface, the other oblique: draw CB; this being a radius, will be perpendicular to the ſurface at the point B; and the oblique ray DB being in this caſe refracted towards the perpendicular, will proceed in ſome line, as BF, meeting the other ray in F, which will therefore be the focal point; produce CB to N, then will DBN, or (El. I. 29.) its equal BCA be the angle of incidence, and FBC that of refraction. Now, ſince any angle has the ſame ſine with its ſupplement to two right ones, the angle of FCB being the ſupplement of ACB, which is equal to the angle of incidence, may here be taken for that angle; and therefore, as the ſides of a triangle have the ſame relation to each other, as the ſines of their oppoſite angles have, FB being oppoſite to this angle, and FC being oppoſite to the angle of refraction, they may here be conſidered as the ſines of the angles of incidence and of refraction; and for the ſame reaſon CB may be conſidered as the ſine of the angle CFB, which angle being, together with the angle FBC, equal to the external one ACB (El. I. 32) it is itſelf equal to the difference between thoſe two laſt angles; and therefore the line FB is to CB, as the ſine of the angle of incidence is to the ſine of an angle which is equal to the difference between the angle of incidence and of refraction. Now becauſe in very ſmall angles as theſe are, for we ſuppoſe in this caſe alſo the diſtance AB to vaniſh, (the reaſon of which will be ſhewn in the note) their ſines have nearly the ſame ratio to each other that they themſelves have, the diſtance FB will be to CB as the ſine of the angle of incidence is to the difference between that ſine and the ſine of the angle of refraction: but becauſe BA vaniſhes, FB and FA are equal, and therefore FA is to CA in that ratio.* Plate 6. Fig. 17.

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* It appears from the above propoſition, that the focal diſtance of the oblique ray DB, is ſuch, that the line BF ſhall be to the line CB or CA as the ſine of the angle of incidence to the ſine of an angle equal to the difference between the angle of incidence and refraction; therefore ſo long as the angles BCA, &c. are ſmall, ſo long the line FB is nearly of the ſame length, becauſe ſmall angles have nearly the ſame ratio to each other that their ſines have. But when the point B is removed far from A, ſo that the ray DB enters the ſurface, ſuppoſe about O, the angles BCA, &c. becoming large, the ſine of the angle of incidence begins to bear a conſiderably leſs ratio to the ſine of an angle which is equal to the difference between the angle of incidence and refraction than before, and therefore the line BF begins to bear a much leſs ratio to BC; wherefore its length decreaſes apace: upon which account thoſe rays which enter the ſurface about O, not only meet nearer the centre of convexity than thoſe which enter at A, but are collected into a more diffuſed ſpace. From hence it is, that the point where thoſe only which enter near A, are collected, is reckoned the true focus; and the diſtance AB in all demonſtrations relating to the foci of parallel rays entering a ſpherical ſurface, whether convex or concave, is ſuppoſed to vaniſh.

Caſe 2. Of parallel rays paſſing out of a denſer into a rarer medium through a concave ſurface of the denſer.

Plate 6. Fig. 17. Let AB be the concave ſurface of the denſer medium, C the centre of convexity, and HA and DB two parallel rays. Through B the point where the oblique ray DB enters the rarer medium, draw the perpendicular CN; and let the ray DB, being in this caſe refracted from the perpendicular, proceed in the direction BM; produce BM back to H; this will be the imaginary focus, and DBN, or its equal ACB, will be the angle of incidence, and CBM, or its equal (El. I. 15.) HBN that of refraction; produce DB to L, and draw BF ſuch, that the angle LBF may be equal to DBH: then becauſe NBD and DBH together are equal to NBH the angle of refraction, therefore BCA which is equal to the firſt, and LBF which is equal to the ſecond, are together equal to the angle of refraction; but LBF is equal to BFA, (El. I. 29) conſequently, BFA and BCA together are equal to the angle of refraction; and therefore ſince one of them, BCA, is equal to the angle of incidence, the other, BFA, is the difference between that angle, and the angle of refraction. Now FB the ſine of the angle FCB, or which is the ſame thing, of its ſupplement to two right ones BCA, the angle of incidence, is to CB the ſine of the angle BFC, as FB to CB, that is, as HB to CB; for the angles DBH and LBF being equal, the lines BF and BH are ſo too; but the diſtance BA vaniſhing, HB is to CB, as HA to CA: that is, the ſine of the angle of incidence is to the ſine of an angle which is the difference between the angle of incidence and refraction, or becauſe the angles are ſmall, to the difference between the ſine of the angle of incidence and that of refraction, as the diſtance of the focus from the ſurface is to that of the centre from the ſame. Plate 6. Fig. 17.

Caſe 3. Of parallel rays paſſing out of a rarer into a denſer medium through a concave ſurface of the denſer.

Let AB be the concave ſurface of the denſer medium, and let LB and FA be the incident rays. Now whereas, when DB was the incident ray, and paſſed out of a rarer into a denſer medium, as in Caſe the firſt, it was refracted into a line BF, this ray LB having the ſame inclination to the perpendicular, will alſo ſuffer the ſame degree of refraction, and will therefore paſs on afterwards in the line FB produced, towards P. So that whereas, in that caſe the point F was the real focus of the incident ray DB, the ſame point will, in this caſe, be the imaginary focus of the incident ray LB: but it was there demonſtrated, that the diſtance FA is to CA, as the ſine of the angle of incidence is to the difference between that and the ſine of the angle of refraction, therefore the focal diſtance of the refracted ray BP is to the diſtance of the centre of convexity in that ratio.

Caſe 4. Of parallel rays paſſing out of a denſer into a rarer medium through a convex ſurface of the denſer.

Let AB be the convex ſurface of the denſer medium, and let LB and FA be the incident rays, as before. Now whereas, when DB was the incident ray paſſing out of a denſer into a rarer medium, it was refracted into BM, as in Caſe the ſecond, having a point as H in the line MB produced for its imaginary focus; therefore LB, for the like reaſon as was given in the laſt caſe, will in this be refracted into BH, having the ſame point H for its real focus. So that here alſo the focal diſtance will be to that of the centre of convexity, as the ſine of the angle of incidence is to the difference between that and the ſine of the angle of refraction.

Cor. 1. Hence, the ſines of the angles of incidence and of refraction of parallel rays being given, and alſo the diſtance of the centre of convexity from the ſurface, the focus of any lens may be eaſily found.

Cor. 2. The diſtance of the centre of a glaſs ſphere, from the principal focus of rays refracted by it, is nearly equal to a radius and a half of the ſphere. For in this caſe it will be $FA : CA :: I : 2I - 2R$, (where I and R ſignify the ſines of incidence and refraction) therefore $FA = \frac{3CA}{6-4} = \frac{3CA}{2}$. And if the ſphere be of water inſtead of glaſs, $FA = \frac{4CA}{2} = 2CA =$ the diameter of the ſphere.

PROP. XXIII. When diverging or converging rays enter into a medium of different denſity through a ſpherical ſurface, the ratio compounded of that which the focal diſtance bears to the diſtance of the radiant point (or of the imaginary focus of the incident rays, if they converge) and of that, which the diſtance between the ſame radiant point (or imaginary focus) and the centre, bears to the diſtance between the centre and the focus, is equal to the ratio, which the ſine of the angle of incidence bears to the ſine of the angle of refraction: that is, $FD \times CA : DA \times CF :: FB : BG$.

Caſe 1. Of diverging rays paſſing out of a rarer into a denſer medium through a convex ſurface of the denſer, with ſuch a degree of divergency, that they ſhall converge after refraction.

Let BD repreſent a ſpherical ſurface, and C its centre of convexity; and let there be two diverging rays AB and AD, proceeding from the radiant point A, the one perpendicular to the ſurface, the other oblique. Through the centre C produce the perpendicular ray AD to F, and draw the radius CB and produce it to K, and let BF be the refracted ray; then will F be the focal point: produce AB to H, and through the point F draw the line FG parallel to CB. AB being the incident ray, and CK perpendicular to the ſurface at the point B, the angle ABK, or which is equal to it, becauſe of the parallel lines CB and FG, FGH is the angle of incidence. Now, ſince the ſupplement of any angle to two right ones has the ſame ſine with the angle itſelf, the ſine of the angle FGB, which is the ſupplement of FGH to two right ones, may be conſidered as the ſine of the angle of incidence; for which ſine the line FB, as the ſides of a triangle have the ſame ratio to each other, that the ſines of their oppoſite angles have, may be taken. Again, the angle FBC is the angle of refraction, or its equal, becauſe alternate to it, BFG, to which BG being an oppoſite ſide, may be taken as the ſine. But FB is to BG in a ratio compounded of FB to BA, and of BA to BG for the ratio that any two quantities bear to each other, is compounded of the ratio which the firſt bears to any other, and of the ratio which that other bears to the ſecond. Now FB is to BA, ſuppoſing BD to vaniſh, as FD to DA; and BA is to BG, becauſe of the parallel lines CB and FG, as AC to CF. That is, the ratio compounded of FD, the focal diſtance, to DA, the diſtance of the radiant point, and of AC, the diſtance

Plate 6. Fig. 19.

between the radiant point and the centre, to CF, the diſtance between the centre and the focus, is equal to that which the ſine of the angle of incidence bears to the ſine of the angle of refraction.*

Caſe 2. Of converging rays paſſing out of a rarer into a denſer medium through a concave ſurface of the denſer with ſuch a degree of convergency, that they ſhall diverge after refraction.

Plate 6. Fig. 19. Let the incident rays be HB and FD paſſing out of a rarer into a denſer medium through the concave ſurface BD, and tending towards the point A, from whence the diverging rays flowed in the other caſe; then the oblique ray HB having its angle of incidence HBC equal to ABK the angle of incidence in the former caſe, will be refracted into the line BL ſuch, that its angle of refraction KBL will be equal to FBC the angle of refraction in the former caſe; that is, it will proceed after refraction in the line FB produced, having the ſame focal diſtance FD with the diverging rays AB, AD, in the other caſe. But, by what has been already demonſtrated, the ratio compounded of FD, the focal diſtance, to DA, in this caſe, the diſtance of the imaginary focus of the incident rays, and of AC, the diſtance between the ſame imaginary focus and the centre, to CF, the diſtance between the centre and the focus, is equal to that which the ſine of the angle of incidence bears to the ſine of the angle of refraction.

Caſe 3. Of diverging rays paſſing out of a rarer into a denſer medium through a convex ſurface of the denſer, with ſuch a degree of divergency as to continue diverging.

Plate 6. Fig. 18. Let AB, AD, be the diverging rays, and let their divergency be ſo great, that the refracted ray BL ſhall alſo diverge from the other; produce LB back to F, which will be the focal point; draw the radius CB, and produce it to K, produce BA likewiſe towards G, and draw FG parallel to BC. Then will ABK be the angle of incidence, whoſe ſine, BF may be taken for, as being oppoſite to the angle BGF, which is the ſupplement of the other to two right ones. And LBC is the angle of refraction, or its equal KBF, or which is equal to this, BFG, as being alternate; therefore BG, the oppoſite ſide to this may be taken for the ſine of the angle of refraction. But BF is to BG, for the like reaſon as was given in Caſe the firſt, in a ratio compounded of BF to BA, and of BA to BG. Now BF is to BA, (DB vaniſhing) as DF to DA, and becauſe of the parallel lines FG and BC, the triangles CBA and AGF are ſimilar, therefore BA is to AG, as CA to AF; conſequently, BA is to BA together with AG, that is, to BG, as CA is to CA together with AF, that is, CF. Therefore the ratio compounded of DF the focal diſtance to DA the

diſtance of the radiant point, and of CA the diſtance between the radiant point and the centre, to CF, the diſtance between the centre and the focus, is equal to that which the ſine of the angle of incidence bears to the ſine of the angle of refraction.

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* Since the focal diſtance of the oblique ray AB is ſuch that the compound ratio of FB to BA and of AC to CF ſhall be the ſame, whatever be the diſtance between B and D; it is evident, that, AC being always of the ſame length, the more the line AB lengthens, the more FB muſt lengthen too, or elſe FC muſt ſhorten; but if BF lengthens, CF will do ſo too, and in a greater ratio with reſpect to its own length than BF will, therefore the lengthening of BF will conduce nothing towards preſerving the equality of the ratio: but as AB lengthens, BF and CF muſt both ſhorten, which is the only poſſible way in which the ratio can be continued the ſame. And it is alſo apparent, that the farther B moves from D towards O, the faſter AB lengthens, and therefore the farther the rays enter from D, the nearer to the refracting ſurface is the place where they meet, but the ſpace they are collected in, is the more diffuſed: and therefore in this caſe, as well as thoſe taken notice of in the two preceding notes, different rays, though flowing from the ſame point, will conſtitute different foci; and none are ſo effectual as thoſe which enter at or very near the point D. And ſince the ſame is obſervable of converging as well as of diverging rays, none, except thoſe which enter very near that point, are uſually taken into conſideration; upon which account it is, that the diſtance DB, in determining the focal diſtances of diverging or converging rays entering a convex or concave ſurface, is ſuppoſed to vaniſh.

SCHOL. By making HB and CD the incident rays, the propoſition may be proved of converging rays paſſing out of a rarer into a denſer medium, through a concave ſurface of a denſer, with ſuch convergency, that they ſhall continue to converge. Alſo, by the ſame method of reaſoning as in the preceding caſes, the propoſition may be proved, of diverging rays paſſing out of a denſer into a rarer medium through a concave ſurface of the denſer, and of converging rays paſſing out of a denſer into a rarer medium through a convex ſurface of the denſer. And all thoſe caſes which are the converſe of the preceding, admit of a ſimilar proof; for, when rays paſs out of one medium into another, the ſine of the angle of incidence has the ſame ratio to the ſine of the angle of refraction, as the ſine of the angle of refraction has to the ſine of the angle of incidence, when they paſs through the ſame lines of direction the contrary way.

Caſe 4. Of rays paſſing out of a denſer into a rarer medium, from a point between the centre of convexity and the ſurface.

Let AB, AD, be two rays paſſing out of a denſer into a rarer medium from the point A, which is taken between C the centre of convexity and the refracting ſurface BD; through B draw CK, and let BL be the refracted ray; produce BL back to F, and draw FG parallel to BC. Then will ABC be the angle of incidence, of which BF, being oppoſite to its alternate and equal angle BGF, is the ſine. LBK will be the angle of refraction, or its equal FBC, of which BG, being oppoſite to its ſupplement to two right angles BFG, is the ſine. But, BF is to BG in the compound ratio of BF to BA, and of BA to BG; and (BD vaniſhing) BF is to BA as DF to DA, and becauſe the lines CB and FG are parallel, BA is to BG as CA to CF. Plate 6. Fig. 20.

Caſe 5. Of rays paſſing out of a rarer into a denſer medium from a point between the centre of convexity and the ſurface.

Let AB, AD, be two diverging rays paſſing out of a rarer into a denſer medium through the refracting ſurface BD, whoſe centre of convexity is C, a point beyond that from whence the rays flow. Through B draw CK, and let BL be the refracted ray, produce it back to F, and draw FG parallel to BC meeting BA in G. ABC will be the angle of incidence, of which BF, being oppoſite to its ſupplement to two right angles BGF, is the ſine. The angle of refraction is LBK, or its equal FBC, of which BG, being oppoſite to its alternate and equal angle BFG, is the ſine. But BF is to BG in the compound ratio of BF to BA and of BA to BG; and (BD vaniſhing) BF is to BA as DF to DA; and becauſe of the parallel lines CB and GF, the triangles AFG and ABC are ſimilar. BA therefore is to AG, as CA to AF; conſequently, BA is to BA—AG, that is, to BG, as CA is to CA—AF, that is, to CF. Plate 6. Fig. 21.

In like manner the Propoſition may be proved of rays paſſing out of a denſer into a rarer medium towards a point between the centre of convexity and the ſurface, and in all other ſuppoſable caſes.

COR. Hence the diſtance of the radiant point, or of the imaginary focus, being given, and alſo the radius of convexity, and the ſines of the angles of incidence and refraction of diverging or converging rays, the focus of any lens may be thus found.

Let it be required to determine the focal diſtance of diverging rays paſſing out of air into glaſs through a convex ſurface, and let the diſtance of the radiant point be 20, and the radius of convexity be 5 : let the focal diſtance be expreſſed by x : then, becauſe by the preceding Propoſition the ratio compounded of that which the focal diſtance bears to the diſtance of the radiant point, (that is, in this ſuppoſition, of x to 20 ;) and of the ratio which the diſtance of the ſame radiant point from the centre bears to the diſtance between the centre and the focus (in this caſe, of 25 to $x-5$) is equal to the ratio which the ſine of the angle of incidence bears to the ſine of the angle of refraction, (that is, of 17 to 11) we ſhall have in the inſtance before us, the following proportion, $\left.\begin{matrix} x : 20 \\ 25 : x-5 \end{matrix}\right\} :: 17 : 11$, and compounding them in one, which is done by multiplying the two firſt parts together, we have $25x : 20x-100 :: 17 : 11$, $x = \frac{1700}{65}$.

SECT. II.—*Of* IMAGES *produced by* REFRACTION.

PROP. XXIV. Rays of light flowing from the ſeveral points of any object, farther from a convex lens than its principal focus, by paſſing through the lens, will be made to converge to points correſponding to thoſe from which they proceeded, and will form an image.

Plate 6. Fig. 22. Let ABC be a luminous or illuminated object. From every point, as A, B, C, rays diverge in all directions. Let ſome of theſe rays fall upon a convex lens GHK placed in a hole GK, in the window ſhutter of a dark room ML, at a greater diſtance from the object than the principal focal diſtance of the lens. BH being the axis, will (by Prop. XIV.) paſs through the lens without refraction in the direction BHE. But the collateral rays BG, BK, made equally convergent by the lens, will croſs the axis at E ; that is, all the rays which come from the point B in the object, will be united behind the lens in the focus E. In like manner, among the rays AG, AH, AK, which diverge from the point A, whilſt AH the axis (as was ſhewn Prop. XVII. Schol.) may be conſidered as if it went ſtraight through the lens, the other rays will be made to converge, and will be united in a focus at F : and alſo, the rays from C will be united in D. The ſame may be ſhewn concerning every other point in the object. Conſequently, there will be as many correſpondent foci in the image as there are radiant points in the object : and theſe foci will be diſpoſed in the ſame manner with reſpect to one another as the radiants, and will therefore form an image. The object muſt be farther from the lens than its principal focus, elſe the rays from the ſeveral radiants would not converge, but either become parallel or diverging, (by Prop. XVII.) whence no image would be formed.

Exp. 1. Let the rays of the ſun paſs through a convex lens into a dark room, and fall upon a ſheet of white paper placed at the diſtance of the principal focus from the lens.

2. The rays of a candle, in a room from which all external light is excluded, paſſing through a convex lens will form an image on white paper.

PROP. XXV. The image produced by rays of light paſſing through a convex lens is inverted.

Plate 6. Fig. 22. The focus in which the rays, that come from any point A, or B, are united, is, in the axis AHF, or BHE, of the beam, whether it fall directly or obliquely upon the lens. But the axis (by Prop. XVII.) is the middle ray of a cone of rays whoſe baſe is the ſurface of the lens, and vertex the radiant point. Every axis, therefore, as AHF, BHE, muſt paſs through H the middle point of the lens, and conſequently muſt croſs one another in that point: from which it is manifeſt, that the rays from the loweſt point C of the object will become the higheſt point of the image D; and that the image will be, with reſpect to the original object, inverted.

PROP. A. An object ſeen through a very ſmall aperture appears inverted. Becauſe the rays from the extreme parts of the object muſt have croſſed at the hole.

The rays of light are probably inflected as they paſs the edge of the aperture, for it renders diſtant objects very diſtinct to a ſhort-ſighted perſon, and has therefore the property of a concave lens. This effect may, perhaps, partly ariſe from the ſmall number of rays admitted to the eye, by which the confuſion to a ſhort-ſighted perſon may, in ſome meaſure, be taken off.

PROP. XXVI. The image will not be diſtinct, unleſs the plane ſurface, on which it is received, be placed at the diſtance of the principal focus of the lens.

For otherwiſe the rays which come from a ſingle point in the object, will not have its correſponding point in the image, but will be ſpread over a larger ſurface.

PROP. XXVII. As an object approaches a convex lens, its image departs from it, and as the object departs the image approaches.

Plate 6. Fig. 22. As the object ABC approaches the lens, the ſeveral radiants approach it: and conſequently (by Prop. XIX.) the ſeveral foci which form the image FED recede; and the reverſe. But the image can never be nearer the lens than its focus of parallel rays, ſince this is the place of the image, when the object is infinitely diſtant.

PROP. XXVIII. When the object is placed parallel to the image, the diameter of the object is to the diameter of the diſtinct image, as the diſtance of the object from the lens, is to the diſtance of the image from the lens.

Plate 6. Fig. 22. The radiant A (as appears from Prop. XXV.) is repreſented by its focus in the point F where the line AH, produced behind the lens, cuts a plane paſſing through the focus of parallel rays parallel to the lens. In like manner the radiant C is repreſented by its correſponding focus in the point D, in which the ſame plane is cut by the line CH produced. If therefore the diſtance of the extremities of the object, or its length, be AC, the length of its image will be DF. Since therefore AC is parallel to DF, the alternate angles ACD, CDF, (El. I. 29.) are equal; and alſo the alternate angles CAF, AFD: whence (El. VI. 4.) AC is to FD, as CH to DH, as BH is to EH, that is, the height of the object is to that of the image, as the diſtance of the object from the lens, to the diſtance of the image from the lens. Any diameter or line drawn acroſs the object may be proved, in like manner, to have the ſame ratio to any correſponding diameter or line drawn acroſs the image.

PROP. XXIX. When the image appears confuſed, it is larger than when it is diſtinct.

For the rays, in this caſe, are not received upon the white ſurface exactly at the diſtance from the lens at which they are brought to a focal point, but either at a diſtance greater or leſs: and in either caſe the rays which come from any radiant points at the extremities A and C, will not be collected into points on the plane at F and D, but be ſpread over a ſmall circular ſpace round theſe points; whence the confuſed image will be larger than the diſtinct image.

PROP. XXX. The object and the diſtinct image are ſimilar ſurfaces.

Though the ſide of any object which is towards the lens be not a plane ſurface, yet the light is reflected from it in the ſame manner as if the figure of the object were drawn upon the plane ſurface of a piece of canvas, and differently ſhaded. Therefore the ſide of the object next to the lens may be conſidered as a plane figure. And ſince (by Prop. XXVIII.) the height of the object is to that of the picture, as the diſtance of the object from the lens, to the diſtance of the image from the lens, and alſo the breadth of the object in any part, to the breadth of the image in the correſponding part in the ratio of theſe diſtances; it follows (El. V. 11.) that the height of the object is to the height of the image, as the breadth of the object in any part is to the breadth of the image in its correſponding part; that is, the object and image are ſimilar ſurfaces.

PROP. XXXI. The diameter of an image formed by rays paſſing from a given object through a convex lens, increaſes as the object approaches the lens, and decreaſes as the object recedes from the lens.

The diameter of the image (by Prop. XXVIII.) increaſes as its diſtance increaſes, and decreaſes as its diſtance decreaſes. And (by Prop. XXVII.) the diſtance of the image increaſes as the diſtance of the object decreaſes, and the reverſe. Whence the diameter of the image increaſes as the diſtance of the object decreaſes, and decreaſes as the diſtance of the object increaſes.

PROP. XXXII. When the diſtance of the object is given, the diameter of the image is as the diameter of the object.

If the object AC remains at the diſtance BH from the lens, the image (by Prop. XXVII.) will remain at the diſtance EH : whence the ratio of BH to EH, and conſequently (by Prop. XXVIII.) the ratio of the diameter AC to its correſpondent diameter DF, is given, or is invariable. Conſequently, if AC increaſes or decreaſes, DF muſt proportionally increaſe or decreaſe, that is, the diameter of the image is directly as the diameter of the object. Plate 6. Fig. 22.

PROP. XXXIII. When the diameter and diſtance of the object are given, the diameter of the image will be as its diſtance from the lens.

If the diameter and diſtance of the object are given, it is manifeſt that the diameter of the image cannot be varied without changing the lens. But if, inſtead of the lens GHK, one leſs convex, or more convex, be uſed, the rays will be brought to a focus, and the image (by Prop. XXIV.) will be formed at a greater or leſs diſtance from the lens. And ſince (by Prop. XXVIII.) the diſtance of the object is always to the diſtance of the image, as the diameter of the object to the diameter of the image ; the firſt and third terms remaining invariable, the ſecond and fourth muſt be increaſed or diminiſhed proportionally, that is, the diameter of the image will be directly as its diſtance from the lens.

PROP. XXXIV. When the diameter and diſtance of the object are given, the area of the image is as the ſquare of its diſtance from the lens.

Becauſe the ſurface of the image (by Prop. XXX.) is ſimilar to the ſurface of the object, whilſt the ſurface of the object remains the ſame, the image, in every variation of its magnitude, muſt be ſimilar to itſelf. But the areas of ſimilar ſurfaces (El. VI. 20. Cor. 1.) are as the ſquares of their homologous ſides, that is, as the ſquares of their heights or breadths. Therefore the area of the image is always as the ſquare of its diameter. And the diameter of the image, when the diameter and diſtance of the object are given, is (by Prop. XXXIII.) as its diſtance from the lens : therefore the ſquare of its diameter, or its area, is as the ſquare of its diſtance from the lens.

PROP. XXXV. Though the diſtance of the object from the lens be varied, the image may be preſerved diſtinct without varying the diſtance of the plane ſurface which receives it.

This will be the caſe, if as much as the image is brought forwards by the removal of the object, it is thrown backwards by diminiſhing the convexity of the lens, and the reverſe : or the image may be preſerved diſtinct without changing the lens, by increaſing or diminiſhing the diſtance of the lens from the plane ſurface which receives the image, in the ſame ratio as the diſtance of the object from the lens is increaſed or diminiſhed ; which may be done either by moving the lens or the plane ſurface.

PROP. XXXVI. The diſtances of the object and image, and the diameter of the object being given, the diameter of the image will not be altered by changing the area of the lens.

Plate 6. Fig. 22. The height of the image DF is the diſtance between the two extreme foci F and D; the former of which is always in the axis AHF of the cone which has A for its vertex, and the latter in the axis CHD of the cone whoſe vertex is C, which axes croſs each other in H. Since therefore DF, the height of the image, is the diſtance between theſe two lines AHF, CHD, where they meet the plane ſurface, the height of the image will be the ſame, whether the whole area, GHK is open, or only a ſmall part of it at H.

PROP. XXXVII. When the object is near the lens, but not ſo near as the principal focus, in order to make the image diſtinct, the area of the lens muſt be ſmall.

Plate 6. Fig. 23. If the object was as near to the lens as the principal focus, or nearer, no image (by Prop. XVII.) could be formed. But let the object A be at a diſtance from the lens NP, very little greater than that of the principal focus: then the extreme rays AN, AP, of the cone NAP, diverging more than the rays, AD, AE, if the plane ſurface which is to receive the rays, is placed where theſe rays are collected to a focus, the extreme rays AN, AP, diverging more, will not be collected, and the image on the plane ſurface will be confuſed. To prevent this, the extreme rays muſt be excluded by diminiſhing the area of the lens, or of the hole where it is placed. If the radiant A were at a greater diſtance, this would be unneceſſary. Suppoſing the lens at SR, the extreme rays AN, AP, would paſs above or below the lens, and only the middle rays, which are brought to a focus on the plane ſurface, would paſs through the lens.

PROP. XXXVIII. The brightneſs of an image, when its diſtance from the lens is given, is as the area of the lens.

Plate 6. Fig. 22. When the whole area GHK is open, the entire cone of rays AGK paſſes through the lens from the point A, and is brought to a focus at F: but when the area is diminiſhed to a ſmall ſurface at H, the greateſt part of the cone is excluded, and no rays, but the axis AH and thoſe which are near it, can paſs through the lens: whence it is manifeſt, that the focal point F muſt be more illuminated by the rays from A when the area of the lens is GHK, than when the area is diminiſhed. The ſame may be ſaid of every other cone of rays, and of every other point in the image. Therefore the whole image, although (by Prop. XXXVII.) made more diſtinct by diminiſhing the area, will be made fainter or leſs bright.

PROP. XXXIX. The brightneſs of the image, when the area of the lens and the diſtance of the object are given, is inverſely as the ſquare of its diſtance from the lens.

The area of the lens and diſtance of the object being given, the number of rays which paſs through the lens and form the image, is given. Now the ſame number of rays ſpread over a larger ſurface will not illuminate it ſo ſtrongly as they would a ſmaller ſurface: that is, the brightneſs will be inverſely as the illuminated area: and the area of the image is (by Prop. XXXIV.) as the ſquare of its diſtance from the lens: whence its brightneſs is inverſely as the ſquare of its diſtance.

PROP. XL. The heat at the focus of a burning glaſs, when the area of the glaſs is given, is inverſely as the ſquare of the focal diſtance.

The diſtance of the burning ſpot, that is, the image of the ſun, from the lens, is the focal diſtance, becauſe the ſun's rays are parallel. And becauſe the heat and the brightneſs at the focus are as the number of rays collected, the heat is as the brightneſs. But the brightneſs (by Prop. XXXIX.) is inverſely as the ſquare of the diſtance of the image from the lens: therefore the heat is in the ſame ratio, that is, in this caſe, inverſely as the ſquare of the focal diſtance of the glaſs.

PROP. XLI. The heat at the focus of a burning glaſs, when the focal diſtance is given, is as the area of the glaſs.

The brightneſs is (by Prop. XXXVIII.) as the area of the lens, and the heat is as the brightneſs: therefore the heat is alſo as the area of the lens.

PROP. XLII. The heat at the focus of a burning glaſs is to the common heat of the ſun, inverſely as the area of the focus to the area of the glaſs.

The brightneſs, or the heat, muſt be inverſely as the ſpace or area over which the rays which cauſe them are ſpread, that is, inverſely as the area of the focus to the area of the glaſs.

Schol. This propoſition ſuppoſes all the rays which fall upon the lens to paſs through it to the focus.

Exp. Moſt of the preceding Propoſitions, from Prop. XIX. to XXXIX. may be confirmed, Plate 12. Fig. 9. in a room from which all external light is excluded, by placing a convex lens *x*, fixed in a frame which moves perpendicularly upon an oblong bar of wood, or table BD, at diſtances, ſuch as the Propoſitions require, from a lighted candle Q placed perpendicularly on the ſame bar of wood, and receiving the images upon white paper *q*. Upon this bar of wood, on one ſide of a line over which the convex lens is placed, let a line perpendicular to the laſt mentioned line be divided into parts 1, 2, 3, 4, &c. each equal to the diſtance of the focus of parallel rays; and on the other ſide of the lens, let a line be divided in the ſame manner, and let the firſt diviſion which is farther from the lens than the focus be ſubdivided into parts reſpectively equal to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of the diſtance of the focus of parallel rays: if a candle be placed over the diviſion 2, it will form a diſtinct image on a paper held over the diviſion $\frac{1}{2}$; if the candle be over 3, the image will be at $\frac{1}{3}$, &c. whence it appears, that the diſtances of the correſpondent foci vary reciprocally. Prop. XL, XLI, XLII, may be confirmed by holding a large double convex lens, or burning glaſs, in the ſun's rays, and receiving the image on white paper, or other ſubſtances.

CHAP.

CHAP. III.

Of REFLECTION.

SECT. I.—*Of the* LAWS *of* REFLECTION.

DEF. XIX. A *Ray of Light*, turned back into the ſame medium in which it moved before its return, is ſaid to be *reflected*.

DEF. XX. The *Angle of Reflection* is that which is contained between the line deſcribed by a reflected ray and a line perpendicular to the reflecting ſurface at the point of reflection.

Plate 6. Fig. 25. Let AC be the incident ray falling upon the reflecting ſurface DE, CB will be the reflected ray, OC the perpendicular, ACO the angle of incidence, and OCB the angle of reflection.

PROP. XLIII. The reflection of light from tranſparent bodies is either partial or total: the partial reflection happens either at the firſt or ſecond ſurface, the total, at the ſecond ſurface only.

When a beam of light falls upon a thick piece of poliſhed glaſs, all the rays will not paſs through it; but ſome of them will be reflected from the firſt ſurface of the glaſs, where the beam enters. At the ſecond ſurface, ſome of the rays will alſo be reflected. Theſe partial reflections happen, whatever is the obliquity of the rays. The total reflection happens when the angle of incidence, or the obliquity, is greater than 41 degrees. All the light which then comes to the ſecond ſurface will be reflected. See Cor. 3. Pr. XIII.

SCHOL. The rays of light are not reflected by ſtriking upon the ſolid parts of bodies.

At leaſt as many rays are reflected from the ſecond ſurface when the light paſſes out of glaſs into air, as from the firſt when it paſſes out of air into glaſs: but if the reflection were cauſed by the ſtriking of the rays upon ſolid parts of bodies, ſince glaſs is denſer than air, that is, has more ſolid parts in a given ſpace, a greater quantity of rays would be reflected from the firſt ſurface than from the ſecond. Beſides, it ſeems improbable that, at the ſecond ſurface, with one degree of obliquity, the rays ſhould meet with nothing but pores or interſtices in the air, and paſs freely into it, and that with a greater degree of obliquity, it ſhould meet with nothing but ſolid parts, and be totally reflected. Again, ſince water is denſer than air, if the reflection were owing to the ſtriking of the rays upon the ſolid body, it might be expected that the light would be more perfectly reflected in paſſing out of glaſs into water than into air; whereas it is found, that if water be placed behind the glaſs inſtead of air, rays will not be reflected at the ſecond ſurface, though their obliquity be greater than 41 degrees: from hence alſo it is manifeſt, that the reflection is not owing to the ſtriking of the rays upon the ſecond ſurface of the glaſs; for then it would be the ſame whatever were the medium beyond it.

PROP. XLIV. Reflection is cauſed by the powers of attraction and repulſion in the reflecting bodies.

Plate 6. Fig. 24.

Suppoſing that bodies attract thoſe rays which are very near them, and repel thoſe a little farther from them; and calling the ſpace contiguous to the ſurface of the glaſs, where the rays are attracted, the attracting ſurface; and the ſpace next to this, the repelling ſurface, the propoſition may be thus proved.

Let GG, MM, be the repelling ſurface of a piece of glaſs, and R*a* a ray falling upon it. This ray when it enters the ſurface will be retarded by the repulſion, and conſequently, refracted from the perpendicular at *a*. And this repulſion increaſing till the ray gets into the ſurface of attraction, the ray will be conſtantly retarded, that is, turned out of its ſtraight courſe at *b*, *c*, *d*, &c. till it becomes parallel to MM at *f* the limit of the repelling ſurface. And in this ſituation of the ray, the repelling force, which had retarded, will now conſtantly accelerate it, and conſequently it will be continually refracted towards the perpendicular, at *g*, *h*, *i*, &c. till it emerge from the ſurface at *l*; when, the repelling force ceaſing to act, the ray will proceed in a right line. Thus the ray, by reflection, is made to deſcribe the curve *a f l*.

PROP. XLV. The angle of reflection is equal to the angle of incidence, and their complements are alſo equal.

In all caſes of reflection, the rays (by Prop. XLIV.) deſcribe ſuch a curve at *a f l*. And ſince they deſcribe one half of this curve by being retarded, and one half by being ſimilarly accelerated, one half will be ſimilar to the other; whence one half will make the ſame angle with a perpendicular at *f* as the other half makes with it. And the bending of the rays is made within ſo very ſmall a compaſs, that is, the curve *a f l* is ſo ſmall that it may be neglected, as in fig. 25. where the angle ACO is equal to the angle BCO, and conſequently, the angle ACD equal to BCE.

EXP. 1. Having deſcribed a ſemicircle on a ſmooth board, and from the circumference let fall a perpendicular biſecting the diameter, on each ſide of the perpendicular cut off equal parts of the circumference; draw lines from the points in which thoſe equal parts are cut off to the centre; place three pins perpendicular to the board, one at each point of ſection in the circumference, and one at the centre; and place the board perpendicular to a plane mirror. Then look along one of the pins in the circumference to that in the centre, and the other pin in the circumference will appear in the ſame line produced with the firſt, which ſhews that the ray which comes from the ſecond pin, is reflected from the mirror at the centre to the eye, in the ſame angle in which it fell on the mirror.

2. Let a ray of light paſſing through a ſmall hole into a dark room, be reflected from a plane mirror; at equal diſtances from the point of reflection, the incident and the reflected ray, will be at the ſame height from the ſurface.

PROP. XLVI. All reflection is reciprocal.

If

Plate 6. Fig. 25. If the ray AC, after it has been reflected in the line CB, is turned back again in the direction CB, it will be reflected (by Prop. XLV.) into AC. Therefore if ACO is the angle of incidence, OCB will be the angle of reflection; and if OCB be the angle of incidence, ACO will be the angle of reflection.

Schol. Sir Iſaac Newton explains the cauſe of reflection by ſuppoſing, that light, in its paſſage from the luminous body is diſpoſed to be alternately reflected by, and tranſmitted through any refracting ſurface it may meet with; that theſe diſpoſitions, which he calls Fits of eaſy reflection and eaſy tranſmiſſion, return ſucceſſively at equal intervals: and that they are communicated to it, at its firſt emiſſion out of the luminous body it proceeds from, probably by ſome very ſubtle and elaſtic ſubſtance diffuſed through the univerſe, in the following manner. As bodies falling into water, or paſſing through the air, cauſe undulations in each, ſo the rays of light may excite vibrations in this elaſtic ſubſtance. The quickneſs of theſe vibrations depending on the elaſticity of the medium, (as the quickneſs of the vibrations in the air, which propagate ſound, depend ſolely on the elaſticity of the air, and not upon the quickneſs of thoſe in the ſounding body) the motion of the particles of it may be quicker than that of the rays; and therefore when a ray, at the inſtant it impinges upon any ſurface, is in that part of a vibration of this elaſtic ſubſtance which conſpires with its motion, it may be eaſily tranſmitted, and when it is in that part of a vibration which is contrary to its motion, it may be reflected. He farther ſuppoſes, that when light falls upon the firſt ſurface of a body, none is reflected there, but all that happens to it there is, that every ray that is not in a fit of eaſy tranſmiſſion, is there put into one, ſo that when they come at the other ſide, (for this elaſtic ſubſtance eaſily pervading the pores of bodies, is capable of the ſame vibrations within the body as without it) the rays of one kind ſhall be in a fit of eaſy tranſmiſſion, and thoſe of another in a fit of eaſy reflection, according to the thickneſs of the body, the intervals of the fits being different in rays of a different kind.

PROP. XLVII. Rays of light reflected from a plane ſurface, have the ſame degree of inclination to each other that their reſpective incident ones have.

Plate 6. Fig. 26. The angles of reflection of the rays AC, AI, AK, being equal to that of their reſpective incident ones, it is evident that each reflected ray will have the ſame degree of inclination to the ſurface DE, and from whence it is reflected, that its incident one has; but it is here ſuppoſed that all thoſe portions of ſurface, from whence the rays are reflected, are ſituated in the ſame plane; conſequently, the reflected rays FC, LI, MK, will have the ſame degree of inclination to each other that their incident ones have, from whatever part of the ſurface they are reflected.

Cor. Parallel rays falling on a reflecting plane ſurface are reflected parallel.

PROP. B. If a plane mirror revolve upon an axis, the angular velocity of the reflected ray is double that of the mirror.

Let DE be a mirror, AC an incident ray, and CB a reflected ray. If the mirror turn upon an axis at C ſo as to come into the ſituation FG, then the incident ray AC will be reflected into the line CH. Now the angle BCH which expreſſes the angular velocity of the reflected ray BC, is double of the angle DCF which is the angular velocity of the mirror. For the angle ACD = BCE = BCH + HCE. (Prop. XLV.) For the ſame reaſon ACF = HCG = HCE + (ECG) FCD. Therefore the angle DCF (ACD — ACF) = BCH — DCF, conſequently the angle BCH = 2DCF; that is, the angular velocity of the reflected ray is double that of the mirror. Plate 6. Fig. 25.

Schol. Upon this Propoſition depend the conſtruction and uſe of Hadley's quadrant.

PROP. XLVIII. Parallel rays reflected from a concave ſurface are made converging.

Let AF, CD, EB, repreſent three parallel rays falling upon the concave ſurface FB, whoſe centre is C. To the points F and B draw the lines CF, CB; theſe being drawn from the centre will be perpendicular to the ſurface at thoſe points. The incident ray CD alſo paſſing through the centre will be perpendicular to the ſurface, and therefore will return after reflection in the ſame line; but the oblique rays AF and EB will be reflected into the lines FM, BM, ſituated on the contrary ſide of their reſpective perpendiculars CF and CB. They will therefore proceed converging after reflection towards ſome point, as M, in the line CD. Plate 6. Fig. 27.

PROP. XLIX. Converging rays falling upon a concave ſurface are made to converge more.

Let GB, HF, be the incident rays. Now, becauſe theſe rays have larger angles of incidence than the parallel ones, AF, EB, in the foregoing caſe, their angles of reflection will alſo be larger than theirs: they will therefore converge after reflection, ſuppoſe in the lines FN, and BN, having their point of concourſe N farther from C than the point M, to which the parallel rays AF and EB converged in the foregoing caſe. Plate 6. Fig. 27.

PROP. L. Diverging rays, falling upon a concave ſurface, if they diverge from the focus of parallel rays, become parallel;—if from a point nearer the ſurface than that focus, will diverge leſs than before reflection;—if from a point between that focus and the centre, will converge after reflection to ſome point, on the contrary ſide of the centre, and farther from the centre than the point from which it diverged;—if from a point beyond the centre, the reflected rays will converge to a point on the contrary ſide, but nearer to it than the point from which they diverged;—if from the centre, they will be reflected thither again.

Plate 6. Fig. 27.

Let the incident diverging rays be MF, MB, proceeding from M the focus of parallel rays; then as the parallel rays AF and EB were reflected into the lines FM and BM, theſe rays will now on the contrary be reflected into them. (By Prop. XLVI.)

Let them diverge from N, a point nearer to the ſurface than the focus of parallel rays, they will then be reflected into the diverging lines HF and BG, which the incident rays GB and HF deſcribed, which were ſhewn to be reflected into them in the foregoing propoſition; but the degree wherein they diverge, will be leſs than that wherein they diverged before reflection.

Let them proceed diverging from X, a point between the focus of parallel rays and the centre, they then make leſs angles of incidence than the rays MF and MB, which became parallel by reflection, they will conſequently have leſs angles of reflection, and proceed therefore converging towards ſome point, as Y; which point will always fall on the contrary ſide of the centre, becauſe a reflected ray always falls on the contrary ſide the perpendicular with reſpect to that on which its incident one falls; and therefore will be farther diſtant from the centre than X.

If the incident ones diverge from Y, they will after reflection converge to X, thoſe which were the incident rays in the former caſe being the reflected ones in this.

Laſtly, if the incident rays proceed from the centre, they fall in with their reſpective perpendiculars, and for that reaſon are reflected thither again.

Exp. Place a concave mirror at proper diſtances from an open orifice, or a convex, or a concave lens, through which a beam of ſolar rays paſſes, according to the three preceding propoſitions.

PROP. LI. Parallel rays reflected from a convex ſurface are made diverging.

Plate 6. Fig. 28.

Let AB, GD, EF, be three parallel rays falling upon the convex ſurface BF, whoſe centre of convexity is C, and let one of them, GD, be perpendicular to the ſurface. Through B, D, and F, the points of reflection, draw the lines CV, CG, and CT, which becauſe they paſs through the centre will be perpendicular to the ſurface at thoſe points. The incident ray GD, being perpendicular to the ſurface, will return after reflection in the ſame line, but the oblique ones AB and EF in the lines BK and FL ſituated on the contrary ſide their reſpective perpendiculars BV and FT. They will therefore diverge after reflection, as from ſome point M in the line GD produced.

PROP. LII. Diverging rays reflected from a convex ſurface are made more diverging.

Plate 6. Fig. 28.

Let GB, GF, be the incident rays. Theſe having larger angles of incidence than the parallel ones AB and EF in the preceding caſe, their angles of reflection will alſo be larger than theirs; they will therefore diverge after reflection, ſuppoſe in the lines BP and FO, as from ſome point N farther from C than the point M; and the degree wherein they will diverge, will exceed that wherein they diverged before reflection.

PROP. LIII. Converging rays reflected from a convex ſurface, if they tend towards the focus of parallel rays, will become parallel;—if to a point nearer the ſurface than that focus, will converge leſs than before reflection;—if to a point between that focus and the centre, will diverge as from a point on the contrary ſide of the centre farther from it than the point towards which they converged;—if to a point beyond the centre, they will diverge as from a point on the contrary ſide of the centre nearer to it than the point towards which they firſt converged;—and if towards the centre, they will proceed, on reflection, as from the centre.

Let the converging rays be KB, LF, tending towards M the focus of parallel rays; then, as the parallel rays AB, EF, were reflected into the lines BK and FL, thoſe rays will now on the contrary be reflected into them. Plate 6. Fig. 28.

Let them converge in the lines PB, OF, tending towards N, a point nearer the ſurface than the focus of parallel rays, they will then be reflected into the converging lines BG and FG, in which the rays GB, GF, proceeded, which were ſhewn to be reflected into them in the laſt propoſition immediately foregoing; but the degree wherein they will converge, will be leſs than that wherein they converged before reflection.

Let them converge in the lines RB and SF proceeding towards X, a point between the focus of parallel rays and the centre; their angles of incidence will then be leſs than thoſe of the rays KB and LF, which became parallel after reflection, their angles of reflection will therefore be leſs, on which account they muſt neceſſarily diverge, ſuppoſe in the lines BH and FI, from ſome point, as Y; which point will fall on the contrary ſide the centre with reſpect to X, and will be farther from it than X.

If the incident rays tend towards Y, the reflected ones will diverge as from X, thoſe which were the incident ones in one caſe, being the reflected ones in the other.

And laſtly, if the incident rays converge towards the centre, they fall in with their reſpective perpendiculars; on which account they proceed after reflection, as from thence.

Exp. Illuſtrate the three preceding propoſitions by receiving upon a convex mirror, a ſolar ray paſſing through an open orifice, or a concave or convex lens.

PROP. LIV. When rays fall upon a plane ſurface, if they diverge, the focus of the reflected rays will be at the ſame diſtance behind the ſurface, that the radiant point is before it:—if they converge, it will be at the ſame diſtance before the ſurface, that the imaginary focus of the incident rays is behind it.

Caſe 1. Of diverging rays. Let AB, AC, be two diverging rays incident in the plane ſurface DE, the one perpendicularly, the other obliquely; the perpendicular one AB will be reflected to A proceeding as from ſome point in the line AB produced; the oblique one AC Plate 6. Fig. 26.

will be reflected into ſome line CF, ſuch that the point G, where the line FC produced interſects the line AB produced alſo, ſhall be at an equal diſtance from the ſurface DE with the radiant A. For the perpendicular CH being drawn, ACH and HCF will be the angles of incidence and reflection, which being equal, their complements ACB and FCE are ſo too: but the angle BCG is equal (El. I. 15.) to FCE: therefore in the triangles ABC and GBC the angles at C are equal, the ſide BC is common, and the angles at B are alſo equal to each other, as being right ones; therefore (El. I. 26.) the lines AB and BG, oppoſite to the equal angles at C, are alſo equal, and conſequently the point G, the focus of the incident rays AB, AC, is at the ſame diſtance behind the ſurface, that the point A is before it.

Caſe 2. Of converging rays. Suppoſing FC and AB to be two converging incident rays, CA and BA will be the reflected ones, (the angles of incidence in the former caſe being now the angles of reflection, and the reverſe) having the point A for their focus; but this, from what was demonſtrated above, is at an equal diſtance from the reflecting ſurface with the point G, which in this caſe is the imaginary focus of the incident rays FC and AB. What is here demonſtrated of the ray AC, holds equally of any other, as AI, AK, &c.

Schol. The caſe of parallel rays incident on a plane ſurface, is included in this propoſition; for in that caſe we are to ſuppoſe the radiant to be at an infinite diſtance from the ſurface, and then by the propoſition, the focus of the reflected rays will be ſo too; that is, the rays will be parallel after reflection, as they were before.

PROP. LV. When parallel rays are incident upon a ſpherical ſurface, the focus of the reflected rays will be the middle point between the centre of convexity and the ſurface.

Plate 7. Fig. 1. Caſe 1. Of parallel rays falling upon a convex ſurface. Let AB, DH, repreſent two parallel rays incident on the convex ſurface BH, the one perpendicularly, the other obliquely; and let C be the centre of convexity; ſuppoſe HE to be the reflected ray of the oblique incident one DH proceeding as from F, a point in the line AB produced. Through the point H draw the line CI, which will be perpendicular to the ſurface at that point, and the angles DHI and IHE, being the angles of incidence and reflection, will be equal. But HCF is equal (El. I. 29.) to DHI, and CHF (El. I. 15.) to IHE; wherefore the triangle CFH is iſoſceles; and conſequently, the ſides CF and FH are equal: but ſuppoſing BH to vaniſh, FH is equal to FB, and therefore upon this ſuppoſition FC and FB are equal, that is, the focus of the reflected rays is the middle point between the centre of convexity and the ſurface.

Plate 7. Fig. 2. Caſe 2. Of parallel rays falling upon a concave ſurface. Let AB, DH, be two parallel rays incident, the one perpendicularly, the other obliquely, on the concave ſurface BH, whoſe centre of concavity is C. Let BF and HF be the reflected rays meeting each other in F; this will be the middle point between B and C. For drawing through C the perpendicular CH, the angles DHC and FHC, being the angles of incidence and reflection, will be equal, to the former of which the angle HCF is equal, as alternate; and therefore the triangle CFH is iſoſceles. Wherefore CF and FH are equal; but if we ſuppoſe BH to

vaniſh, FB and FH are alſo equal, and therefore CF is equal to FB; that is, the focal diſtance of the reflected rays is the middle point between the centre and the ſurface.[*]

Schol. The converſe of theſe two caſes may be demonſtrated in a ſimilar manner, by making the incident rays the reflected ones.

PROP. LVI. When rays fall upon any ſpherical ſurface, if they diverge, the diſtance of the focus of the reflected rays from the ſurface is to the diſtance of the radiant point from the ſame (or, if they converge, to that of the imaginary focus of the incident rays) as the diſtance of the focus of the reflected rays from the centre is to the diſtance of the radiant point (or imaginary focus of the incident rays) from the ſame.

Plate 7, Fig. 4.

Caſe 1. Of diverging rays falling upon a convex ſurface. Let RB, RD, repreſent two diverging rays flowing from the point R as from a radiant, and falling the one perpendicularly, the other obliquely, on the convex ſurface BD, whoſe centre is C. Let DE be the reflected ray of the incident one RD; produce ED to F, and through R draw the line RH parallel to FE, till it meets CD produced in H. Then will the angle RHD be equal (El. I. 29.) to EDH the angle of reflection, and therefore equal alſo to RDH, which is the angle of incidence; wherefore the triangle DRH is iſoſceles, and conſequently DR is equal to RH. Now the lines FD and RH being parallel, (El. VI. 2.) FD is to RH, or its equal RD, as CF to CR; but BD vaniſhing, FD and RD differ not from FB and RB; wherefore FB is to RB alſo, as CF to CR; that is, the diſtance of the focus from the ſurface is to the diſtance of the radiant point from the ſame, as the diſtance of the focus from the centre is to the diſtance of the radiant from thence.

Plate 7. Fig. 4.

Caſe 2. Of converging rays falling upon a concave ſurface. Let KD and CB be the converging incident rays, having their imaginary focus in the point R, which was the radiant in the foregoing caſe. Then as RD was in that caſe reflected into DE, KD will in this be reflected into DF; for, ſince the angles of incidence in both caſes are (El. I. 15.) equal, the angles of reflection will be ſo too; ſo that F will be the focus of the reflected rays: but it was there demonſtrated, that FB is to RB as CF to CR, that is, the diſtance of the focus from the ſurface, is to the diſtance (in this caſe) of the imaginary focus of the incident rays, as the diſtance of the focus from the centre is to the diſtance of the imaginary focus of the incident rays from the ſame.

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[*] It is here obſervable, that the farther the line DH is taken from AB, the nearer the point F falls to the ſurface. For the farther the point H recedes from B, the larger the triangle CFH will become; and conſequently, ſince it is always an iſoſceles one, and the baſe CH, being the radius, is every where of the ſame length, the equal legs CF and FH will lengthen; but CF cannot grow longer unleſs the point F approach towards the ſurface. And the farther H is removed from B, the faſter F approaches to it. This is the reaſon, that whenever parallel rays are conſidered, as reflected from a ſpherical ſurface, the diſtance of the oblique ray from the perpendicular ray is taken ſo ſmall with reſpect to the focal diſtance of that ſurface, that without any phyſical error, it may be ſuppoſed to vaniſh. From hence it follows, that if a number of parallel rays, as AB, CD, EG, &c. fall upon a convex ſurface, and if BA, DK, the reflected rays of the incident ones AB, CD, proceed as from the point F, thoſe of the incident ones CD, EG, namely, DK, GL, will proceed as from N, thoſe of the incident ones EG, HI, as from O, &c. becauſe the farther the incident ones CD, EG, &c. are from AB, the nearer to the ſurface are the points F, *f*, *f*, in the line BF, from which they proceed after reflection; ſo that properly the foci of the reflected rays BA, DK, GL, &c. are not in the line AB produced, but in a curve line paſſing through the points F, N, O, &c.

Plate 7, Fig. 3.

The ſame is applicable to the caſe of parallel rays reflected from a concave ſurface, as expreſſed by the dotted lines on the other half of the figure, where PQ, RS, TV, are the incident rays; QF, S*f*, V*f*, the reflected ones interſecting each other in the points X, Y, and F; ſo that the foci of thoſe rays are not in the line FB, but in a curve paſſing through thoſe points.

Plate 7. Fig. 5. Caſe 3. Of converging rays falling upon a convex ſurface, and tending to a point between the focus of parallel rays and the centre. Let BD repreſent a convex ſurface, whoſe centre is C, and focus of parallel rays is P; and let AB, KD, be two converging rays incident upon it, and having their imaginary focus at R, a point between P and C. Now becauſe KD tends to a point between the focus of parallel rays and the centre, the reflected ray DE will diverge from ſome point on the other ſide the centre, ſuppoſe F; as was ſhewn Prop. LIII. Through D draw the perpendicular CD, and produce it to H, then will KDH and HDE be the angles of incidence and reflection, which being equal, their verticle ones RDC and CDF will be ſo too, and therefore the vertex of the triangle RDF is biſected by the line DC: whence (El. VI. 3.) FD and DR, or, BD vaniſhing, FB and BR are to each other as FC to CR; that is, the diſtance of the focus of the reflected rays is to that of the imaginary focus of the incident ones, as the diſtance of the former from the centre is to the diſtance of the latter from the ſame.

Plate 7. Fig. 5. Caſe 4. Of diverging rays falling upon a concave ſurface, and proceeding from a point between the focus of parallel rays and the centre. Let RB, RD, be the diverging rays incident upon the concave ſurface BD, having their radiant in the point R, the imaginary focus of the incident rays in the foregoing caſe. Then as KD was in that caſe reflected into DE, RD will now be reflected into DF. But it was there demonſtrated that FB and RB are to each other as CF to CR; that is, the diſtance of the focus is to that of the radiant, as the diſtance of the former from the centre is to the diſtance of the latter from the ſame.*

Schol. 1. If the reflected ray be made the incident one, thoſe caſes which are reſpectively the converſe of the foregoing may be demonſtrated in the ſame manner.

Plate 7. Fig. 6. * In the caſe of diverging rays falling upon a convex ſurface, the farther the point D is taken from B, the nearer the point F, the focus of the reflected rays, approaches to B, while the radiant R remains the ſame. For it is evident from the curvature of a circle, that the point D may be taken ſo far from B, that the reflected ray DE ſhall proceed as from F, G, H, or even from B, or from any point between B and R, and the farther it is taken from B, the faſter the point, from which it proceeds, approaches towards R. The like is applicable to any of the other caſes of diverging or converging rays incident upon a ſpherical ſurface. This is the reaſon that, when rays are conſidered as reflected from a ſpherical ſurface, the diſtance of the oblique rays from the perpendicular one is taken ſo ſmall, that it may be ſuppoſed to vaniſh. From hence it follows, that if a number of diverging rays are incident upon the convex ſurface BD at the ſeveral points B, D, D, &c. they ſhall not proceed after reflection as from any one point in the line RB produced, but as from a curve line paſſing through the ſeveral points F, f, f, &c. The ſame is applicable in all the other caſes.

Schol. 2. Let it be required to find the focal diſtance of diverging rays incident upon a convex ſurface, whoſe radius of convexity is 5 parts, and the diſtance of the radiant from the ſurface is 20.

Call the focal diſtance ſought x, then will the diſtance of the focus from the centre be $5-x$, and that of the radiant from the ſame 25; therefore (by Prop. LVI.) we have the following proportion, $x : 20 : : 5-x : 25$, and $x = \frac{100}{45}$.

If in any caſe it ſhould happen, that the value of x ſhould be a negative quantity, the focal point muſt then be taken on the contrary ſide of the ſurface.

SECT. II.—*Of* Images *produced by* Reflection.

PROP. LVII. When any point of an object is ſeen by reflected light, it appears in the direction of that line which the ray deſcribes after its laſt reflection.

Since reflection gives the ſame direction to the rays as if they had originally come from the place from which the reflected rays diverge, an object ſeen by reflection muſt appear in that place. The viſible image muſt therefore conſiſt of imaginary radiants diverging from thence.

PROP. LVIII. In all mirrors, plane or ſpherical, the place of the imaginary radiant, when it is determined, is the interſection of the perpendicular from the radiant to the mirror, and any reflected ray.

Plate 7. Fig. 7.

Let D be a radiant in any object DE, and DF a ray from this radiant reflected in the line FC; draw DI the perpendicular from the radiant to the mirror, and produce CF, DI, till they met in L; this point will be the imaginary radiant. Becauſe the ray DI falls perpendicularly upon the mirror, it will be reflected back in the ſame line ID, and therefore will appear to come from ſome point in DI produced. And ſince (from Prop. LVII.) all rays which diverge from the ſame real radiant before reflection, muſt diverge from the ſame imaginary radiant after reflection, any other ray from D, as FC, muſt appear to diverge from the ſame imaginary radiant with the ray DI, that is, from ſome point in DI: but the ray FC (by Prop. LVII.) appears after reflection to proceed in the line FC: It muſt therefore appear to come from ſome point in FC, and alſo from ſome point in DI, that is, from the point L, in which DI interſects FC. The imaginary radiant of the rays DI, DF, after reflection is therefore L, the interſection of the perpendicular and the reflected ray.

Def. XXI. The paſſage of reflection is the incident ray added to the reflected ray: As DF+FC.

PROP. LIX. In plane mirrors, the diſtance of the laſt image from the mirror is equal to the diſtance of the object from it, and the diſtance of any point in the laſt image from the eye is equal to the paſſage of reflection.

Plate 7. Fig. 7. The diſtances of the imaginary radiants L, M, behind the mirror, are (by Prop. LIV.) reſpectively equal to the diſtances of the correſponding real radiants D, E, before the mirror; therefore the diſtance of the laſt image L, M, made up of imaginary radiants between L and M, correſponding to real radiants in the object DE, is equal to the diſtance of that object. The diſtance of L, the higheſt point of the image, from C, any given place of the eye, is CFL, equal to DFC the paſſage of reflection, becauſe (by Prop. LIV.) LF is equal to DF. The ſame may be ſhewn of M, or any other point in the image.

PROP. LX. In plane mirrors, the image is equal and ſimilar to the object.

Plate 7. Fig. 7. If D is the higheſt point of the object, the higheſt point of the image is (by Prop. LVII.) in the perpendicular DIL; and if E be the loweſt point of the object, the loweſt point of the image is in the perpendicular EZM. But DIL and EZM are parallel (El. XI. 6.) becauſe they are both perpendicular to the plane ſurface AB. Conſequently, the diſtance between theſe lines, that is, the heights of the object and image, DE, LM, are equal. In like manner it may be ſhewn, that any diameter of the object is equal to its correſponding diameter in the image: whence the object and image are in all reſpects equal, and conſequently ſimilar, ſurfaces.

CHAP. IV.

Of VISION.

SECT. I.—*Of the* LAWS *of* VISION.

PROP. LXI. When the rays which come from the ſeveral points in any object enter the eye, they will paint an inverted image upon the *retina*.

Plate 7. Fig. 10. Let ABA be a ſection of an eye. AB, BA, is the *tunica ſclerotica*, a white coat which encompaſſes the globe of the eye, except the fore-part between A and A. The fore-part AA is covered by a tranſparent coat, a little more protuberant than any other part of the eye called the *tunica cornea*. In the cavity of the eye is placed a convex lens C*e*, conſiſting of a hard tranſparent ſubſtance called the *chryſtalline humour*. This humour is kept in its place and fixed to the coats by certain ligaments all around it at *ee* called *ligamenta ciliaria*. Under the *tunica cornea*, and at ſome little diſtance from it, is the *iris*, *o*, *o*, which has in it a ſmall orifice, called the pupil of the eye. This iris is tinged with variety of colours, from which the eye is ſaid to be blue, hazel, black, &c. It conſiſts of muſcular fibres, which can contract or dilate the pupil. The remaining part between the *cornea* and the chryſtalline humour is filled with a thin tranſparent fluid, like water, called the *aqueous humour*. S*e*N, is a white coat which conſiſts of the fibres of the optic nerve woven together like a net: this coat is called the *retina*. Between the *ſclerotica* and the *retina* is another coat which is called the *choroides*. The cavity of the eye, between the chryſtalline humour and the *retina*, is filled with a tranſparent

a tranſparent ſubſtance, neither ſo fluid as the aqueous humour nor ſo hard as the chryſtalline, called the *vitreous humour*. The *optic nerve* MBB is inſerted at N.

Mr. Harris has given a table of the dimenſions of the parts of the human eye, of which the following are the principal particulars.

	In.
Diameter from the cornea to the choroides - - -	.95
Radius of the cornea - - - - - -	.335
Diſtance of the cornea from the firſt ſurface of the chryſtalline -	.106
Radius of the firſt ſurface of the chryſtalline - - -	.331
Radius of the back ſurface of the chryſtalline - - -	.25
Thickneſs of the chryſtalline - - - - -	.373

Through the pupil the rays which diverge from the ſeveral points of any object ABC paſs into the cavity. The rays firſt paſs into the *cornea* IK, which is a meniſcus like a watch glaſs, and therefore has no conſiderable effect upon the rays. The fore-part of the aqueous humour under the cornea is convex; and therefore, being denſer than the air, it will make the ſeveral diverging beams from ABC (by Prop. XVII.) diverge leſs. The rays then paſs through the pupil; and next, through the chryſtalline humour, which, being a convex lens, will make them (by Prop. XVII.) converge to as many points upon the retina LDF. Conſequently at DEF, or ſomewhere upon the retina, as upon a piece of white paper in a dark room, an inverted image of the object (by Prop. XXV.) will be painted. Fig. 11.

SCHOL. The refractive powers of the aqueous and vitreous humours, have been found by experiment to be about the ſame as common water; and that of the chryſtalline is a little greater: that is, the ſine of incidence, is to that of refraction, out of air into the aqueous humour as 4 to 3, out of the aqueous into the chryſtalline as 13 to 12, and out of the chryſtalline into the vitreous as 12 to 13.

EXP. Take off the *ſclerotica* from the back part of the eye of an ox, or other animal, and place the eye in the hole of the window ſhutter of a dark room, with its fore-part towards the external objects; a perſon in the room will, through the tranſparent coat, ſee the inverted image painted upon the *retina*.

DEF. XXII. The optic axis, is the axis of the chryſtalline humour continued to the object at which we look. The axis PO of the chryſtalline humour GPH, continued to the point B, is the optic axis directed towards that point. Plate 7. Fig. 11.

DEF. XXIII. That point of the *retina*, upon which the optic axis continued back would fall, is called the middle of the *retina*. If OP be continued back to E, the point E is the middle of the retina.

PROP. LXII. The images upon the *retina* are the cauſe of viſion.

It is found from experience, that when the image upon the *retina* is bright, the object is clearly ſeen; and when the image is faint, the object appears faint: alſo, that when the image is diſtinct, the object is ſeen diſtinctly; and when the image is confuſed, the object appears confuſed. Hence it may be concluded, that theſe images are the cauſe of viſion.

Cor. It is manifeſt that a different conformation of the eye, or ſome parts of the eye, is neceſſary for diſtinct viſion at different diſtances. Some think the change is in the length of the eye; others that it is a change in the figure or poſition of the chryſtalline humour, and others that it is a change in the cornea. Any of theſe changes would produce the effect.

PROP. LXIII. The point in any object towards which the optic axis is directed, is ſeen more diſtinctly than the reſt.

It is known from experience, that when the eye is turned directly towards any part of an object, that is, when the optic axis is directed towards that point, though the whole object, if it be not very large, will be ſeen, that part on which we look directly will appear moſt diſtinct, and the other portions of the object being drawn on parts of the *retina*, ſomewhat nearer to the chryſtalline humour than the middle point of the retina will appear a little confuſed.

PROP. LXIV. Objects appear erect, although their images on the *retina* are inverted.

This is known by experience, and is not inconſiſtent with the explanation above given of viſion. For it is not the image in the *retina*, but the object itſelf, which we ſee, and we judge of its relative poſition, by moving the point of diſtinct viſion along the object, and determine that part to be the higheſt which requires the eye to be the moſt lifted up in order to ſee it diſtinctly.

PROP. LXV. An object may appear ſingle, although it is ſeen by both eyes at once.

Plate 7. Fig. 12.

If both eyes are turned directly to the object C, that is, if the optic axes AC, BC, meet in the object, it will appear ſingle. But if, whilſt one eye is turned towards C, the corner of the other is preſſed with the finger ſo as to alter the poſition of its optic axis, the object will then appear double. For when one eye is turned towards an object, and the other turned a different way, the ſame object will be ſeen by each eye in a different direction; that is, one eye will ſee it in one place, and the other in another, from whence it muſt appear double. But, if both eyes are directed the ſame way, that is, to the place of the object, though two objects may be ſaid to be ſeen, yet as both of them are alike, and ſeen in the ſame place, they will appear but as one. If whilſt both eyes are directed towards C, another object D be placed at ſome conſiderable diſtance directly beyond it, the object D will appear double; for ſince the eyes ſee the object D without being turned directly towards it, the place of D is indeterminate; to the right eye it appears on the right hand of C, and to the left eye on the left of C, that is, being ſeen in two places, it muſt appear double. If the ſight be directed to the farther object D, the nearer object C will appear double. For, the object C is ſeen by the right eye in the direction of a line which paſſes on the left ſide of D, and by the left eye in the direction of a line which paſſes on the right of D. In both caſes, one of the objects appears double, when the eyes are not directly fixed upon it, that is, when the optic axes do not meet in it; and the other object appears ſingle, when the eyes are both directly fixed upon it, that is, when the optic axes meet in the object.

EXP. 1. View a nearer and a more diſtant object at the ſame time, according to the propoſition.

2. Let a paſteboard, having a hole in it, be fixed between the eyes of a ſpectator and two candles, ſo placed, that when the right eye is ſhut, the left eye may ſee only one candle, and when the left eye is ſhut, the right eye can ſee only the other, although both candles are viſible; if both eyes be fixed ſteadfaſtly upon the hole, they will appear as one candle placed at the hole.

PROP. LXVI. There is one part of the *retina* where no perception of the object is conveyed to the mind by the image formed upon it.

This is found by experiment. Place two ſmall circles of white paper upon a dark coloured wall at the height of the eye, and at the diſtance of near two feet from each other. If the ſpectator, at a proper diſtance, ſhuts his right eye and looks with the left directly at the paper on his right hand, he will not ſee the left hand paper, although the objects round it are viſible. Hence it is to be inferred, that the rays from the left hand paper fall upon a part of the *retina* which is inſenſible.

SCHOL. It is ſuppoſed that this part of the *retina* is that where the opic nerve is inſerted: and becauſe the coat called the choroides touches the *retina* in all other parts but is diſcontinued here, it has been conjectured that the ſeat of viſion is not the optic nerve, but the choroides; but this point remains undetermined.

COR. Hence an object cannot become inviſible to both eyes at once; becauſe the image cannot fall upon the optic nerve of each eye at the ſame time. An object ſeen with both eyes, is ſaid to appear about $\frac{1}{10}$ or $\frac{1}{11}$ brighter than with one eye alone. Harris's Optics, p. 116.

PROP. LXVII. If the chryſtalline humour has either too much or too little convexity, the ſight will be defective.

In perſons who are ſhort-ſighted, the humours of the eye are too convex, and bring the rays to a focus before they reach the retina, unleſs the object be brought near to it; in which caſe (by Prop. XXVII.) the image is caſt farther back. In others, the humours of the eye have ſo little convexity, that the focal point lies behind the *retina*; whence, unleſs the object is removed to a great diſtance from the eye, the viſion will be indiſtinct.

DEF. XXIV. The *optic angle* in viewing any object is the angle at the eye ſubtended by the diameter of the object. The angle AOC, ſubtended by AC, the diameter of the object, is the optic angle. Plate 7. Fig. 11.

PROP. LXVIII. The apparent diameter of any object, is proportional to the diameter of the image of that object in the *retina.*

Plate 7. Fig. 11. To an eye placed at O, the apparent magnitude of the object ABC is that viſible extenſion which lies between A and C. If two lines AO, CO, are drawn from theſe points croſſing one another at the eye, they will be the axes of the pencils which come from A and C, and will contain between them the diameter AC; and the points A, C, will be repreſented on the *retina* (by Prop. LXI.) at F and D: conſequently, DF will be the diameter of the image; and this diameter is contained between the two lines AO, CO, produced to the retina. Now it is manifeſt, that the viſible extenſion contained between AO and CO, that is, the apparent diameter of the object is as the angle AOC; and that the diameter of the image contained between DO and FO is as the angle DOF. But the angles DOF, AOC, (El. I. 15.) are equal. Therefore the apparent diameter of the object, and the diameter of the image, are each of them proportional to the ſame angle, and conſequently proportional to one another.

Schol. 1. When we ſpeak of an optic angle, it is not meant that we ſee the point in which the optic axes meet; but, ſince by experience we learn to judge of ſuch diſtances as are not very great, by the ſenſations accompanying the different inclinations of the eye, which are analogous to the optic angle, we expreſs theſe different inclinations of the eye, by that angle. In like manner, although the eye does not ſee a pencil of rays, whilſt the breadth of the pupil bears a ſenſible proportion to the diſtance of the focus from which the rays diverge to the eye, we have ſenſations from which experience enables us to judge of the place of that focus. So, the magnitude of an image upon the retina being always proportional to that of the viſual angle of the object, though that angle is not actually meaſured by the eye, the difference of ſenſations accompanying different magnitudes of the image enable us to diſtinguiſh different viſual angles. Thus it appears, that the uſe of lines and angles in optics, has its foundation in nature.

Schol. 2. The angle ſubtended by the leaſt viſible object, called by the writers on optics the *minimum viſibile*, cannot be accurately aſcertained, as it depends upon the colour of the object, and the ground upon which it is ſeen; it depends alſo upon the eye. Mr. Harris thinks the leaſt angle for any object to be about 40″; and at a medium not leſs than two minutes.

To the generality of eyes the neareſt diſtance of diſtinct viſion is about 7 or 8 inches. Taking 8 inches for that diſtance, and 2 minutes for the leaſt viſible angle, a globular object of leſs than $\frac{1}{100}$ part of an inch cannot be ſeen. Harris's Optics, p. 120—4.

PROP. LXIX. When the diameter of an object is given, its apparent diameter is inverſely as its diſtance from the eye.

The apparent diameter of an object (Prop. LXVIII.) is as the diameter of its image upon the retina: and (Prop. XXXI.) the diameter of the image, when the object is given, is inverſely as the diſtance of the object. Therefore the apparent diameter of the object is alſo inverſely as the diſtance of the object. The ſame may be proved of any apparent length whatſoever.

COR. 1. Hence the apparent diameter of an object may be magnified in any proportion; for the leſs the diſtance of the eye from the object, the greater will be its apparent diameter. But without the help of glaſſes, an object brought nearer the eye than about five inches, though it appears larger, will at the ſame time be ſeen confuſedly.

COR. 2. Hence parallel lines, as the ſides of a long room, or two rows of trees, as ABC, DEF, ſeen obliquely, appear to converge more and more, as they are farther extended from the eye: for the apparent magnitude of their perpendicular intervals, as AD, BE, CF, &c. are perpetually diminiſhed. Plate 7. Fig. 15.

COR. 3. An horizontal plane AI ſeems to aſcend. For the viſual rays cut a perpendicular DR to the horizon, in points that are higher and higher, or nearer to the horizontal line OG, according as they proceed from points in AI that are more remote from A.

COR. 4. It is for a like reaſon that a ceiling DH appears to deſcend, and that faſter than the floor aſcends, as the diſtance of the eye of the ſpectator to the ceiling is greater than the diſtance of the eye from the floor.

PROP. LXX. The apparent diameter of an object, whoſe diſtance is given, is directly as its real diameter.

The apparent diameter of an object (by Prop. LXVIII.) is as the diameter of its image: and the diameter of the image (by Prop. XXXIII.) when the diſtance of the object is given, is as the diameter of the object. Therefore the apparent diameter of an object whoſe diſtance is given, is as its real diameter.

PROP. LXXI. The apparent diameters of different objects at different diſtances from the eye will be equal, if their real diameters are as their diſtances.

For (by Prop. XXXIV.) the diameters of their reſpective images upon the *retina* will be equal; and their apparent diameters (by Prop. LXVIII.) are as the diameters of their images.

PROP. LXXII. The apparent length of an object ſeen obliquely, is as the apparent length of a ſubtenſe of the optic angle perpendicular to the optic axis.

If DF is an object ſeen obliquely, and DG an object ſeen directly, that is, if DF is oblique, and DG perpendicular to the optic axis OR, then ſuppoſing them to ſubtend the ſame angle DOF, their images upon the *retina* (Def. XXIV.) will be equal, whence (by Prop. LXVIII.) their apparent diameters will be equal. Conſequently, the greater the ſubtenſe GD is, the greater will be the apparent length of the object DF; and the reverſe. Plate 7. Fig. 13.

COR. Hence an object appears ſhortened by being ſeen obliquely.

PROP.

PROP. LXXIII. When equal objects are ſeen obliquely, their apparent lengths are inverſely as the ſquares of their diſtances from the eye.

Plate 7. Fig. 13.

Let the eye be at O ; and in the line BC at different diſtances from the eye take equal ſpaces DF, df. The apparent length of DF (by Prop. LXXII.) is proportional to the apparent length of GD, the ſubtenſe of the optic angle DOF, perpendicular to the optic axis OR. In like manner, the apparent length of df is proportional to that of gd, the ſubtenſe of dOf. Now GD, gd, are ſubtenſes alſo of the angles GFD, gfd : and as the ſide Of is to the ſide OF, ſo is the ſine of the angle OFf, that is, of its ſupplement OFB, to the ſine of the angle OfF, or OfB. Hence, ſince ſmall angles are to one another nearly as their ſines, if theſe are ſmall angles, Of will be to OF as the angle OFB to the angle OfB ; that is, Of will be to OF, as the ſubtenſe GD to the ſubtenſe gd, or GD is to gd inverſely as OF to Of ; that is, the ſubtenſes of the optic angles, and conſequently, from what has been ſhewn, the apparent diameters DF, df, are inverſely as their diſtances from the eye. This proportion ariſes from the different degrees of obliquity at which the eye ſees the equal ſpaces DF, df. But, if their obliquities with reſpect to the eye were the ſame, the apparent length of DF (by Prop. LXIX.) would be to that of df inverſely as their diſtances. Since then the apparent lengths of DF, df, are inverſely as their diſtances on account of their different obliquities, and alſo inverſely as their diſtances on account of their different diſtances ; on both accounts taken together, they are in the ratio compounded of the inverſe ratio of their diſtances, and the ſame, that is, inverſely as the ſquares of their diſtances.

PROP. LXXIV. The apparent diameter of an object is not changed by contracting or dilating the pupil.

For when the diſtance is given, the diameter of the image (by Prop. XXXVI. Schol. 2.) remains the ſame whatever be the area of the pupil, and conſequently (by Prop. XXXIII.) the apparent diameter of the object.

PROP. LXXV. An object appears larger when it is ſeen confuſedly, than when it is ſeen diſtinctly.

For the confuſed image (by Prop. XXIX.) is larger than the diſtinct image, and conſequently (by Prop. LXVIII.) the apparent magnitude of the object is greater when it is ſeen confuſedly than when it is ſeen diſtinctly.

Cor. Hence objects appear magnified when ſeen through a miſt ; the drops of which refract the rays ſo differently, that they cannot be collected into one focus.

PROP. LXXVI. A ſpectator in motion ſees an object at reſt, moving the contrary way.

Plate 7. Fig. 14.

If while an object at P is at reſt, the eye be carried parallel to PQ in the direction from Q towards P, its image on the *retina* will move from p to q : the ſame effect will be produced,

duced, if the object moves from P towards Q; and if the velocity of the object and the eye, in each caſe, be the ſame, the apparent velocity will be the ſame alſo.

COR. 1. An object moving along a line PK will appear at reſt to a ſpectator moving along the line QG, parallel to PK: if the motion of the object be quicker or ſlower than that of the ſpectator, it will have an apparent motion direct or retrograde: if the two motions are in contrary directions, the apparent motion of the object will vary with the real motion of the ſpectator.

COR. 2. If the earth be ſuppoſed to move round its axis from weſt to eaſt, while the heavenly bodies are at reſt, they appear to us to move the contrary way, there being nothing in this caſe to indicate to us our own motions. And therefore no argument drawn from the apparent diurnal motions of the ſtars and planets can be of any ſupport to either the *Ptolemaic* or *Pythagorean* ſyſtems.

SCHOL. A perſon riding, or walking ſlowly, though he perceives the change of ſituation of adjacent bodies, yet being ſenſible of his own motion, and having time to reflect in the intervals of theſe apparent changes, thoſe bodies appear to keep their places. But if he runs or rides very ſwiftly, he cannot help fancying, that the bodies, which he is looking at, are moving towards him. The deception is ſtill ſtronger when he ſits at his eaſe in a ſwift ſailing veſſel.

PROP. LXXVII. The ſame degree of velocity appears greateſt, when the motion is in a line perpendicular to the optic axis; and when the motion is in other directions, the apparent velocity will be as the coſine of the angle of inclination to the ſaid perpendicular.

Plate 7, Fig. 14. If two bodies ſet out at the ſame time from P, the one moving along the line PQ, perpendicular to the optic axis Q*q*, and the other along the line PS, oblique to it, and if their velocities be ſuch, as to paſs over the lines PQ, PS, in the ſame time, it is manifeſt, that their apparent velocities will be the ſame; for the images of each will paſs over the ſame ſpace *pq*, on the *retina*, in the ſame time. The real velocities being, in this caſe, as PQ to PS, it is manifeſt, that when the velocities are equal, the apparent velocity of the body which moves in PQ is to that of the body moving in PS, as PS to PQ, that is, as radius to the coſine of the angle QPS: but PS is always greater than PQ: whence the propoſition is manifeſt.

PROP. LXXVIII. If objects at different diſtances from the eye, move in parallel lines, nearly at right angles to the optic axis, and if their velocities are proportional to theſe diſtances, their apparent velocities will be equal; and if their real velocities are equal, their apparent velocities will be reciprocally as their diſtances from the eye.

Let

Plate 7. Fig. 14. Let an object move from Q to P, in the ſame time that another moves from G to H, their real velocities are as QP to GH, that is, (El. VI. 2.) as QO to GO, the diſtances from the eye; and their apparent velocities will be equal; for the ſpace qp upon the *retina* will be paſſed over in the ſame time by the image of each. If the velocities of the objects G, Q, be equal, the object G will arrive at K, and its image deſcribe the ſpace gk upon the *retina*, in the ſame time that the image of the object Q deſcribes the ſpace qp; whence the apparent velocities of theſe two objects are as qk and qp, or as GK (or QP) and GH; that is, (El. VI. 2.) the apparent velocity of the object G is to that of Q, as QO to OG.

Schol. It is here ſuppoſed, that the ſpectator makes no allowance for the different diſtances.

PROP. LXXIX. The apparent velocities of bodies moving in parallel lines at different diſtances from the eye, are directly as the real velocities, and reciprocally as the diſtances.

Plate 7. Fig. 14. Let two bodies move from Q, G, in the parallel lines QP, GH; let the velocity of the object Q be called V, and that of G, v; and let their apparent velocities be called A, a. If the two objects be conceived to move in the ſame line GK, whatever be their velocities, $V : v :: A : a$; and ſuppoſing the velocity of the object Q, to be the ſame in QP as before in GK, A in QP : A in GK :: GO : QO, by Prop. LXXVIII. and A in GK : $a :: V : v$; whence, compounding theſe ratios, $A : a :: GO \times V : QO \times v$, that is,

$$:: \frac{V}{QO} : \frac{v}{GO}.$$

Schol. 1. We judge of the *diſtance* of any object by the viſible length of the plane which lies between the eye and the object. When this method fails us, we compare the known magnitude of the object, with its preſent apparent magnitude; or we compare the degrees of diſtinctneſs with which we ſee the ſeveral parts of an object; or we obſerve whether the change of the apparent place of an object when viewed from different ſtations, or its *parallax*, be great or ſmall, this change being always in proportion to the diſtance of the object. On this principle, we may judge of the diſtance of a near object by obſerving the change which is made in its apparent ſituation, upon viewing it ſucceſſively, with each eye ſingly. Or, ſince it is the difference of the apparent place of an object, as viewed by each eye ſeparately, which makes an object to be ſeen double unleſs we turn both eyes directly towards it, and ſince in doing this, where the diſtance is very ſmall, we turn the eyes very much towards each other, and leſs at a greater diſtance; the different ſenſations accompanying the different degrees in which the eyes are turned towards each other, afford, by habit, a rule for judging of the diſtance of objects.

Schol. 2. In objects placed at ſuch diſtances as we are uſed to, and can readily allow for, we know by experience how much an increaſe of diſtance will diminiſh their apparent *magnitude*, and therefore inſtantly conceive their real magnitude, and, neglecting the apparent, ſuppoſe them of the ſize they would appear if they were leſs remote; but this can only be done, where we are well acquainted with the real magnitude of the object; in all other caſes, we judge of magnitudes by the angle which the object ſubtends at the known, or ſuppoſed, diſtance, that is, we infer the real magnitude from the apparent magnitude in compariſon with the diſtance of the object.

SECT.

SECT. II.—*Of* Vision *as affected by* Refraction.

PROP. LXXX. When any ſmall object is ſeen through a refracting medium, it appears in the direction of that line which the rays deſcribe after their laſt refraction.

The ray DE from any ſmall object D, in paſſing through a glaſs priſm, the end of which is ACB, will be refracted towards a perpendicular, and will deſcribe the line EF; and when it goes out of the priſm, it will be refracted from a perpendicular, into the line FG, which is its direction after its laſt refraction; and the object D will be ſeen in this direction at L inſtead of D. For the image in the *retina* will be in the place in which it would have been, if the naked eye had been looking at an object really placed at L, in the laſt direction of the rays. Plate 7. Fig. 16.

Cor. Hence an object ſeen through a glaſs cut into different ſurfaces inclined to one another, will appear at one view in many different places. The object A ſeen from the point F through the glaſs CEDB, will appear at H, A, G; the laſt direction of the rays AC, AD, AB, after refraction. Fig. 17.

Exp. View any object through a multiplying glaſs.

Schol. If a hair be placed acroſs a ſmall hole made in a thin board, and an eye, ſituated in the dark, look through the hole at a number of candles, the hair will appear multiplied: for a ſhadow of the hair is caſt upon the eye by each of the candles.

PROP. LXXXI. An object ſeen through a medium terminated by plain and parallel ſurfaces, appears nearer, brighter, and larger, than with the naked eye.

Let AB repreſent the object, CDEF the medium, and GH the pupil of an eye. Let RK, RL, be two rays proceeding from the point R, and entering the denſer medium at K and L; theſe rays will here by refraction be made to diverge leſs (by Prop. XVII.) and to proceed afterwards, ſuppoſe in the lines *Ka*, *Lb*; at *a* and *b*, where they paſs out of the denſer medium, they will be as much refracted the contrary way, proceeding in the lines *ac*, *bd*, parallel to their firſt directions; (by Prop. XI.) produce theſe lines back till they meet in *e*, this will be the apparent place of the point R, and it is evident from the figure, that it muſt be nearer the eye than that point; and becauſe the ſame is true of all other pencils flowing from the object AB, the whole will be ſeen in the ſituation *fg*, nearer to the eye than the line AB. As the rays RK, RL, would not have entered the eye, but have paſſed by it in the directions K*r*, L*t*, had they not been refracted in paſſing through the medium, the object appears brighter. The rays A*h*, B*i*, will be refracted at *h* and *i* into the leſs converging lines *hk*, *il*, and at the other ſurface into *k*M, *l*M, parallel to A*h* and B*i* produced, (by Prop. XI.) ſo that the extremities of the object will appear in the lines M*k*, M*l*, produced, namely, in *f* and *g*, and under as large an angle *f*M*g*, as the angle A*q*B, under which an eye at *q* would have ſeen it, had there been no medium interpoſed to refract the rays; and therefore Plate 7. Fig. 18.

it appears larger to the eye at GH, being ſeen through the interpoſed medium, than otherwiſe it would have done. But it is here to be obſerved, that the nearer the point *e* appears to the eye on account of the refraction of the rays RK, RL, the ſhorter is the image *fg*, becauſe it is terminated by the lines M*f* and M*g*, upon which account the object is made to appear leſs; and therefore the apparent magnitude of an object is not much augmented by being ſeen through a medium of this form.

Farther, it is apparent from the figure, that the effect of a medium of this form depends wholly upon its thickneſs: for the diſtance between the lines R*r* and *ec*, and conſequently the diſtance between the points *e* and R depends upon the length of the line K*a*: again, the diſtance between the lines AM and *f*M, depends on the length of the line *hk*; but both K*a* and *hk* depend on the diſtance between the ſurfaces CE and DF, and therefore the effect of this medium depends upon its thickneſs.

PROP. LXXXII. In viewing objects through a convex or concave lens, the object itſelf is not ſeen, but its laſt image, conſiſting of all the imaginary radiants from which the rays appear to diverge after the laſt refraction.

Plate 7. Fig. 8. If AC be an object nearer the convex lens GIL than its principal focus, the rays which diverge from any point B in this object, will, in paſſing through the lens (by Prop. XVII.) be made to diverge leſs, and the imaginary radiant (by Prop. VI.) will be more diſtant than the real one. Hence the rays BG, BL, after refraction, will appear to diverge from the radiant E, farther from the lens than the real radiant B. The ſame happening to the rays from every other point of the object, there will be in DEF as many imaginary radiants as there are real radiants in the object ABC, which, taken together, will compoſe the laſt image. And ſince all the rays fall upon the eye as if they had diverged from this laſt image, the eye will be affected by the object ABC in the ſame manner in which it would be affected without the lens, by an object in all reſpects like DEF, that is, the eye perceives the laſt image.

PROP. LXXXIII. An object ſeen through a convex or concave lens will appear erect, if the object and its image are on the ſame ſide of the lens, but inverted, if they are on contrary ſides.

Plate 7. Fig. 8, 9. It appears from the laſt Prop. that all the rays which diverge from B before refraction, will appear to diverge from E after refraction: and the like may be ſaid of any other points, A and D, or C and F. Now AI is the axis of the beam which comes from A, and therefore, with the reſt of the rays of the beam, after refraction, will appear to diverge from the point D in the ſame right line with A. The ſame may be ſhewn of FCI. Now theſe right lines only croſs one another at the lens. Conſequently, the higheſt point both of the object and image is in DI the upper ſide of the angle DIF, and the loweſt points of both in FI, the lower ſide of the ſame angle; that is, the image, which is viſible, is erect, or in the ſame poſition with the object.

If the object ABC is more remote than the principal focus of the convex lens E, there will be (by Prop. XXIV.) a diſtinct image formed on the other ſide of the lens. If the rays thus collected are not received upon a plane ſurface, they will proceed ſtraight forwards; thoſe which had converged diverging from the focus; whence an inverted image will be preſented to an eye placed beyond the focus.—In the ſame manner this, and the preceding propoſition, may be proved concerning an object ſeen through a concave lens.

EXP. 1. Obſerve the image of a candle whoſe rays have paſſed through a convex lens, and are received at the focus on a white ſurface, whilſt the object is on the ſame, or on the contrary ſide of the lens.

2. An inverted image will be produced without a lens, by ſolar rays paſſing through a very ſmall hole into a darkened room; and if the edge of a knife be applied to one ſide of the hole, and moved ſlowly over it, the parts of the image ſituated oppoſite to the covered ſide will be firſt concealed; from whence it is manifeſt, that the rays croſs one another in paſſing through the hole.

PROP. LXXXIV. The apparent magnitude of an object ſeen through a lens placed cloſe to the eye, or to the object, is equal to its apparent magnitude when ſeen without the lens.

If the eye be placed cloſe to the lens at I, the diameter of the object of refracted viſion DF, is to the diameter of the object of plane viſion AC (by Prop. XXVIII.) as EI to IB, that is, as their reſpective diſtances from the vertex of the lens. Therefore (as appears from Prop. LXIX.) their apparent diameters when ſeen from I, are equal. Plate 7. Fig. 8.

If the lens is placed cloſe to the object, the real radiants touching the lens, the imaginary radiants, that is, the laſt image, will alſo touch the lens: whence their diameters, or apparent magnitudes, will be equal.

PROP. LXXXV. If an object ſeen through a convex lens is nearer to the lens than its principal focus, it will appear brighter than to the naked eye, diſtinct, and in an erect poſition.

In this caſe, the rays which come from any point (by Prop. XVII.) will be brought nearer by refraction, and conſequently a greater number will enter the eye than if there had been no refraction; whence it is manifeſt, that the object will appear brighter. Becauſe the rays of each pencil diverge after refraction, but leſs than before, they come to the eye, as they would if the object were at a moderate diſtance and no lens were uſed, and therefore will be ſeen diſtinctly. And becauſe the refracted rays (by Prop. XVII.) diverge leſs than the incident rays, that is, (by Prop. VI.) the imaginary radiants are more remote than the real ones, the laſt image, as DEF, which is formed by theſe imaginary radiants, is farther from the lens than the object, and on the ſame ſide of the lens; and conſequently, ſince the extreme axes DAI, FCI, only croſs one another at the lens, the image will be in the ſame poſition with the object, and appear erect.

PROP. LXXXVI. If an object ſeen through a convex lens is farther from the lens than its principal focus, the object will appear brighter than to the naked eye, confuſed, and in an erect poſition.

If the eye be placed between the lens and the diſtinct image, whilſt the eye is nearer the lens than the place of the image, the rays being made convergent by the lens, will be cloſer together, and therefore a greater number of them will enter the eye, and the object will appear brighter, than if it had been ſeen without a lens: becauſe the rays come to the eye in a converging ſtate, which from one and the ſame point they do not in plane viſion, they will give a confuſed image. And becauſe no image is formed till the rays come to the *retina*, the object will appear erect.

PROP. LXXXVII. If an object ſeen through a convex lens is in the principal focus, it will appear brighter than to the naked eye, and erect.

The rays of each particular beam becoming in this caſe (by Prop. XVII.) parallel after refraction, are brought nearer together than if there had been no lens; conſequently, more rays will enter the eye from every point, and the object will appear brighter: and no image being formed before the rays come to the *retina*, the object will appear erect.

PROP. D. A minute object, when ſeen through a lens of *very ſmall* principal focal length, appears magnified and diſtinct, if the object be placed in the principal focus.

The angle under which the object appears, will be to that which it ſubtends, when ſeen by the naked eye; as the diſtance at which it is viewed by the naked eye diſtinctly, is to the principal focal length of the lens.

If the principal focal length of the lens be $\frac{1}{30}$ of an inch, and the diſtance at which the eye can ſee diſtinctly be eight inches, it follows that the lens will magnify 240 times in diameter.

PROP. LXXXVIII. When an object, ſeen through a convex lens, is nearer than the principal focus, it is magnified, unleſs the eye or the lens touches the object; and as the eye departs from the lens, its apparent magnitude will decreaſe.

Plate 7. Fig. 8. If the object ABC continues in the ſame place, or does not change its diſtance BI from the lens, the laſt image DF will (by Prop. XXVII.) remain at the ſame diſtance EI: therefore the real diameter of DF (by Prop. XXXIII.) will be invariable, wherever the eye is placed. If the eye be at the vertex of the lens I, the apparent diameters of the laſt image DEF, ſeen through the lens, and that of the object ABC, ſeen with the naked eye, are manifeſtly the ſame. Both theſe apparent diameters decreaſe as the eye recedes from the lens

towards O; but the apparent diameter of the laſt image decreaſes in the inverſe ratio of OE to IE, and that of the object in the inverſe ratio of OB to IB. But OE has a leſs ratio to IE than OB to IB; for IE and IB are unequal quantities, of which IE the diſtance of the image, is always (by Prop. XVII. and VI.) the greater, which are equally increaſed, but not proportionally: therefore the apparent diameter of the image decreaſes ſlower than that of the object, as the eye recedes from the lens. Conſequently, when the eye is at any diſtance from the lens, the laſt image, or the object of refracted viſion, will appear greater than the object ſeen by the naked eye. As the eye departs from the lens, the apparent magnitude of the object, from what has been ſaid, muſt continually decreaſe, till at an infinite diſtance it vaniſhes.

PROP. LXXXIX. If an object ſeen through a convex lens is more remote than the principal focus, and the eye on the other ſide of the lens is nearer than the place of the image, the object appears magnified, and its apparent magnitude will be inverſely as the diſtance of the eye from the image.

Plate 7. Fig. 9.

Suppoſe the eye at the ſide of the lens GL; it might ſucceſſively ſee both the object and the image without looking through the lens; and in this ſituation (by Prop. XXVIII.) the real diameter of the object is to that of the image, as their reſpective diſtances from the lens or the eye; conſequently (by Prop. XXXIV.) their apparent diameters will be equal. Next, ſuppoſe the eye cloſe to the image at F, E, or D, the apparent diameter of the image would manifeſtly be infinite. Alſo in this ſituation of the eye, the apparent diameter of the object would be infinite; for, if the eye be at F, the rays from the point C are the only rays collected into the eye, which appear diffuſed over the whole ſurface, and would do ſo if the lens were ever ſo large; and the ſame would be true of the points B or A, if the eye was at E or D; that is, the apparent diameter of the object ſeen through the lens is infinite. Since then the object and the image appear equal when the eye is cloſe to the lens, and that both appear infinite when the eye is cloſe to the image; they muſt have increaſed equally as the eye was moving from the lens to the image, and their apparent diameters muſt always have been equal. Hence, the object in every ſtation of the eye when it does not touch the lens is magnified. And becauſe the apparent diameter of the object ſeen through the lens is every where equal to that of the image, and that of the image (by Prop. XXXI.) inverſely as the diſtance of the eye from the picture, the apparent diameter of the object ſeen through the lens, is inverſely as the diſtance of the eye from the image.

PROP. XC. If an object ſeen through a convex lens is placed in the principal focus, its apparent magnitude will not be altered by withdrawing the eye from the lens.

Since in this caſe the rays from the object come parallel to the eye, both the imaginary radiants and the image (by Prop. XVII.) are infinitely diſtant. Therefore the apparent magnitude of the object cannot be diminiſhed by receding from the imaginary radiants, nor increaſed by approaching to the image, but will always remain the ſame.

PROP. XCI. If a convex lens is moved whilſt the eye and object remain fixed, the apparent magnitude of the object will increaſe, till the lens is at the middle point between them, after which it will decreaſe till the lens reaches the object; provided the eye is never farther from the lens than the place of the image.

When the lens is at either extreme (by Prop. LXXXIV.) the object is not magnified; but between the extremes (by Prop. LXXXVIII. and LXXXIX.) it is magnified; therefore when it is equally diſtant from the two extremes, it is moſt magnified, and muſt increaſe in its apparent magnitude as the lens moves from the eye towards the middle ſtation, and decreaſe, as it moves from that middle ſtation towards the object.

PROP. XCII. The apparent magnitude of an object, ſeen through a concave lens, decreaſes as the eye, or the object, departs from the lens.

Plate 7. Fig. 9. If the eye touches the vertex of the lens I, the apparent diameters of the object and the laſt image are equal. As the eye recedes from the lens, its diſtance both from the object ABC and laſt image DEF increaſes, and conſequently, the apparent magnitude of both decreaſes. But the diſtance IE from the laſt image increaſes faſter than the diſtance IB from the object, as was ſhewn in Prop. LXXXVIII. Therefore (by Prop. LXIX.) the apparent diameter of the laſt image, or the object of refracted viſion, is diminiſhed as the eye recedes from the lens, more than that of the object ſeen by the naked eye.—Again, as the object departs from the lens, the image departs with it; whence its viſible diameter decreaſes.

PROP. XCIII. When the eye and object are fixed, if a concave lens be moved from the eye, the apparent magnitude of the object will decreaſe till it reaches the middle point between them, and increaſe as it moves on towards the object.

When the lens is at each extreme, the apparent magnitude of the object ſeen through the lens (by Prop. LXXXIV.) is the ſame as when ſeen with the naked eye. In all other ſtations of the lens, the object appears diminiſhed: therefore it muſt appear moſt of all diminiſhed when the lens is in the middle ſtation, and it muſt decreaſe whilſt it is approaching to that ſtation, and increaſe whilſt it is departing from thence towards the object.

Exp. View a candle through a convex or concave lens, in the manner deſcribed Prop. XLII. varying the poſition of the object, or lens, according to the preceding Propoſitions, from Prop. LXXXIV.

PROP. XCIV. Convex lenſes aſſiſt the ſight of thoſe perſons whoſe eyes are not ſufficiently convex, and concave lenſes, that of thoſe whoſe eyes are too convex.

For convex lenſes enable the former to bring the rays from objects to a focus nearer to the chryſtalline than can be done by their eyes; and concave lenſes enable the latter to bring the rays to a focus at a greater diſtance; and thus to produce a diſtinct image upon the *retina*.

Schol. Were there no other uſe of the ſcience of dioptrics, ſays Mr. Molyneux, than that of ſpectacles, the advantage that mankind receive thereby is inferior to no other benefit, not abſolutely requiſite to the ſupport of life. For as the ſight is the moſt noble and extenſive of all our ſenſes;—as we make the moſt frequent uſe of our eyes in all the actions and concerns of life, ſurely that inſtrument which relieves the eyes when decayed, and ſupplies their defects, muſt be eſtimated as the greateſt of all advantages. Forlorn indeed muſt have been the ſituation of many young, and almoſt all old people, before this admirable invention. The ſame author concludes, that ſpectacles were firſt uſed about the end of the 13th century; and he aſcribes the invention to Friar Bacon.

SECT. III.—*Of* Vision *as affected by* Reflection.

PROP. XCV. If a plane mirror and the object ſeen in it are both perpendicular to the horizon, the object appears erect.

The object DE, and the mirror AB, being both perpendicular to the horizon, the lines DI, EZ, in which (by Prop. LVII.) the higheſt and the loweſt points of the object appear, being both perpendicular to the ſurface AB, are parallel to each other, and do not meet. Therefore the line DI, which is higheſt at the object, is alſo higheſt at the image, and EZ will be loweſt at both; therefore the image is not inverted with reſpect to the object; and each point of the image LM (by Prop. LIV.) is equally diſtant from the ſurface of the mirror with its correſponding point in the object DE: therefore LM, DE, are parallel, (El. I. 30.) and ſince the object is erect, the image will be ſo too. Plate 7. Fig. 7.

PROP. XCVI. When the object is parallel to a plane mirror, the length or breadth of that part of the mirror upon which the image appears, is to the length or breadth of the object, as any reflected ray is to the paſſage of reflection.

If the object DE is parallel to the mirror AB, and the image LM is ſeen by the eye at C, then FN, the length of that part of the mirror which is taken up by the image, ſubtends the angle LCM, under which the image appears. For ſince all the viſible length of the image is manifeſtly included within the angle LCM, there cannot be more of the mirror taken up by that viſible length, than is included within the ſame angle. Now the length of the image LM is equal (by Prop. LX.) to the length of the object DE. And (El. VI. 2.) FN is to LM, as FC to CL, or (by Prop. LIX.) CFD; that is, the length of that part of the mirror which is taken up by the image is to the length of the image, or (by Prop. LX.) the Plate 7. Fig. 7.

the length of the object, as any reflected ray is to the paſſage of reflection of that ray. In the ſame manner it may be ſhewn, that the breadth of that part of the mirror taken up by the image, is to the breadth of the object in the ſame ratio.

Cor. Hence, in plane mirrors the object, and the part of the ſurface on which it appears, are ſimilar.

PROP. XCVII. If, at a certain diſtance from the mirror, the whole object cannot be ſeen by reflection, the whole will become viſible either by bringing the eye nearer to the mirror, or removing the object farther from it.

Plate 7. Fig. 7.

For (by Prop. XCVI.) the leſs the ratio of the reflected ray is to the paſſage of reflection, ſo much the leſs will be the ratio of the length of that part of the mirror on which the whole object will appear, to the length of the object. If therefore the reflected ray CF decreaſes by bringing the eye nearer to the mirror, ſince it is diminiſhed faſter than the greater quantity DFC, the paſſage of reflection, when equal parts are taken from each, the ratio of the reflected ray CF to the paſſage of reflection DFC (El. V. 8.) diminiſhing, the ratio of the length of that part of the mirror, on which the whole object will be ſeen, to the object, is diminiſhed; that is, (the object being given) the length of the part of the mirror on which the whole object is ſeen, will be diminiſhed: conſequently, the whole object, which at a certain diſtance of the eye from the mirror was not viſible, on a nearer approach of the eye may become viſible.

If the object DE be removed farther from the glaſs, the ratio of the reflected ray FC to the paſſage of reflection DFC will alſo be diminiſhed, becauſe DFC will be increaſed whilſt FC remains the ſame; and conſequently the length of the part of the mirror on which the whole image is ſeen, is diminiſhed, and a leſs ſurface of glaſs is required in order to ſee the whole image.

PROP. XCVIII. If a ſpectator ſees himſelf entirely in a plane mirror placed parallel to him, the mirror muſt be half as long as himſelf.

When a ſpectator is looking at himſelf, the incident ray is his diſtance from the mirror, and the reflected ray is equal to it, and is the diſtance of the mirror from him. The paſſage of reflection is therefore equal to twice his diſtance from the mirror; and conſequently the reflected ray is to the paſſage of reflection as 1 to 2: whence (by Prop. XCVI.) the length of the glaſs, in which he can ſee himſelf entirely, muſt be to his own length as 1 to 2, or the length of the glaſs muſt be half his own length.

If the mirror be at all ſhorter than this, the ſpectator will not be able to ſee himſelf, whether he is nearer to the glaſs, or farther from it. If he approaches towards the glaſs, the object, being himſelf, approaches as faſt as the eye, ſo that though (by Prop. XCVII.) he might ſee more of himſelf by the approach of the eye, he will ſee juſt as much leſs of himſelf on account of the approach of the object. In the ſame manner it may be ſhewn, that if he recedes from the mirror, he will not be able to ſee himſelf entirely.

PROP.

PROP. XCIX. Objects perpendicular to the horizon, ſeen in a plane mirror parallel to the horizon appear inverted.

By Prop. LIV. each imaginary radiant is at the ſame diſtance behind the mirror, that the real radiant is before it; hence, if the mirror be below the object and the eye, the object will have its loweſt part neareſt the ſurface of the mirror, and its higheſt part fartheſt from it, and therefore will in this ſituation appear inverted: if the mirror be above the object and the eye, the object will have its higheſt part neareſt the ſurface, and its loweſt part fartheſt from it, and therefore will, in this ſituation alſo, appear inverted.

PROP. C. If a plane mirror is inclined to the horizon at an angle of forty-five degrees, an object parallel to the horizon will appear erect in the mirror, and an object perpendicular to the horizon will appear parallel to it.

Let the object CD parallel to the horizon be ſeen in AB, a mirror ſo placed as to incline to the horizon in an angle of forty-five degrees. At whatever diſtance any radiant C is from the mirror, at the ſame diſtance (by Prop. LIV.) is the correſponding radiant C in the image, or CE will be equal to *c*E. In like manner, DB will be equal to *d*B. Thus every radiant in the image is at the ſame diſtance behind the mirror, as the object is before it; whence the image *cd* makes half a right angle *c*BA with the mirror on one ſide, whilſt the object makes with it half a right angle CBA on the other: whence *c*BC is a right angle, that is, the image is perpendicular to the object or horizon, and appears erect in the mirror. Plate 7. Fig. 19.

By making *cd* the object, and CD the image, it may be ſhewn in like manner that when the object is erect, it will appear in the mirror parallel to the horizon.

PROP. CI. If an object is placed between two plane mirrors inclined to one another at any angle, ſeveral images may be ſeen.

Let the object F be placed between the two plane mirrors CB, CA, making with one another the angle BCA. From the object F, draw FD perpendicular to the mirror CA, meeting CA in K, and make KD equal to FK. The image of F will (by Prop. LIV.) appear at D. In like manner, if FG be drawn perpendicular to CB, the object will be ſeen in G, as far behind the mirror as F is before it. Thus two images of the ſame object, but of different ſides or ſurfaces of it, will be ſeen. Plate 7. Fig. 20.

Again, ſince ſome of the reflected rays, which diverge from the image D in all directions fall upon the oppoſite mirror CB, the image D may be conſidered as an object placed before the mirror CB: and conſequently, when the rays which diverge from D are reflected from CB, if DHE be drawn perpendicular to CB, and if EH is taken equal to HD, theſe reflected rays will (by Prop. LVIII.) repreſent the image of D at E, as far behind the mirror as D is before it. In like manner, ſome of the rays from this ſecond image E will fall upon the mirror CA, and the image of E, or a third image of the object, will appear. And thus, as long as the image repreſented in one mirror is before the other, ſo long a new image of the laſt image will be produced. And all theſe images, beginning from CA, and being ſucceſſive repreſentations of D, will be images of the ſide of the object F towards CA. Beſides theſe, there will be another ſet of images beginning from CB, which will be formed in the ſame manner, and repreſent the ſide of the object towards CB, the firſt of which will be G, and the ſecond L.

PROP. CII. The images which appear in two plane mirrors inclined to one another, are in the circumference of a circle, the radius of which is the diſtance of the object from the vertex of the angle contained between the mirrors.

Plate 7. Fig. 20. Since FK is equal to KD, KC common, and the angles at K right angles, CF, the diſtance of the object from the vertex of the angle made by the inclination of the mirrors to one another, is (El. I. 4.) equal to CD. In like manner it may be proved that CE is equal to CD. Therefore CF and CE are equal to one another. Thus all the ſtraight lines drawn from C to G, L, or any other image in the mirrors, may be proved to be equal to CF. Conſequently, if C be made the centre of a circle, and CF its radius, the circumference will paſs through the points D, E, G, L, and every other image which appears in the mirrors, that is, all the images, are in the circumference of a circle, whoſe radius is the diſtance of the object from the vertex of the angle made by the inclination of the mirrors to one another.

PROP. CIII. If two plane mirrors are parallel to one another, and an object is placed between them, innumerable images of that object may be ſeen in each, ſtanding in a right line.

Plate 7. Fig. 20. If the two mirrors CA, CB, were ſeparated at C, ſo as to be leſs inclined, or nearer parallel to one another, the angle of inclination being diminiſhed, it is manifeſt from Prop. CII. that the number of images will be increaſed. At the ſame time the circumference of the circle in which the images are placed will be enlarged, becauſe the vertex C is farther removed from F, or FC is increaſed. Conſequently, if the mirrors CA, CB, are ſo far ſeparated, that the vertex is infinitely diſtant, the images become innumerable, and they are placed in a ſtraight line.

PROP. CIV. In ſpherical mirrors, concave or convex, when the place of the image is determinate, the object and the image are in the ſame ſituation, if they are both on the ſame ſide of the centre and in contrary ſituations if they are on oppoſite ſides.

Plate 7. Fig. 23. Let AFB be an object placed nearer the concave mirror SGV than its principal focus, and let C be the centre of concavity. The rays from A, F, B, being reflected, will, (by Prop. LII.) diverge, and the diſtances of the correſponding imaginary radiants I, E, M, may be determined by Prop. LVI. The real and the imaginary radiants are, in this caſe, on

the ſame ſide of C the centre of concavity. Now, the imaginary radiant which correſponds to the real one A, is (by Prop. LVIII.) in CAI the perpendicular drawn from A to the ſurface; and the ſame with reſpect to B, and all the other radiants. And theſe perpendiculars, and all the reſt, being drawn from the centre, do not croſs each other but at the centre; conſequently they are in the ſame poſition with reſpect to each other at the object and the image, and that which is the higheſt at the object will be the higheſt at the image, and the reverſe. Since therefore (by Prop. LVIII.) every point of the object appears in its perpendicular at the image, the higheſt point in the object will appear the higheſt in the image, and the reverſe: that is, the object and image will be in the ſame ſituation. In like manner it may be ſhewn, that if the object AFB is placed before a convex mirror, and its image IM is on the ſame ſide of the centre, they will be both in the ſame ſituation. If the object AFB is farther from the concave mirror SGV than its principal focus, and if M, E, I, be the places of the ſeveral foci, to which the rays from A, F, B, (by Prop. L.) will converge, a diſtinct image of the object will appear upon a paper placed at M, E, I: and if the paper is taken away, and the eye is more remote from the mirror than MEI, the rays will diverge from theſe foci, and become the laſt image. But, becauſe the extreme perpendiculars ICH, MCK, in which (by Prop. LVIII.) the points A and B will appear, croſs each other at the centre C, between the object and image, A the higheſt point of the object will appear at I the loweſt point of the image, and the reverſe; that is, the image with reſpect to the object will be inverted, or they will be in contrary ſituations. The ſame may be ſhewn in like manner with reſpect to the convex mirror. Plate 7. Fig. 22. Fig. 21.

PROP. CV. In ſpherical mirrors, concave or convex, the diameter of the object is to the diameter of the image, as the diſtance of the object from the centre to the diſtance of the image from the centre, and alſo as the diſtance of the object from the ſurface to the diſtance of the image from the ſurface.

If the eye is any where in the line FG, or that line produced, FG is the optic axis; whence the viſible length of the object AB, and alſo of the image IM, is proportional (as appears from Prop. LXVIII.) to a ſubtenſe of the optic angle perpendicular to FG. The viſible extenſions, or lengths of the object and of the image being then perpendicular to the ſame line FG, are parallel to one another. Hence in all the caſes, the angles ACB, ICM, are equal, and alſo the angles CAB, CIM. Therefore (El. VI. 4.) AB the viſible length of the object is to MI that of the image, as AC the diſtance of the object from the centre is to IC the diſtance of the image. Plate 7. Fig. 21, 23.

Again, ſince the object AFB conſiſts of real radiants, and the image MEI of imaginary radiants when the rays diverge, and of foci when they converge, after reflection; and ſince when they diverge, FG the diſtance of the object from the ſurface is (by Prop. LVI. to EG the diſtance of the image from the ſurface, as FC, the diſtance of the object from the centre is to EC the diſtance of the image from the centre; but (by El. VI. 2.) AB is to MI, as FC to EC: therefore AB is likewiſe to MI as FG to EG, that is, the diameter of the object is to that of the image, as the diſtance of the object from the ſurface to that of the image from the ſurface.

PROP. CVI. If the eye is cloſe to a concave or convex mirror, the apparent diameter of the object is equal to the apparent diameter of the image.

If the eye is at G, the real diameters of the object AFB and image MEI are (by Prop. CV.) as their reſpective diſtances from the eye. Therefore (by Prop. LXXI.) their apparent diameters will ſubtend the equal angles AGB, IGM, and will be equal.

PROP. CVII. If the eye is placed in the centre of a concave mirror, it can ſee nothing in the mirror but its own image.

For the eye, in this ſituation, is in the place of its own image, and therefore rays will be reflected to it from every point of the ſurface.

PROP. CVIII. If an object is nearer to a concave mirror than its principal focus, the image appears behind the mirror, farther from the mirror and larger than the object, erect, and diſtinct.

Plate 7. Fig. 23. The object AFB being nearer to the concave mirror than the principal focus, the rays which diverge from each point in the object before the mirror will diverge after reflection (by Prop. L.) leſs than before from the imaginary radiants M, E, I; whence the image formed by them will (Prop. VI.) be farther from the mirror than the object; and conſequently (by Prop. CV.) it will be larger than the object. Becauſe the object is nearer the mirror than its principal focus, it is likewiſe nearer than the centre; whence (by Prop. CIV.) the image will be erect. Laſtly, the image will be ſeen diſtinctly, becauſe the rays from it diverge as from objects at a moderate diſtance.

PROP. CIX. If an object touches a concave mirror, the image will touch it likewiſe, and they will be equal.

For the real radiants being cloſe to the mirror, the imaginary radiants are ſo too; whence (by Prop. CV.) their diameters will be equal.

PROP. CX. If an object is placed in the focus of a concave mirror, the image is at an infinite diſtance behind the mirror, larger than the object, erect, and diſtinct.

When the object AFB is in the principal focus of the concave mirror SGV, the image is at an infinite diſtance. It will (by Prop. CV.) be larger than the object, on account of its remoteneſs. It will be ſeen erect (by Prop. CIV.) becauſe it is on the ſame ſide of the centre with the object. And ſince the rays of each beam are parallel, it will be ſeen as diſtinctly as the naked eye ſees very remote objects.

PROP. CXI. If the object is farther from a concave mirror than its focus, and the eye is nearer than the place of the image, the object will appear confuſed, behind the mirror, erect, and magnified.

Plate 7. Fig. 21.

The rays which diverge from A, F, B, in an object more remote from the concave mirror SGV than its focus, will (by Prop. L.) be collected, and on a ſurface of white paper form an inverted image. If the eye is any where between B and G, the rays from every radiant are converging when they come from the mirror to the eye, whence it will appear confuſed, becauſe the eye is not accuſtomed to ſee rays in this ſtate. The rays AH, AG, diverging from A, will, after reflection, converge towards I: but if the eye is nearer to the mirror than I, the reflected ray GI will not croſs its perpendicular HI in any place before the eye, ſince they are in a ſtate of convergency; conſequently, the apparent place of this, or any other point of the image, will be indeterminate. It will be ſeen erect, becauſe MEI, the inverted image of the object, will be drawn inverted on the *retina*. Laſtly, becauſe the rays of each beam converge after reflection, as HI, GI, they will appear to come, not from points, but from circular ſpots larger than the points of the object, the image will appear confuſed, and be larger than the object.

PROP. CXII. If an object is farther from a concave mirror than its principal focus, and the eye is farther from the mirror than the place of the image, the image appears before the mirror, inverted, and diſtinct.

Plate 7. Fig. 21.

Rays coming from M, E, I, an object at a greater diſtance than the principal focus from the mirror SGV, will (by Prop. L.) converge to AFB, and would paint an inverted image upon a ſurface of white paper placed there. From thence they will diverge, (the paper being taken away;) whence to the eye placed any where beyond AFB, the image will appear inverted. And it will be ſeen diſtinctly, becauſe the rays come to the eye diverging, as from an object at a moderate diſtance.

Schol. 1. The inverted images of objects may be repreſented in a dark room by a concave mirror, which receives rays paſſing from external objects through a hole in a window-ſhutter, and collects them into a focus on a ſurface of white paper.

Schol. 2. A concave mirror, collecting the parallel rays of the ſun into a focus, will act as a burning glaſs.

PROP. CXIII. When an object is placed before a convex mirror, its image appears behind the mirror, nearer the mirror, and leſs than the object, diſtinct, and erect.

Plate 7. Fig. 22.

If AFB is an object placed before the convex mirror SGV, the rays which before reflection diverge from A, F, B, will (by Prop. LVIII.) after reflection diverge from as many radiants I, E, M, behind the mirror, forming the image. And becauſe the reflected rays (by Prop. LII.) diverge more than the incident ones, the image IM (by Prop. VI.) will be nearer the mirror than the real radiants or object AFB: whence (by Prop. CV.) the image will be leſs than the object. And becauſe the reflected rays come to the eye in a ſtate of divergency, the image will be ſeen as diſtinctly as any viſible object, ſeen by ſuch diverging rays, at the ſame diſtance. Laſtly, if the object AFB were at an infinite diſtance from the mirror, the rays proceeding from any point in it would fall parallel upon the mirror, and therefore would upon reflection form the image in the principal focus, that is, in the middle point between G and C, or on the ſame ſide of C with the object; whence (by Prop. CIV.) it would be erect. At any finite diſtance of the object, the image, being ſtill nearer the ſurface, muſt therefore be erect.

PROP. CXIV. When either the eye or the object departs from a convex mirror, the apparent diameter of the image decreaſes.

Plate 7. Fig. 22. If the object AFB continues in its place, the image IM will (by Prop. LVI.) be always at the ſame diſtance from the mirror; and (by Prop. CV.) the real diameter will be invariable: conſequently (by Prop. LXIX.) the apparent diameter of the image will be inverſely as the diſtance of the eye.

If the object AFB departs from the mirror, the ratio of FG, the diſtance of the object, to GE, that of the image, (El. V. 8.) will increaſe: whence (by Prop. CV.) the ratio of the diameter of the object to that of the image will likewiſe increaſe, that is, the image will become leſs with reſpect to the object: but, the eye remaining in the ſame place, the apparent diameter of the image will (by Prop. LXX.) be as its real diameter: conſequently, the apparent diameter will decreaſe.

EXP. The Propoſitions in this ſection may be confirmed, by placing an object before a Plane, Concave, or Convex Mirror, according to the terms of the reſpective Propoſitions.

CHAP. V.

Of COLOURS.

SECT. I.—*Of the different* REFRANGIBILITY *of* LIGHT.

DEF. XXV. Rays of light are *differently refrangible*, when at the ſame or equal angles of incidence, ſome are more turned out of the way than others.

DEF. XXVI. Rays are *differently reflexible*, when ſome are more eaſily reflected than others.

DEF. XXVII. Light is called *homogeneous*, when all the rays are equally refrangible; and *heterogeneous*, when ſome rays are more refrangible than others.

Def. XXVIII. The *Colours* of homogeneous rays, are called *primary* or *ſimple* colours; thoſe of heterogeneous, *ſecondary* or *mixed.*

PROP. CXV. The rays of the ſun are not all equally refrangible; and thoſe rays which have a different degree of refrangibility, have likewiſe a different colour.

Plate 7. Fig. 24.

If a beam of light SF from the ſun paſſes into a dark room through F a round hole in a window-ſhutter EG, and is received upon a white ſurface, a white round image will be ſeen. If a glaſs priſm ABC is ſo placed as to receive the beam of light, the rays of this beam, from their refraction in paſſing through the priſm, will be turned upwards, and the refracted image PT will be oblong, having its breadth equal to the diameter of the circular picture O. If all the rays were equally refracted upwards, it is manifeſt that ſuch a refraction would not change the form of the picture. Since therefore the refracted image is oblong, it muſt be formed by rays differently refrangible, which fall with equal angles of obliquity upon BC the firſt ſide of the priſm, but are ſome of them, in refraction, turned more out of the way than others; thoſe rays which go to P, the upper part of the image, being moſt refrangible, and thoſe which go to T, the lower part, being leaſt refrangible.

This oblong image is of different colours in different parts; the whole image being made up of rays of ſeven different colours, in the following order, beginning with thoſe which are moſt refrangible; violet, indigo, blue, green, yellow, orange, red. This refracted picture conſiſts of ſeveral round pictures ſo near each other, that each higher circle mixes in part with that below it, whence the colours near the upper and lower edge of each circle are blended. The ſides of theſe circles being very near to each other appear like right lines.

Exp. 1. Obſerve the priſmatic image formed by the refraction of the rays in paſſing through a ſingle priſm.

Plate 7. Fig. 27.

2. To ſeparate the ſeveral colours as much as poſſible, make the hole F in the window-ſhutter very ſmall, and collect the rays which paſs through it, into a focus L, by a convex lens MN. Let the rays which have paſſed through the lens be now received upon a priſm placed near the lens; the rays will be refracted upwards into an oblong image.

Plate 7. Fig. 25.

3. To prove that the priſmatic image is produced by the different refrangibility of the rays, and by no other cauſe, let a ſecond priſm DH be placed beyond the firſt *abc*, at right angles to it. The rays paſſing through this ſecond priſm are refracted ſideways; thoſe which were moſt refracted upwards by the firſt priſm, are moſt refracted ſideways by the ſecond; but, the rays not being ſpread in breadth, the image remains of the ſame form.

PROP. CXVI. Thoſe rays of light which are moſt refrangible, are alſo moſt reflexible.

Plate 7. Fig. 26.

If the beam of light paſſing through F falls upon a priſm ABC, whoſe ſides AC, AB, are equal, and the angle at A a right angle; when the obliquity of theſe rays, as they are to paſs out of the priſm at its baſe BC, is leſs than 40 degrees, (by Cor. 3. Prop. XIII.) the greateſt part of the beam will paſs through, but ſome rays will be reflected at the ſurface BC. Thoſe rays which paſs through the baſe (by Prop. CXV.) form an oblong coloured image at HG, between the moſt refrangible ray MH, and the leaſt refrangible ray MG. If the rays which are reflected from M are made to paſs through another priſm XYV, they will alſo form a faint oblong coloured image *pt*. Now, if the priſm ACB is turned ſlowly round upon its axis in the direction ACB, the obliquity of the rays FM to the baſe BC, will continually increaſe, till all the rays will be reflected at M. Conſequently, the image *pt* will become much brighter than before. And this total reflection will not be produced at once, but the moſt refrangible rays MH will be firſt entirely reflected ; for the violet colour in HG will firſt diſappear, and the ſame colour at *p* will firſt become brighter. In like manner, as the priſm ABC is turned round, each different ſort of ray will be reflected ſooner, as it has a greater degree of refrangibility. Hence it appears, that the rays of the ſun have different degrees of reflexibility, and that thoſe which are moſt refrangible are alſo moſt reflexible.

PROP. CXVII. Homogeneous light is refracted regularly without any dilatation of the rays.

Exp. When the rays of any colour in the oblong image, as green, are ſeparated from the reſt, in the manner deſcribed in Prop. CXV. if ſome of theſe rays are tranſmitted through a ſmall hole in a thin board, and refracted by a priſm placed in the other ſide, the image formed by theſe rays after refraction will not be oblong, but circular.

PROP. CXVIII. The confuſed appearance of objects ſeen through refracting bodies, is owing to the different refrangibility of light.

Exp. Small objects placed in a ſun beam and viewed through a priſm, will be ſeen confuſedly ; but if they are placed in a beam of homogeneous light ſeparated by a priſm, they will appear as diſtinct through a priſm, as when viewed by the naked eye.

Schol. 1. Although the 13th Propoſition (in which it was ſhewn that when a ray of the ſun is paſſing out of one medium into another, the ratio of the ſine of incidence to the refracted ſine will not be changed by changing the obliquity of the incident ray) proceeds upon the ſuppoſition that all rays are equally refrangible, and therefore is not exactly true ; the demonſtration is ſtrictly applicable to any one ſort of rays, as the red ones, which are equally refrangible.

Schol. 2. Since, all other circumſtances being equal, the ſame cauſe, namely, the paſſing of the rays out of one given medium into another, will turn the violet rays more out of the way than the red rays ; the attracting force which acts upon both being the ſame, it is probable that any ſingle ray of the leaſt refrangible ſort contains a greater quantity of matter than any ſingle ray of the moſt refrangible ſort.

PROP. CXIX. The colours of homogeneous light can neither be changed by refraction nor reflection.

EXP. 1. Let a beam of homogeneous light paſs through a round hole in a paſteboard, and then be refracted by a priſm on the other ſide, the colour of the rays will remain the ſame.

2. Red lead, viewed in homogeneous red light, will be red, but if placed in green, or any other homogeneous light, it will take the colour of the rays which fall upon it.

PROP. CXX. The whiteneſs of the ſun's light ariſes from a due mixture of all the primary colours.

EXP. If the oblong picture PT fall upon the convex lens MN, the rays, which were ſeparated at PT, will, by paſſing through the lens, be collected into a focus at G, and form a round image of the ſun upon a piece of paper DE. This image formed of all the primary ſorts of rays, is white. That the whiteneſs of the image is owing to the due mixture of all the ſorts of rays, appears from hence, that, if any of the colours be intercepted at the lens, the image loſes its whiteneſs. The paper being removed from DE to *de*, the rays, having croſſed at G, will form the priſmatic image *tp*, inverted, but diſtinct; from whence it appears, that the colours are not changed by being mixed at the focus.

Plate [illegible],
Fig. 28.

PROP. CXXI. The colours of all bodies are either the ſimple colours of homogeneous light, or ſuch compound colours as ariſe from a mixture of homogeneous light.

Each ſort of light having a peculiar colour of its own, which no refraction or reflection can alter, ſince bodies appear coloured only by reflected light, their colours can be no other than the colour of ſome ſingle homogeneous light, or of a mixture of different ſorts of light.

PROP. CXXII. Water, air, glaſs, or any other tranſparent ſubſtance, when drawn into thin plates, become coloured.

EXP. 1. If a ſoap-bubble be blown up, and ſet under a glaſs that the motion of the air may not affect it, as the water glides down the ſides and the top grows thinner, ſeveral colours will ſucceſſively appear at the top, and ſpread themſelves from thence in rings down the ſide of the bubble, till they vaniſh in the ſame order in which they appeared. At laſt a black ſpot appears at the top, and ſpreads till the bubble burſts.

2. If a piece of plane poliſhed glaſs is placed upon the object glaſs of a long teleſcope, and the interval between them is filled up with water, as the glaſſes are preſſed together the ſame colours ariſe at the point of contact, and ſpread themſelves in circular rings round that point in the ſame order as the ſoap-bubble.

3. A convex and concave lens, of nearly the ſame curvature, being preſſed cloſely together, exhibit rings of colours about the point where they touch. Between the colours there are dark rings, and when the glaſſes are very much compreſſed, the central ſpot is dark. Sir I. Newton found the thickneſs of the air between the glaſſes, where the colours appeared, to be as 1, 3, 5, 7, &c. and the thickneſs where the dark rings appeared, to be as 0, 2, 4, 6, 8, &c. The coloured rings muſt have appeared from the reflection of the light; and the dark rings from the tranſmiſſion of it. Conſequently, the rays were tranſ-

mitted when the thickneſs of the air was, 0, 2, 4, 6, 8, &c. and reflected at the thickneſſes 1, 3, 5, 7, &c. Sir I. Newton, therefore, ſuppoſed, that every ray of light, in its paſſage through any refracting ſurface, is put into a certain ſtate, which, in the progreſs of the ray, returns at equal intervals, and diſpoſes the ray, at every return, to be eaſily tranſmitted through the next refracting ſurface, and, between the returns, to be eaſily reflected by it. Theſe he calls fits of eaſy tranſmiſſion and reflection. See Schol. Prop. XLVI.

4. Two pieces of plate glaſs wiped clean, and rubbed together, will ſoon adhere with a conſiderable force, and exhibit various ranges of colours. One of the moſt remarkable circumſtances attending this experiment, is the facility with which the colours may be removed, or even made to diſappear, by heats too low to ſeparate the glaſſes. A touch of the finger immediately cauſes the irregular rings of colours to contract towards their centre, in the part touched.

Cor. 1. From theſe experiments it appears plain, that the colours of bodies depend in ſome degree, upon the thickneſs and denſity of the particles that compoſe them.

Cor. 2. Hence, if the denſity, or ſize of the particles in the ſurface of a body be changed, the colour is likewiſe changed.

Schol. 1. When the thickneſs of the particles of a body is ſuch, that one ſort of colour is reflected, other colours will be tranſmitted, and therefore the body will appear of the firſt colour. And, in general, a leſs thickneſs is found to be neceſſary to reflect the moſt refrangible rays, as violet and indigo, than thoſe which are leaſt refrangible, as red and orange.

Schol. 2. Sir I. Newton, from a great variety of experiments on light and colours, concludes that every ſubſtance in nature is tranſparent, provided it be made ſufficiently thin. Gold, when reduced into thin leaves, tranſmits a bluiſh-green light. If we ſuppoſe any body, therefore, as gold, for inſtance, to be divided into a vaſt number of plates, ſo thin as to be almoſt perfectly tranſparent; it is evident, that all, or the greateſt part of the rays will paſs through the upper plates, and when they loſe their force, will be reflected from the under ones. They will then have the ſame number of plates to paſs through, which they had penetrated before; and thus, according to the number of thoſe plates through which they are obliged to paſs, the object appears one colour or another, juſt as the rings of colour appeared in Exp. 3. according to the diſtance of the glaſſes, or the thickneſs of the plates of air between them.

The philoſopher who has of late years moſt diſtinguiſhed himſelf on the ſubject of light and colours, is Mr. Delaval, who, by a great variety of well conducted experiments, has ſhewn that colours are exhibited, not by reflected, but by tranſmitted light. This he proved by covering coloured glaſs, and other tranſparent coloured media, on the further ſurface, with ſome ſubſtance perfectly opake, when he found that they reflected no colour, but appeared perfectly black.

He concludes, therefore, that, as the fibres of mineral and animal ſubſtances are found, when cleared of heterogeneous matters, to be perfectly white, the rays of light are reflected from theſe white particles, through the coloured media with which they are covered; that theſe media ſerve to intercept and impede certain rays in their paſſage through them, while a free paſſage being left to others, they exhibit, according to theſe circumſtances, different colours.

Mr. Delaval inſtituted other experiments with coloured fluids put into phials of flint glaſs, in the form of a parallelopiped. The bottom, and three ſides of each phial, were covered with a black varniſh, the neck and the front being left uncovered. On expoſing them to the incident light, he found, that, from the parts of the phials which were covered, no light was reflected, but it was perfectly black, while the light tranſmitted through the uncoated parts of the phials was of different colours. The ſame fluids ſpread thinly on a white ground, exhibited their proper colours; the light, indeed, being, in this caſe, reflected from a white ground, and tranſmitted through a coloured medium.

From theſe, and many other experiments, Mr. Delaval concludes, (1.) That the colouring particles do not reflect any light. (2.) That a medium, ſuch as Sir I. Newton deſcribed, is diffuſed over the anterior and further ſurfaces of the plates, whereby objects are reflected equally and regularly, as in a mirror.

Our author next conſiders the colouring particles themſelves, unmixed with other media. For this purpoſe, he reduced ſeveral tranſparent-coloured liquors to a ſolid conſiſtence by evaporation with a gentle heat, which does not injure the colouring matter; and in this ſtate alſo the colouring particles reflect no light, but are perfectly black.

To determine the principle on which opake bodies appear coloured, it muſt be recollected, firſt, that all the coloured liquors appeared ſuch only by tranſmitted light; and, ſecondly, that theſe liquors ſpread thinly upon white ground, exhibited their reſpective colours; he therefore concludes, that all coloured bodies, which are not tranſparent, conſiſt of a ſub-ſtratum of ſome white ſubſtance, which is thinly covered with the colouring particles.

On extracting, by means of ſpirits of wine, the colouring matter from the leaves, wood, and other parts of vegetables, he found that the baſis was a ſubſtance perfectly white. He alſo extracted the colouring matter from different animal ſubſtances, as fleſh, feathers, &c. when the ſame concluſion was obtained. Fleſh conſiſts of fibrous veſſels, containing blood, and is perfectly white when diveſted of blood by ablution, and the red colour proceeds from the light which is reflected from the white fibrous ſubſtance, through the red tranſparent covering formed by the blood. The reſult was the ſame from an examination of the mineral kingdom.

Some portions of light are reflected from every ſurface of a body, or from every different medium into which it enters. Thus tranſparent bodies reduced to powder, and water in the ſhape of froth, appear white, which is no other than a copious reflection of light from all the ſurfaces of the minute parts, and from the air interpoſed between them.

For a full inveſtigation of this curious and intereſting ſubject, the reader muſt be referred to the Memoirs of the Mancheſter Society, Vol. II.

SECT. II.—*Of the* RAINBOW.

PROP. CXXIII. When the rays of the ſun fall upon a drop of rain and enter into it, ſome of them, after one reflection and two refractions, may come to the eye of a ſpectator, whoſe back is towards the ſun, and his face towards the drop.

Plate 8. Fig. 1. If the ſun ſhines upon XY, a drop of rain, in any lines SF, SD, SA, &c. the greateſt part of the rays will enter the drop, and paſſing on to the ſecond ſurface, will be tranſmitted through the drop. But at PG in the ſecond ſurface ſome few rays will be reflected, and proceed in ſome ſuch lines as NR, NQ; and coming out of the drop in the lines RV, QT, they may fall upon the eye of a ſpectator, placed in thoſe lines with his face towards the drop. Theſe rays are refracted when they enter the drop, reflected from the ſecond ſurface, and again refracted when they come out of the drop.

Def. XXIX. When rays of light reflected from a drop of rain come to the eye, thoſe rays which excite a perception of light, are called *effectual.*

PROP. CXXIV. When rays of light come out of a drop of rain, they will not be effectual, unleſs they are parallel and contiguous.

Moſt of the rays, which enter the drop between X and A, paſſing out of the hinder ſurface between P and G, only a few rays are reflected, and come out of the drop through the nearer ſurface between A and Y. Now of theſe, only the rays which are parallel to one another will be effectual, becauſe if they diverge they will be ſo far aſunder when they come to the eye, that only a very few of them can enter the pupil, and no perception of colours will be excited. Alſo, unleſs ſeveral parallel rays be very near each other, the rays will be too few to create any perception.

PROP. CXXV. When rays of light come out of a drop of rain after one reflection, thoſe will be effectual which are reflected from the ſame point, and enter the drop near one another.

Plate 8. Fig. 2. Any rays AB, CD, when they have paſſed out of the air into a drop of water, will be refracted towards the perpendiculars BL, DL, by Prop. XI. And as the ray AB falls farther from the axis than the ray CD, AB will be more refracted than CD: ſo that theſe rays, though parallel to one another at their incidence, may deſcribe the lines BE, DE, after refraction, and be both of them reflected from one and the ſame point E. Now all rays which are thus reflected from one and the ſame point, when they have deſcribed the lines EF, EG, and after reflection emerge at F and G, will be ſo refracted, when they paſs out of the drop into the air, as to deſcribe the lines FH, GI, parallel to one another. If theſe rays were to return from E in the lines EB, ED, and were to emerge at B and D, they would be refracted into the lines of their incidence BA, DC, (by Prop. XII.) But if theſe rays, inſtead of being returned in the lines EB, ED, are reflected from the ſame point E in the lines EG, EF, the lines of reflection EG and EF will be inclined both to one another and to the ſurface of the drop, juſt as much as the lines EB and ED are. Firſt, EB and EG make juſt the ſame angle with the ſurface of the drop; for the angle BEX, which EB makes with the ſurface of the drop, is the complement of incidence, and the angle GEY, which EG makes with the ſurface, is the complement of reflection; and theſe two are equal to one another, by Prop. XLV.

In the ſame manner we might prove that ED and EF make equal angles with the ſurface of the drop. Secondly, the angle BED is equal to the angle FEG, or the reflected rays EG, EF, and the incident rays BE, DE, are equally inclined to each other. For the angle of incidence BEL is equal to the angle of reflection GEL, and the angle of incidence DEL is equal to the angle of reflection FEL, by Prop. XII. Conſequently, the difference between the angles of incidence is equal to the difference between the angles of reflection, or BEL—DEL is equal to GEL—FEL, or BED to GEF. Since therefore either the lines EG, EF, or the lines EB, ED, are equally inclined both to one another and to the ſurface of the drop, the rays will be refracted in the ſame manner, whether they were to return in the lines EB, ED, or are reflected in the lines EG, EF. But if they were to return in the lines EB, ED, the refraction, when they emerge at B and D, would make them parallel. Therefore if they are reflected from one and the ſame point E in the lines EG, EF, the refraction, when they emerge at G and F, will likewiſe make them parallel.

Farther, in order to render the rays which emerge at F and G effectual, they muſt not only emerge in a direction parallel to each other, but muſt enter the drop nearly at the ſame place.

Let XY be a drop of rain, AG the axis or diameter of the drop, and SA a ray of light, that comes from the ſun and enters the drop at the point A. This ray SA, becauſe it is perpendicular to both the ſurfaces, will paſs ſtraight through the drop in the line AGH without being refracted, by Prop. XIV. But any collateral rays that fall about SB, as they paſs through the drop, will be made to converge to their axis, and paſſing out at N will meet the axis at H, by Prop. XI. Rays which fall farther from the axis than SB, ſuch as thoſe which fall about SC, will likewiſe be made to converge; but then their focus will be nearer to the drop than H, by Prop. XI. Suppoſe therefore I to be the focus to which the rays that fall about SC will converge; any ray SC, when it has deſcribed the line CO within the drop, and is tending to the focus I, will paſs out of the drop at the point O. The rays, that fall upon the drop about SD more remote ſtill from the axis, will converge to a focus ſtill nearer than I, ſuppoſe at K, by Prop. XXI. note. Theſe rays therefore go out of the drop at P. The rays, that fall ſtill more remote from the axis, as SE, will converge to a focus nearer than K, as ſuppoſe at L; and the ray SE, when it has deſcribed the line EO within the drop, and is tending to L, will paſs out at the point O. The rays, that fall ſtill more remote from the axis, will converge to a focus ſtill nearer. Thus the ray SF will, after refraction, converge to a focus at M, which is nearer than L, and having deſcribed the line FN within the drop, it will paſs out at the point N. Now here we may obſerve, that as any rays SB or SC fall farther above the axis SA, the points N or O, where they paſs out behind the drop, will be farther above G, or, that as the incident ray riſes from the axis SA, the arc GNO increaſes, till we come to ſome ray SD, which paſſes out of the drop at P, and this is the higheſt point where any ray, that falls upon the ſide AX, can paſs out: for any rays SE, or SF, that fall higher than SD, will not paſs out in any point above P, but at the points O, or N, which are below it. Conſequently, though the arc GNOP increaſes, whilſt the diſtance of the incident ray from the axis SA increaſed, till we came to the ray SD; yet afterwards the higher the ray falls above the axis SA, this arc PONG will decreaſe.

Plate 8. Fig. 1.

As there are many rays which paſs out of the drop between G and P, ſo, by Prop. XLIII. ſome few rays will be reflected from thence; and conſequently, the ſeveral points between G and P, which are the points where ſome of the rays paſs out of the drop, are likewiſe the points of reflection for the reſt, which do not paſs out. Therefore in reſpect of thoſe rays which are reflected, we may call GP the arc of reflection, and may ſay that this arc of reflection increaſes, as the diſtance of the incident ray from the axis SA increaſes, till we come to the ray SD; the arc of reflection is GN for the ray SB, it is GO for the ray SC, and GP for the ray SD. But after this, as the diſtance of the incident ray from the axis SA increaſes, the arc of reflection decreaſes; for OG, leſs than PG, is the arc of reflection, for the ray SE, and NG is the arc of reflection for the ray SF.

From hence it is obvious, that ſome one ray, which falls above SD may be reflected from the ſame point with ſome other ray, which falls below SD. Thus, for inſtance, the ray SB will be reflected from the point N, and the ray SF will be reflected from the ſame point; and conſequently, when the reflected rays NR, NQ, are refracted as they paſs out of the drop at R and Q, they will be parallel, by what has been ſhewn in the former part of this Prop. But ſince the intermediate rays which enter the drop between SF and SB, are not reflected from the ſame point N, theſe two rays alone will be parallel to one another when they come out of the drop, and the intermediate rays will not be parallel to them. And conſequently, theſe rays RV, QT, though they are parallel, after they emerge at R and Q, will not be contiguous, and for that reaſon will not be effectual, by Prop. CXXIV. The ray SD is reflected from P, which has been ſhewn to be the limit of the arc of reflection; ſuch rays, as fall juſt above SD and juſt below SD, will be reflected from nearly the ſame point P, as appears from what has been already ſhewn. Theſe rays therefore will be parallel, becauſe they are reflected from the ſame point P; and they will likewiſe be contiguous, becauſe all of them enter the drop at one and the ſame place, very near to D. Conſequently ſuch rays, as enter the drop at D and are reflected from P the limit of the arc of reflection, will be effectual, by Prop. CXXIV. ſince when they emerge at the part of the drop between A and Y, they will be both parallel and contiguous.

PROP. CXXVI. When rays which are effectual emerge from a drop of rain after one reflection and two refractions, thoſe which are moſt refrangible will, at their emerſion, make a leſs angle with the incident rays than thoſe do which are leaſt refrangible; by which means, the rays of different colours will be ſeparated from one another.

Plate 8, Fig. 2. Let FH, GI, be effectual violet rays emerging from the drop at F, H; and FN, GP, effectual red rays emerging from the ſame drop at the ſame points. The violet rays (by Prop. CXXIV.) are parallel among themſelves, becauſe they are effectual: for the ſame reaſon the red rays are parallel among themſelves: but on account of the difference of refrangibility of the violet and red rays, the violet ray GI is not parallel to the red ray GP, but they diverge from the point G; and ſo of the reſt. Both the violet ray GI and the red ray GP are refracted from the perpendicular LO, but (by Prop. CXV.) GI more than GP; whence the angle IGO is greater than the angle PGO. If the incident ray AB be continued in the direction ABK, and if IG and PG be continued backward till they meet AB in K and W, the angle IKA is that which the violet or moſt refrangible ray makes at its emerſion with the incident ray, and PWA that which the red or leaſt refrangible ray makes with the ſame. And the angle IKA (El. I. 16.) is leſs than the exterior angle PWA. The ſame may be proved concerning the rays FH, FN, or any other rays which emerge reſpectively parallel to GI and GP. But (by Prop. CXXIV.) all the effectual violet rays are parallel to GI, and all the effectual red rays are parallel to GP. Therefore the effectual violet rays at their emerſion make a leſs angle with the incident rays than the effectual red rays. And univerſally the more refrangible rays, at their emerſion, make a leſs angle with the incident rays than thoſe which are leſs refrangible. And ſince the effectual rays GI, GP, of different colours make different angles with the incident ray AK at their emerſion, they will be ſeparated from one another: ſo that if the eye was placed in the beam FGHI, it would receive only rays of one colour from the drop XY, and in FGNP only rays of another colour.

Schol. The angle which the effectual red rays make with the incident rays is found to be 42°. 20′. that of the violet rays 40°. 17′.

Exp. Let a glaſs globe filled with water be expoſed to the rays of the ſun: let the eye of the ſpectator be ſo ſituated, that the leaſt refracted ray from the drop, coming to the eye, ſhall make an angle of about 42°. with the line paſſing through the eye and the ſun, the red rays only will be ſeen: if the place of the eye be changed, ſo as to enlarge this angle, the red will diſappear; but if the angle be leſſened, the colours of the more refrangible rays will appear.

PROP. CXXVII. If a line is ſuppoſed to be drawn from the centre of the ſun through the eye of the ſpectator, the angle which any effectual ray after two refractions and one reflection makes with the incident ray, will be equal to the angle which it makes with that line.

Let I be the place of the eye of the ſpectator; QT a line drawn from the centre of the ſun through the eye; and AB a ray coming from the centre of the ſun. Theſe two lines AB, QT, on account of the great diſtance of the ſun, may be looked upon as parallel. Therefore (El. I. 29.) the alternate angles AKI, KIT, or GIT, are equal. Plate 8. Fig. 2.

PROP. CXXVIII. When the ſun ſhines upon the drops of rain as they are falling, the rays which come from thoſe drops to the eye of the ſpectator, after one reflection and two refractions, produce the innermoſt or primary rainbow.

Let TFY be the innermoſt or primary rainbow, the outer part of which TFY is red, the inner part VDX violet, and the intermediate parts reckoning from the red to the violet, orange, yellow, green, blue, indigo. Suppoſe the ſpectator's eye at A; and let AI be an imaginary line from the centre of the ſun to the eye of the ſpectator. If a beam of light S coming from the ſun falls upon any drop F, and the effectual rays which emerge at F make an angle FAI of 42°. 2′. with the line AI, theſe rays (by Prop. CXXVII.) make the ſame angle with the incident rays, and conſequently are red. Hence the drop F will appear red; for all the other rays which emerge from F, and would be effectual if they fell upon the eye, being refracted more than the red rays, will paſs above the eye. If another beam of light S falls upon the drop D, and the effectual rays emerging at H make an angle of 40°. 17′. Plate 8. Fig. 3.

with the incident rays, the drop D will be of a violet colour: for all the other rays which emerge from H, and would be effectual if they came to the eye, being refracted leſs than the violet rays, will paſs below the eye. The intermediate drops between F and D will for the ſame reaſons be of the intermediate colours. And that which has been proved concerning the drops in the line FD, may be ſhewn of any other ſet of drops in which the angles made by the emerging and incident rays are equal. Thus, wherever a drop of rain is placed, if the angle which the effectual rays make with AI is equal to the angle FAI, or is 42°. 2′. any ſuch drop will appear red. If FAI was turned round upon the line AI, ſo that one end of this line ſhould always be at the eye, and the other at I oppoſite to the ſun, in this revolution the drop F would deſcribe a circle, of which I would be the centre, and TFY an arc. And ſince in this revolution the angle FAI continues the ſame, if the ſun was to ſhine upon this drop as it revolves, the effectual rays (by Prop. CXXVII.) would make the ſame angle with the incident rays in whatever part of the arc TFY the drop may happen to be; and conſequently in whatever part of the arc the drop F is, it will appear red. Now as innumerable drops are falling at once in right lines from the cloud, whilſt one drop is at F, there will be others at T, Y, and every other part of the arc, which will appear red in the ſame manner that F would have done in the ſuppoſed circular revolution. Therefore when the ſun ſhines upon the rain, there will be a red arc TFY produced oppoſite to the ſun. In like manner a violet arc VDX will be produced, and other intermediate arcs of the ſeveral intermediate colours, which will together make up the primary rainbow.

Schol. Caſcades and fountains, whoſe waters, in their fall, are divided into drops, will exhibit rainbows to a ſpectator, properly ſituated, during the time of the ſun's ſhining. This appearance is alſo ſeen by moon light, though ſeldom ſufficiently vivid to render the colours diſtinguiſhable. Dr. Gregory, in his excellent Economy of Nature, ſays, that he once ſaw a lunar bow, it was in autumn, the night was uncommonly light but ſhowery, and the colours much more vivid than he could have conceived. There were not ſo many colours diſtinguiſhable as in the ſolar bow. Coloured bows have been ſeen on graſs formed by the refraction of ſun's rays in the morning dew.

Artificial rainbows may be produced by candle light on the drops of water ejected by a ſmall fountain, or jet d'eau, or from the ſtream emitted from an œolipile. But the moſt natural and pleaſing is by means of the air fountain, the jet of which is perforated with a great number of very fine holes from which the water ſpouts ſo as to form a kind of fluted column. The rainbow is formed by the ſun's rays, for the ſpectator has only to place the ſpouting ſtreams directly in the ſun's beams, with his own back to the ſun, and being in a direct line with the ſun and centre of the jet, by ſtooping his head to a certain degree, he will diſcover the beautiful appearance of the natural priſmatic colours, and a ſmall rainbow, on the ſame principle as thoſe which are ſeen in the time of rain and ſun-ſhine.

PROP. CXXIX. The primary rainbow is never a greater arc than a ſemicircle.

Plate 8. Fig. 3. Since the line AI is drawn from the ſun through the eye of the ſpectator, and through I the centre of the rainbow, this centre is always oppoſite to the ſun. And ſince the angle FAI is an angle of 42°. 2′. F the higheſt part of the bow is 42°. 2′. from I its centre. If

therefore the ſun is more than 42°. 2′. above the horizon, I, which is oppoſite to it, muſt be more than 42°. 2′. below the horizon, and no primary rainbow will be ſeen. As much as the altitude of the ſun is leſs than 42°. 2′. ſo much will the higheſt point F of the rainbow be above the horizon: and when the ſun is in the horizon, I the centre of the bow will alſo be in the horizon on the oppoſite ſide, and half the circle will be viſible; but when the ſun is ſet, no bow can be ſeen.

PROP. CXXX. When the rays of the ſun fall upon a drop of rain, ſome of them after two reflections and two refractions may come to the eye of a ſpectator, who has his back towards the ſun and his face towards the drop.

Plate 8. Fig. 5.

If parallel rays from the ſun, ZV, YW, fall upon the lower part of the drop of rain BGW, they will be refracted towards the perpendiculars VL, WL, in entering the drop, and proceed in the direction VH, WI. At HI ſome part of theſe rays will (by Prop. XLIII.) be reflected into the directions HF, IG. And ſome of theſe rays will be again reflected at F, G, into the directions FD, GB; which rays, when they emerge out of the drop at B and D, will be refracted from the perpendiculars, and may come to the eye of a ſpectator whoſe back is towards the ſun and his face towards the drop.

PROP. CXXXI. Thoſe rays which are parallel to one another after they have been once refracted and once reflected in a drop of rain, will be effectual when they emerge after two refractions and two reflections.

Plate 8. Fig. 5.

The contiguous rays ZV, YW, being refracted towards the perpendiculars VL, WL, when they enter the drop, will (by Prop. XVIII.) become convergent; and becauſe theſe rays fall upon the drop very obliquely, their focus will not be far from the ſurface VW. If this focus is at K, the rays after they have paſſed the focus, will diverge from thence in the directions KH, KI: and, if KI is the focal diſtance of the concave reflecting ſurface HI, the reflected rays HF, IG, (by Prop. L.) will be parallel. Theſe rays are reflected again from the concave ſurface FG, and will meet in a focus at E, ſo that GE will be the focal diſtance of this reflecting ſurface: and becauſe HI, FG, are parts of the ſame ſphere, the focal diſtances GE, KI, are equal. When the rays have paſſed the focus E, they will diverge in the lines EB, ED. Now, if the rays VK, WK, when they have met at K, were to be turned back in the directions KV, KW, on emerging at V and W, they would (by Prop. XX.) be refracted into the lines of incidence, and become parallel. But ſince GE is equal to IK, the rays ED, EB, which diverge from E, fall in the ſame manner upon the drop at D and B, as the rays KV, KW, would fall upon it at V and W, and ED, EB, have the ſame inclination to the refracting ſurface DB, as KV, KW, would have to VW: whence the rays ED, EB, emerging at D and B, will be refracted in the ſame manner, and will have the ſame ſituation with reſpect to one another, as KV, KW, would have, that is, will be parallel to one another: having been contiguous before their entrance into the drop, they will therefore (by Prop. CXXIV.) be effectual.

PROP. CXXXII. When effectual rays emerge from a drop of rain after two reflections and two refractions, thoſe which are moſt refrangible will at their emerſion make a greater angle with the incident rays than the leaſt refrangible will make with them; by which means the rays of different colours will be ſeparated.

Plate 8. Fig. 5. Let BM, BA, a violet and a red ray, emerge from B; the angle which the violet ray BM makes with the incident ray YW is Y*r*M; and that which the red ray BA makes with the ſame is YSA. And ſince BSY, the external angle of the triangle B*r*S, is (El. I. 16.) greater than the internal angle B*r*S or B*r*Y; Y*r*M, the complement of B*r*S, is greater than YSA the complement of BSY. Conſequently, ſince the emerging rays make different angles with the ſame incident ray, the refraction which they ſuffer at emerſion will ſeparate them from one another.

Schol. The angle which the violet rays make with the incident ones is found to be 54°. 7′. and that of the red rays 50°. 57′.

PROP. CXXXIII. If a line is ſuppoſed to be drawn from the centre of the ſun through the eye of the ſpectator, the angle which, after two refractions and two reflections, any effectual ray makes with the incident ray, will be equal to the angle which it makes with that line.

Plate 8. Fig. 5. If YW be an incident ray, and BA an effectual ray, and AO a line drawn from the centre of the ſun through A the eye of the ſpectator, YW and AO may be conſidered as parallel; whence the alternate angles YSA, SAO, (El. I. 29.) will be equal.

PROP. CXXXIV. When the ſun ſhines upon the drops of rain as they are falling, the rays which come from thoſe drops to the eye of the ſpectator after two reflections and two refractions, produce the outermoſt or ſecondary rainbow.

Plate 8. Fig. 4. When the ſun ſhines upon a drop of rain E in the outer edge of the ſecondary rainbow CBD, the effectual violet ray EA (by Prop. CXXXII. Schol.) makes an angle EAI of 54°. 7′. with AI a line drawn from the ſun through the eye of the ſpectator, and therefore (by Prop. CXXXIII.) makes the ſame angle with the incident ray SB. Therefore if the ſpectator's eye is at A, all the rays except the violet, will (by Prop. X.) make a leſs angle with AI than EA, and fall above the ſpectator's eye. In like manner it may be ſhewn, that from the drop F only red rays will come to the ſpectator's eye, the reſt falling below it; and that the rays emerging from the intermediate drops between E and F, and coming to A, will emerge at intermediate angles, and preſent to the eye the intermediate colours. If EAI be conceived to turn round upon the line AI, in ſuch a revolution of the drop E, the angle EAI would remain the ſame, and conſequently the emerging rays would make the ſame angle

with the incident rays. But in ſuch a revolution the drop E would deſcribe a circle, of which I would be the centre, and CBD an arc. Conſequently, ſince the emerging rays make the ſame angle with the incident ones when the drop is at any other part of the arc as at E, the colour of the drop will be violet to an eye placed at A, in whatever part of the arc the drop is placed. Now, ſince there are innumerable drops of rain falling at once, whilſt one drop is at E, there will be others in all parts of the arc, which will all appear violet-coloured, for the ſame reaſon that E would have appeared of this colour in any other part of the arc. In like manner, as the drop F appears red at F, and at any part of the arc FD, ſo will any other falling drop when it comes to any part of that arc. The intermediate arcs are formed in the ſame manner with the violet arc CBD, and the red arc FD; and thus the whole ſecondary rainbow is produced.

PROP. CXXXV. The colours of the ſecondary rainbow are fainter than thoſe of the primary, and are ranged in the contrary order.

At every reflection many rays paſs out of the drop without being reflected; conſequently, the ſecondary rainbow which is produced after two reflections, is formed by fewer rays than the firſt, which is produced after one reflection.

Again, in the primary bow, the violet rays, when they emerge effectually, make a leſs angle with the incident rays (by Prop. CXXVI.) and therefore (by Prop. CXXVII.) with the line AI, than the red rays. But the rays are here only once reflected, and the angle which the effectual rays make with AI is the diſtance of the coloured drop from I the centre of the bow. Therefore the violet arc in the primary bow will be nearer to the centre of the bow than the red arc, that is, the innermoſt colour will be violet, and the outermoſt red. But in the ſecondary rainbow, the rays are twice reflected; and (by Prop. CXXXII.) the violet rays, which emerge ſo as to be effectual after two reflections, make a greater angle with the incident rays, that is, with the line AI than the red ones; which angle is the diſtance of the violet arc from I the centre of the bow. Therefore the violet arc in the ſecondary bow will be farther from the centre of the bow than the red arc; that is, the outermoſt colour is violet, and the innermoſt red. Plate 8. Fig. 3.

PROP. CXXXVI. The ſecondary rainbow is never a greater arc than a ſemicircle.

This is proved in the ſame manner as Prop. CXXIX. with this difference, that, ſince the rays of the higheſt colour in the ſecondary bow make an angle of 54°. 7'. with AI, this bow will begin to appear when the altitude of the ſun is leſs than 54°. 7'. and when the ſun is in the horizon on one ſide, this bow will have its centre in the horizon on the other ſide at the diſtance of 54°. 7'. from its higheſt point.

CHAP. VI.

Of Optical Instruments.

SECT. I.—*Of* Telescopes.

Def. XXX. An *Aſtronomical Teleſcope* conſiſts of two convex lenſes, whoſe diſtance from each other is equal to the ſum of their principal foci: that lens which is towards the object, is called the *object-glaſs;* that which is next the eye, is called the *eye-glaſs.*

Plate 8. Fig. 7.

If NL is one convex lens, whoſe focal diſtance is MF, and BD another, whoſe focal diſtance is CF; and if theſe are ſo placed that the diſtance between them is equal to MF added to CF, that is, MC, they form an aſtronomical teleſcope.

PROP. CXXXVII. Very remote objects, ſeen through an aſtronomical teleſcope, appear diſtinct and inverted.

Plate 8. Fig. 7.

Let PM, PL, PN, be rays coming (by Prop. VIII.) parallel from the middle point in a very diſtant object: let AN, AM, AL, come from the higheſt point, and QN, QM, QL, come from the loweſt point. Theſe parallel rays will (by Def. XVIII.) be collected into the focus, and there form an image of the object, which (by Prop. LXXXII.) forms the object of refracted viſion. But, by the conſtruction of the teleſcope, GFE is the focus of the eye-glaſs. Conſequently, the rays which diverge from any point G in this image will, (by Prop. XX.) after they have paſſed through the eye-glaſs, become parallel. Therefore if the eye is at any point on the other ſide of the eye-glaſs, the object of refracted viſion may be ſeen as diſtinctly as any very remote object can be ſeen by the naked eye; and becauſe the image is the object of viſion (by Prop. XXV.) it will be ſeen inverted.

PROP. CXXXVIII. The apparent diameter of an object ſeen through an aſtronomical teleſcope, is to the apparent diameter of the ſame object ſeen by the naked eye at the ſtation of the object-glaſs, as the diſtance of the image from the object-glaſs is to its diſtance from the eye-glaſs.

Plate 8. Fig. 7.

If the image, formed by the object-glaſs NL, were received upon a paper at EFG, the apparent diameter of the object ſeen by the naked eye at M, the ſtation of the object-glaſs, would be (by Prop. LXXXIV.) equal to the apparent diameter of the image ſeen from the ſame ſtation. Now the real diameter of the image is given, becauſe its diſtance MF from the lens is given. Conſequently, the apparent diameter of the image (by Prop. LXIX.) will be inverſely as the diſtance of the eye from it. If the eye is placed at C the ſtation of the eye-glaſs, and conſequently its diſtance from the image is FC, the image will appear to the eye in that ſtation bigger than at the ſtation M (by Prop. LXXXIX.) in the inverſe ratio of the diſtances FC, MF; that is, the apparent magnitude of the image at C will be to that at M, as MF to FC. But the apparent magnitude of the image ſeen from M is equal to that of the object ſeen by the naked eye. Therefore the image ſeen from C appears bigger than the object, in the ratio of MF to FC. This would ſtill be the caſe (by Prop. LXXXIV.) if the eye-glaſs were placed between the eye and the image, touching the eye. And ſince the image is in the focus of the eye-glaſs, the apparent magnitude (by Prop. XC.) is the ſame, whether the eye is cloſe to the lens, or at any diſtance from it. Therefore wherever the eye is, the apparent diameter of the object ſeen with the teleſcope, is to the apparent diameter of the ſame object ſeen by the naked eye at the ſtation of the object-glaſs, as MF to FC, or as the diſtance of the diſtinct image from the object-glaſs, to its diſtance from the eye-glaſs, that is, as the focal diſtance of the object-glaſs, is to the focal diſtance of the eye-glaſs, conſequently if the former be divided by the latter, the quotient will expreſs the magnifying power, thus if MF : FC : : 10 : 1, the teleſcope will magnify 10 times, in diameter.

PROP. CXXXIX. A teleſcope will not magnify an object, unleſs the focal diſtance of the object-glaſs is greater than the focal diſtance of the eye-glaſs.

The rays which come from diſtant objects being nearly parallel, the image GFE (by Def. XVIII.) will be in the focus of the object-glaſs, which, by the conſtruction of the teleſcope, is alſo the focus of the eye-glaſs. But the apparent diameter of an object ſeen through a teleſcope, is to its apparent diameter when ſeen by the naked eye (by Prop. CXXXVIII.) as the diſtance of the image from the object-glaſs, to its diſtance from the eye-glaſs; that is, by what has been juſt proved, as the focal diſtance of the object-glaſs, to the focal diſtance of the eye-glaſs. Conſequently, if MF, the focal diſtance of the object-glaſs, is greater than FC, the focal diſtance of the eye-glaſs, the object will be magnified: if MF be equal to FC, the object will appear as to the naked eye; if MF be leſs than FC, the object will appear diminiſhed. Plate 8. Fig. 7.

COR. 1. Hence the object-glaſs of a teleſcope ſhould be leſs convex than the eye-glaſs.

COR. 2. An object will be equally magnified by two teleſcopes of very different lengths, if the ratio of the focal diſtances of the object-glaſs and eye-glaſs be the ſame in each.

COR. 3. If a teleſcope is inverted, objects ſeen through it will be diminiſhed: for the object-glaſs which has the greater focal diſtance then becomes the eye-glaſs.

PROP. CXL. The viſible area, or ſpace which may be ſeen at one view through a teleſcope, is as the area of the eye-glaſs.

If GFE is any image, its diſtance from the object-glaſs being equal to the focal diſtance of the lens, the area of the image (by Prop. XXXVI.) is given; but the quantity of this image which can be ſeen at one view muſt be greater or leſs, according to the magnitude of the hole through which it is ſeen; that is, muſt be as the area of the eye-glaſs. Plate 8. Fig. 7.

PROP. CXLI. The brightneſs of an object ſeen through a teleſcope depends upon the area of the object-glaſs, but not the viſible area.

The brightneſs of the image, that is, of the object of refracted viſion, is (by Prop. XXXVIII.) as the area of the lens which forms it, that is, of the object-glaſs. But (by Prop. XXXVI. Schol. 2.) the magnitude of the image is the ſame, whether the area of the object-glaſs is great or ſmall; and conſequently, if we look at it through an eye-glaſs of a given area, the quantity to be ſeen at once will not be altered by any change in the area of the object-glaſs.

PROP. CXLII. The diſtance of the eye from the eye-glaſs, ſhould be equal to the principal focal diſtance of the eye-glaſs.

Plate 8. Fig. 7. Since the image GFE is in the focus of the lens DCB, wherever the eye is placed on the other ſide of the glaſs, the image will appear equally magnified. But when the eye is juſt as far from the eye-glaſs as its focal diſtance, the viſible area will be the greateſt: for, in that caſe, (by Def. XVIII.) none but rays parallel, before the refraction, to MC the axis of the teleſcope, and therefore to the ſides of the cylindrical tube in which the lenſes are placed, can reach the eye, and conſequently, no rays can come from the inner ſurface of this tube to the eye to make it viſible: whereas in any other ſtation of the eye, oblique rays from that ſurface would make the ſides of the tube viſible; whence the area of viſion, which remains the ſame, being (by Prop. CXL.) always as the area of the eye-glaſs, will be in part occupied by the ſides of the tube, and the object will be ſeen only through the remaining part.

Def. XXXI. A teleſcope conſiſting of four convex lenſes is a *double Aſtronomical Teleſcope.*

Plate 8. Fig. 8. Let the two lenſes NML, and B, placed at the diſtance MB, equal to the ſum of their focal diſtances, form one teleſcope, and the two lenſes C, D, placed at the diſtance CD, equal to the ſum of their focal diſtances, form another. If theſe two teleſcopes are fixed at the diſtance CB from each other, ſo as to be both uſed together, they form a double teleſcope: the lens LMN next to the object, is called the object-glaſs, and the lens B next the object-glaſs is called the firſt eye-glaſs, C the ſecond, and D next to the eye the third.

PROP. CXLIII. An object ſeen through a double teleſcope appears diſtinct and erect.

Plate 8. Fig. 8. The parallel rays which fall upon the object-glaſs NML (by Prop. CXXXVII.) form a diſtinct inverted image at GFE the focus of the object-glaſs. This image being alſo in the focus of the firſt eye-glaſs B, the rays of each beam from the ſeveral points of this image will become parallel by paſſing through B: whence, falling parallel on the ſecond eye-glaſs C, they will form a diſtinct inverted image at KIH the focus of this ſecond eye-glaſs: and becauſe KIH is alſo the focus of the third eye-glaſs D, the rays from this image, after paſſing through this third eye-glaſs, will come to the eye parallel to each other. Conſequently,

the object will be ſeen diſtinctly: and becauſe the ſecond image is inverted with reſpect to the firſt, which is inverted with reſpect to the object, the ſecond image, or object of refracted viſion, is in the ſame ſituation as the object itſelf.

PROP. CXLIV. A double teleſcope magnifies an object in the ratio of the focal diſtance of the object-glaſs, to the focal diſtance of the firſt eye-glaſs.

The firſt teleſcope MB magnifies the object in the ratio of MF to FB: and the ſecond teleſcope CD is commonly made up of two lenſes of equal convexities, which will not alter the apparent magnitude of the objects. Therefore, when both are uſed together, the object is only magnified by the firſt in the ratio of MF the focal diſtance of the object-glaſs, to FB the focal diſtance of the firſt eye-glaſs; conſequently, the magnifying power is found by dividing MF by FB. Plate 8. Fig. 8.

SCHOL. 1. The different refrangibility of the rays of light makes refracting teleſcopes imperfect: for thoſe rays which are moſt refracted by paſſing through the lens, will be brought to a focus, and form an image nearer to the object-glaſs than thoſe which are leſs refracted; and conſequently the ſeveral ſorts of rays are not properly collected in one focus to produce a perfectly white image, but each has its own focus, producing a confuſed and coloured image.

Of two refracting teleſcopes which magnify equally, the ſhorter will give a more imperfect image than the longer. For the image appearing equal in both, but being farther from the object-glaſs in the longer than the ſhorter, muſt be in reality larger or more magnified: whence the defect ariſing from the different refrangibility of the rays will be more viſible in the longer than in the ſhorter teleſcope. Hence, reflecting teleſcopes are more perfect than refracting ones; for when all the rays are reflected, their angles of incidence and reflection being equal, they will all meet in a focus at the ſame diſtance.

SCHOL. 2. To remedy the defect of refracting teleſcopes, ariſing from the different refrangibility of rays of light, a compound object glaſs was invented by Mr. Dollond, conſiſting partly of white flint glaſs, and partly of crown glaſs, which have different refracting powers. Theſe refract contrary ways; and the exceſs of refraction in the crown glaſs is made ſuch, as to deſtroy the colour cauſed by the flint glaſs. A teleſcope thus formed is called achromatic. Let ABED repreſent a double concave lens of white flint glaſs, and AGDF a double convex of crown glaſs; then the part of the lenſes which are on the ſame ſide of the common axis, viz. AEB and AFG may be conceived to act like two priſms which refract contrary ways; and if the exceſs of refraction in the crown glaſs, be ſuch as preciſely to deſtroy the divergency of colour cauſed by the flint glaſs, the incident ray SH will be refracted to X without any production of colour: the ſame is true of the ray *sh*, and of all the other incident rays; and conſequently the whole focal image formed by this compound object-glaſs, will be achromatic, or free from colour which might ariſe from refraction. It will therefore bear a larger aperture, and greater magnifying power, and of courſe enlarge objects much more than a common refracting teleſcope of the ſame length. The great impediment to the conſtruction of large achromatic teleſcopes, is the want of a flint glaſs of an uniform denſity. Plate 12. Fig. 10.

Dr. Blair has, within theſe few years, diſcovered that certain fluids, particularly thoſe which contain the muriatic acid, may be formed into lenſes. With theſe he has produced achromatic teleſcopes, which ſeem as perfect as the thing will admit of. See Tranſactions of the Edinburgh Royal Society. Alſo, Encyc. Brit. Art. Teleſcope. Vol. XVIII. Part 1.

PROP. CXLV. To explain the conſtruction and uſe of ſeveral kinds of teleſcopes.

I. Of Galileo's Teleſcope.

Galileo's teleſcope conſiſts of a convex object-glaſs and a concave eye-glaſs, ſo placed that the diſtance between them is the difference of their focal diſtances.

Plate 8. Fig. 9. In this teleſcope ZYX, a convex lens, is placed at the diſtance from BA a concave lens of YC, the difference between YF the focal diſtance of ZX, and CF the focal diſtance of BA.

From a diſtant object let rays fall upon the convex lens YZ, from which they will proceed towards the focus of this lens at FG. But the concave lens AB, the focus of which is at FG, renders the converging rays parallel when they reach the eye; whence an image will be formed upon the *retina*. And the pencils of rays being made more diverging by paſſing through the concave lens, the viſible image is ſeen under a larger angle than the object, and appears magnified. Alſo, becauſe the pencils which form the image only croſs one another once, the image appears erect.

II. Of Sir Isaac Newton's Teleſcope.

Plate 8. Fig. 10. In a tube ABCD, towards the end BC, let the concave mirror GH be placed perpendicular to DC the lower ſide of the tube. If an object, which is at ſuch a diſtance that rays coming from the ſame point may be conſidered as parallel to one another, be placed before the open end of the tube AD, theſe parallel rays will be reflected from the concave mirror GH, and becoming convergent, would (by Prop. CXII.) form an inverted picture of the object upon a paper held at the focus of the mirror. But if the converging rays, before they reach the focus, fall upon a plane mirror K, placed at an angle of 45 degrees with DC the ſide of the tube, or with the axis of the teleſcope, they will be reflected from thence, and meet before it at L, forming an image perpendicular to the object, or parallel to the axis of the teleſcope. If this image be placed in the focus of the convex lens L, fixed in the ſide of the teleſcope, the eye will ſee it diſtinctly through the lens.

The image ſeen from the ſtation of the eye-glaſs L, either with or without the glaſs, will (as in the refracting teleſcope, ſee Prop. CXXXVIII.) appear as much larger than when ſeen from the concave mirror, that is, as much larger than to the naked eye, as the diſtance of the image from the eye-glaſs, is leſs than its diſtance from the mirror, or as its diſtance from the mirror is greater than its diſtance from the lens.

III. Of Gregory's Teleſcope.

Plate 8. Fig. 11. In the tube TTYY, let a concave mirror EN be placed. Any parallel rays OO, PP, form an object A, falling upon this mirror, will, after reflection, (by Prop. CXI.) form an

inverted image at C its focus. Let C be more remote from a ſecond ſmaller concave mirror PO (placed parallel and oppoſite to the firſt mirror EA in ſuch manner that their axes ſhall be in the ſame ſtraight line) than its focus. The rays which diverge from the ſeveral points of the image at C, and fall upon the mirror PO, will (by Prop. L.) converge after reflection; and conſequently, if they paſs through a hole NM in the firſt mirror EA, they will form a ſecond image which will be inverted in reſpect of the firſt, and in the ſame poſition with the object. If, whilſt theſe rays are converging, they paſs through a plano-convex lens *Spo* (placed in a ſmaller tube joined to the larger) they will be brought to a focus, ſooner than they would otherwiſe have been, forming the ſecond image F. This erect image is ſeen by the eye at O, through a meniſcal eye-glaſs LL, whoſe convexity is greater than its concavity. For the magnifying power of this teleſcope, ſee Muſchenbroek. Intro. ad Phil. Nat. or Prieſtley's Optics, page 736.

SCHOL. 1. In the teleſcopes made by Dr. Herſchel, the object is reflected by a mirror, as in the Gregorian teleſcope, and the rays are intercepted by a lens at a proper diſtance, ſo that the obſerver has his back to the object, and looks through the lens at the mirror. The magnifying power will be the ſame as in the Newtonian teleſcope; but there being no ſecond reflector, the brightneſs of the object viewed in the Herſchel teleſcope, is greater than that in the Newtonian teleſcope.

The tube of Dr. Herſchel's grand teleſcope, is 39 feet 4 inches in length, 4 feet 10 inches in diameter, every part of which is made of iron. The concave ſurface of the great mirror is 48 inches of poliſhed ſurface in diameter, its thickneſs is $3\frac{1}{2}$ inches, and its weight is upwards of 2000lb. This noble inſtrument was, in all its parts, conſtructed under the ſole direction of Dr. Herſchel: it was begun in the year 1785, and completed Auguſt 28, 1789, on which day was diſcovered the ſixth ſatellite of Saturn. It magnifies 6000 times.

SCHOL. 2. Dr. Prieſtley obſerves, that the eaſieſt method of finding the magnifying power of any teleſcope, by experiment, is to meaſure the diameter of the aperture of the object-glaſs, and that of the little image of it which is formed at the place of the eye. For the proportion between theſe gives the ratio of the magnifying power, provided no part of the original pencil be intercepted by the bad conſtruction of the teleſcope. For, in all caſes, the magnifying power of teleſcopes or microſcopes, is meaſured by the proportion of the original pencil, to that of the pencil which enters the eye. Another method, is to obſerve at what diſtance you can read any book with the naked eye; and then removing the book to the fartheſt diſtance at which you can diſtinctly read it by the help of the teleſcope. The book choſen for this purpoſe ſhould be ſuch, that the connexion of the ſubject ſhould not aſſiſt the obſerver; as tables of logarithms, &c. Much depends on the ſteadineſs with which the inſtrument is fixed.

SECT. II.—*Of* MICROSCOPES.

DEF. XXXII. A *ſingle Microſcope* is one convex lens placed between a ſmall object and the eye. Plate 8. Fig. 6.

DEF. XXXIII. A *double Microſcope* conſiſts of two convex lenſes, of which the object-glaſs is more convex than the eye-glaſs; and the diſtance between them is equal to the diſtance of the image from the object-glaſs, added to the focal diſtance of the eye-glaſs.

Plate 8. Fig. 12. Let AB, a convex lens, be the object-glaſs, and EF, another convex lens, be the eye-glaſs. Let the ſmall object KL be farther from the object-glaſs than its focus; an image MDN of the object will (by Prop. XXIV.) be formed behind the glaſs; let the diſtance of this image from the object-glaſs be ID, and let its diſtance from the eye-glaſs be equal to the focal diſtance of the eye-glaſs; the diſtance of the two glaſſes from each other will be $ID+DX$, or IX, that is, the diſtance of the image from the object-glaſs, added to the focal diſtance of the eye-glaſs.

Some compound microſcopes are made with three glaſſes, ſo that the rays after paſſing through AB the object-glaſs, and EF the eye-glaſs, are again made converging by a ſecond eye-glaſs, and therefore brought ſooner to a focus, than by the firſt, and the field of viſion will be much greater than if only one lens was uſed.

Cor. Hence it appears, that the difference between the microſcope and teleſcope is, that in the teleſcope the rays of each pencil fall upon the object-glaſs nearly parallel, and are united in its focus; but in the microſcope they fall upon it very much diverging from one another, and therefore form the image in a place beyond the focus, and conſequently larger than the object.

PROP. CXLVI. An object ſeen through a double microſcope appears diſtinct and inverted.

Plate 8. Fig. 12. The pencils of rays iſſuing from the objects KL, being tranſmitted through the object-lens AB, their foci will be in MN; where there will be an inverted image of the object, which is viewed through another lens, or eye-glaſs EF, the focus of which is at MN; hence a diſtinct and direct image is formed upon the *retina*, and it is ſeen inverted.

PROP. CXLVII. The apparent diameter of an object ſeen through a double microſcope is to that of the ſame object ſeen by the naked eye from the ſtation of the object-glaſs, in the compound ratio of the diſtance of the image from the object-glaſs, to its diſtance from the eye-glaſs, and of the limit of diſtinct viſion to the diſtance of the object-glaſs from the object.

The firſt part of this Propoſition is demonſtrated as Prop. CXXXVIII. And it is manifeſt, from Prop. LXIX. that if AB the diſtance of the object-glaſs from the object is leſs than the limit of diſtinct viſion, the apparent diameter of the object will be as much greater than that of the object at the diſtance at which the naked eye can ſee it diſtinctly, as IC is leſs than that diſtance. Therefore the object is magnified, becauſe the diſtance of the image from the object-glaſs is greater than its diſtance from the eye-glaſs, and alſo becauſe the diſtance from the object is leſs than the limit of diſtinct viſion. The magnifying power of the microſcope is then in the ratio compounded of theſe two ratios. Suppoſe $ID=6IC$; and the eye-glaſs EF to be one inch focus, and the limit of diſtinct viſion to be ſeven inches, then the diameter of the object KL, will be magnified $6 \times 7 = 42$; conſequently 1764 times in ſurface.

PROP. CXLVIII. When the ſame eye-glaſs is uſed, the magnifying power of the microſcope will be increaſed by increaſing the convexity of the object-glaſs.

For in order to keep the image in the focus of the eye-glaſs, when the convexity of the object-glaſs is increaſed, the object-glaſs muſt be brought nearer to the object: the conſequence of which will be that the ratio of the limit of diſtinct viſion to the diſtance of the object-glaſs from the object will (El. V. 8.) be increaſed: whence (by Prop. CXLVII.) the ratio of the apparent diameter of the object of refracted viſion to that of the object ſeen by the naked eye will alſo be increaſed.

Schol. The aperture of the object-glaſs in a microſcope muſt be ſmall, elſe the outermoſt rays, diverging too much, will hinder the diſtinctneſs of viſion: but, on account of the ſmallneſs of the aperture, the object will appear faint, and it will appear neceſſary, in order to remedy this, to illuminate the object as much as poſſible.

PROP. CXLIX. To deſcribe the conſtruction and uſe of the Solar Microſcope.

In a dark room, let a round hole be made in a window-ſhutter about three inches in diameter, through which the ſun may caſt a cylinder of rays into the room. In this hole let a tube be fixed, containing a convex lens of about two inches in diameter, and three inches focal diſtance; the object, placed between two concave glaſſes, at the diſtance of about two inches and a half from the firſt convex lens; and a ſecond convex lens, whoſe focal diſtance is a quarter of an inch, placed at this diſtance from the object. Let a plane mirror, connected with the tube, and moveable by means of a wheel, receive the ſun's rays on the outſide of the ſhutter, and convey them into the tube. The rays, paſſing through the firſt lens, will ſtrongly illuminate the object, from which they will paſs through the ſecond lens, and form an inverted image of the object, magnified in the ratio of the diſtance of the object from the lens to that of the image from the lens, that is, in this caſe, ſuppoſing the diſtance of the diſtinct picture to be twelve feet or 144 inches, in the ratio of $\frac{1}{4}$: 144. Conſequently, the diameter of the object will be magnified 576 times.

SECT. III.—*Of the* Magic Lantern.

PROP. CL. To deſcribe the conſtruction of the Magic Lantern.

In the ſide of a lantern let a tube be inſerted, conſiſting of two parts, one moveable upon the other. In the moveable part let a convex lens GG be fixed; in the immoveable part let an object EE, painted with tranſparent colours upon a piece of thin glaſs, be placed; and in the Plate 8. Fig. 13.

the ſixed part of the tube, a convex lens DD. This lens will caſt a ſtrong light from the candle upon the object EE. And when the rays which diverge from the ſeveral points of the object are, by the lens GG, made to converge, they will (by Prop. XXV.) form an inverted image of the object at KL, upon any white ſurface; provided that the object is farther from the lens than its focus, and that the whole apparatus is placed in a dark room. The image KL will be larger than the object EE, in proportion as the diſtance of the image from the lens is greater than the object. A concave reflector AB may be placed within the lantern, behind the candle, to increaſe the illumination of the picture EE. If the object be placed in an inverted poſition, its image will appear erect.

SECT. IV.—*Of the* Camera Obscura.

PROP. CLI. To deſcribe the conſtruction and uſe of the Camera Obſcura.

Plate 8. Fig. 14.

Let CD be a convex lens, and HK a plane mirror inclined at an angle of 45 degrees. An inverted image of the object AB would be formed at EG, where the foci of the rays from the object are found after refraction; but the rays being intercepted by the plane mirror HK, are reflected (by Prop. C.) to NM, the focal diſtance before it, making an angle with the mirror of 45 degrees; whence the image will be in a poſition perpendicular to the object, at the top of the box, where, if the rays be received on a ſheet of oiled paper, or a plate of glaſs unpoliſhed on one ſide, it will be diſtinctly viſible.

BOOK VII.

OF ASTRONOMY.

PART I.

Of the MOTIONS *of the* HEAVENLY BODIES.

CHAP. I.

Of the SOLAR SYSTEM *in General.*

DEF. I. THE *Solar Syſtem* conſiſts of the Sun, which is a luminous body; ſeven primary planets, Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Herſchel; eighteen ſecondary planets, the Earth's Moon, Jupiter's four Satellites, Saturn's ſeven, and ſix belonging to the Herſchel; and an uncertain number of comets: all which are opaque.

SCHOL. Upon entering the ſubject of Aſtronomy, it will be proper briefly to deſcribe the different ſyſtems which have been invented, in order to ſolve the natural appearances of the heavenly motions.

PTOLEMY ſuppoſed the earth to be perfectly at reſt, and the ſun, moon, planets, comets, and fixed ſtars to revolve about it every day; but that beſides this diurnal motion, the ſun, moon, planets, and comets, had a motion in reſpect to the fixed ſtars, and were ſituated, in reſpect to the earth, in the following order: the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn. The revolutions of theſe bodies he ſuppoſed to be made in circles about the earth placed a little out of the centre. This ſyſtem will not ſolve the phaſes of Venus and Mercury, and therefore cannot be true.

The ſyſtem received by the Egyptians was this: the earth was ſuppoſed immoveable in the centre, about which revolved, in order, the Moon, Sun, Mars, Jupiter and Saturn; and about the Sun revolved Mercury and Venus. This diſpoſition will account for the phaſes of Mercury and Venus, but not for the apparent motions of Mars, Jupiter, and Saturn.

Another ſyſtem was that of Tycho Brahe, a Daniſh nobleman, who was anxious to reconcile the appearances of nature with ſome paſſages of the Scriptures, taken in their literal interpretation. In his ſyſtem, the earth is placed immoveable in the centre of the orbits of the ſun and moon, without any rotation about its axis; but he made the ſun the centre of the orbits of the other planets which, therefore, revolved with the ſun about the earth. Objections to this ſyſtem are, the want of that ſimplicity by which all the apparent motions may be ſolved; and the neceſſity of ſuppoſing that all the heavenly bodies revolve about the earth every day: alſo to ſuppoſe that a body ſhould revolve in a circle about its centre without any central body is phyſically impoſſible.

Some of Tycho's followers, ſeeing the abſurdity of a diurnal revolution of the heavenly bodies about the earth, gave a rotatory motion to the earth, and this was called the Semi-Tychonic ſyſtem.

The ſyſtem which is now univerſally received, is called the Copernican. It was formerly taught by Pythagoras, 500 years before Chriſt; and afterwards rejected, till revived by Copernicus in the ſixteenth century. Here the ſun is placed in the centre of the ſyſtem, about which the planets revolve from weſt to eaſt, in the following order: Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Herſchel planet: beyond which, at immenſe diſtances, are placed the fixed ſtars. The moon revolves round the earth; and the earth turns about an axis. The other ſecondary planets move round their reſpective primaries from weſt to eaſt at different diſtances, and in different periodical times.

Plate 9. Fig. 1. According to this doctrine, the Sun S is the centre of the ſyſtem; Mercury *a*, Venus *b*, the Earth *t*, Mars *e*, Jupiter *f*, and Saturn *h*, revolve in elliptical orbits round the ſun; the moon *d* revolves about the earth, and the ſatellites of Jupiter, Saturn, and the Herſchel, revolve about their primaries; and the planes of their orbits are inclined to one another.*

This doctrine, being admitted as true, will account for the apparent motions, and other phenomena, of the heavenly bodies, as will be ſeen in the following Chapters.

CHAP. II.

Of the EARTH.

SECT. I.—*Of the* GLOBULAR FORM *of the* EARTH, *and its* DIURNAL MOTION *about its* AXIS, *and of the Appearances which ariſe from theſe.*

PROPOSITION I.

The earth is of a globular form.

For, 1. The ſhadow of the earth projected on the moon in an eclipſe is always circular; which appearance could only be produced by a ſpherical body. 2. The convexity of the ſurface of the ſea is viſible; the maſt of an approaching ſhip being ſeen before its hull. 3. The north pole becomes more elevated by travelling northward, in proportion to the ſpace paſſed over. 4. Navigators have ſailed round the earth, and by ſteering their courſe continually weſtward, arrived, at length, at the place from whence they departed.

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* Dr. Herſchel, on the 13th of March, 1781, while purſuing a plan which he had formed of obſerving, with teleſcopes, of his own conſtruction, every part of the heavens, diſcovered, in the neighbourhood of H Geminorum, a planet far beyond the orbit of Saturn, which had never before been viſible to mortal eyes. He has ſince diſcovered ſix ſecondaries belonging to this new planet, which planet is called either the HERSCHEL, from the name of its indefatigable and truly great diſcoverer; or the Georgium Sidus, or Georgian Planet, in honour of the preſent king, who has diſtinguiſhed himſelf as the patron of Dr. Herſchel. The planet is denoted by this character ♅; an H, as the initial of the name, the horizontal bar being croſſed by a perpendicular line, forming a kind of croſs, the emblem of chriſtianity, denoting, perhaps, that its diſcovery was made in the Chriſtian era, as all the other planets were known long before that period.

Def. II. The *Axis* of the earth is an imaginary line paſſing through the centre, about which its diurnal revolution is performed.

Def. III. The *Poles* of the earth are the extremities of this axis.

Def. IV. The *Equator* is the circumference of an imaginary great circle paſſing through the centre of the earth, perpendicular to the axis, and at equal diſtances from the poles.

Def. V. If the axis of the earth be produced both ways, as far as the concave ſurface of the heavens, in which all the heavenly bodies appear to be placed, it is then called the *Axis of the Heavens*; its extremities are called the *Poles of the Heavens*; and the circumference produced by extending the plane of the equator to the ſame concave ſurface, is called the *Equator in the Heavens*.

Def. VI. Circles drawn through the poles of the earth or heavens perpendicular to the plane of the equator, are called *Secondaries* of the equator.

Def. VII. The *ſenſible Horizon* is an imaginary circle, which, touching the ſurface of the earth, ſeparates the viſible part of the heavens from the inviſible. The *rational Horizon* is a circle parallel to the former, the plane of which paſſes through the centre of the earth.

Schol. Since (by Book VI. Prop. LXIX.) the apparent diameter of an object is inverſely as its diſtance, if the diſtance be increaſed in ſuch manner that it may be looked upon as infinite, the apparent magnitude becomes a point. Hence AF, the ſemidiameter of the earth, viewed at the different diſtances *o*, O, R, diminiſhes, till at the diſtance of O, a fixed ſtar, it becomes a point, and the ſtar appears in the ſame place in the heavens, whether viewed from the viſible horizon SET, or rational horizon HBR. Plate 9. Fig 4.

Def. VIII. The *Poles* of the *Horizon* are two points, the one of which, over the head of the ſpectator, is called the *Zenith*; the other, which is under his feet, is called the *Nadir*.

Def. IX. Circles drawn through the zenith and nadir of any place, cutting the horizon at right angles, are called *Vertical Circles*.

Def. X. A verticle circle paſſing through the poles of the heavens, is a *Meridian*, and is ſaid to be the meridian of any place through which it paſſes.

Def. XI. The meridian of any place paſſing through the poles, and falling perpendicularly upon the horizon, cuts it in two oppoſite cardinal points, called *North* and *South*.

Def. XII. A *Meridian Line*, is the common interſection of the plane of the meridian and the plane of the horizon.

Cor. Hence any line which lies due north and ſouth in an horizontal plane, may be conſidered as part of the meridian line.

Schol. 1. To draw a meridian line, perpendicular to an horizontal plane, erect a wire, or ſtile, ſeven or eight inches long, and as it is not eaſy to determine preciſely the extremity of the ſhadow, it will be beſt to make the ſtile flat at top, and to drill a ſmall hole through it, noting the lucid point projected by it, mark, at ſeveral different times before noon, theſe lucid points, and through them draw concentric circles about the middle point of the wire's ſtation; obſerve in the afternoon when the lucid points again touch theſe circles; and find the middle point of each arc between the points already taken: a line drawn through theſe middle points, and the common centre, will be the meridian line; for, ſince at equal diſtances from noon, the ſun is at the ſame height, or in verticals equally diſtant from the meridian, the circle drawn through the zenith at equal diſtances from theſe verticals is the meridian. This ſhould be done about the ſummer ſolſtice, between the hours of 9 and 11 in the morning, and 1 and 3 in the afternoon.

Schol. 2. To obſerve the tranſit of any heavenly body over the plane of the meridian; place, in this plane, a teleſcope, having two croſs hairs before its object-glaſs, one vertical, the other horizontal, and obſerve when the vertical hair paſſes through the centre of the heavenly body: or, hanging two plumb-lines exactly over the meridian line, place your eye cloſe to one of the threads in ſuch manner, as that it ſhall cover the other thread, and obſerve when the body is behind the threads.

Def. XIII. The *Altitude* or *Depreſſion* of any heavenly body above or below the horizon, is the arc of a vertical circle intercepted between the body and the horizon, or the angle at the centre meaſured by that arc.

Schol. The altitude of any heavenly body is found by the help of a quadrant thus: bring the quadrant into ſuch a ſituation that the ſtar may be ſeen through the ſights; then the angle, contained between the ſtring of the plummet and the ſide of the quadrant on which the ſights are not placed, is the altitude of the ſtar.

Def. XIV. The *Prime Vertical*, is that which croſſes the meridian at right angles in the zenith and nadir, cutting the horizon in the cardinal points *Eaſt* and *Weſt*.

Def. XV. The *Azimuth* of a heavenly body, is the arc of the horizon intercepted between the meridian and a verticle circle paſſing through that body; it is eaſtern or weſtern as the body is eaſt or weſt of the meridian.

Schol. The azimuth of any ſtar may be thus found. Let AC be a given meridian line. Above any point A in this line, let a cord with a plummet be hung: let another cord with a plummet be hung at E, ſo that the ſtar and the two cords ſhall lie in one and the ſame right line. Let the perpendiculars AD, BE, repreſent the cords, and draw AB. From the point B, to any point C, in the meridian line AC, taken at pleaſure, draw the right line BC; then with a ſcale of equal parts meaſure the three lines AB, AC, BC. In the triangle, therefore, ABC, there will be given all the ſides, from whence will be found the angle BAC, equal to the azimuth required. Plate 9. Fig. 5.

For if the meridian line be ſuppoſed to be continued to F, and the line BA to G, the angle FAG will be the azimuth of the ſtar; but the angle FAG will be equal to the angle at the vertex BAC; therefore the angle BAC will be equal to the azimuth.

Def. XVI. The *Amplitude* of a heavenly body at its riſing, is the arc of the horizon intercepted between the point where the body riſes, and the eaſt; its amplitude at ſetting, is the arc of the horizon intercepted between the point where the body ſets, and the weſt: it is northern, or ſouthern, as the body riſes, or ſets, to the north or ſouth of eaſt or weſt.

Def. XVII. If an heavenly body riſes, or ſets, when the ſun riſes, it is ſaid to riſe or ſet *coſmically*; if it riſes, or ſets, when the ſun ſets, it is ſaid to riſe or ſet *achronically*; it is ſaid to ſet or riſe *heliacally*, when it approaches ſo near the ſun as to become inviſible, or recedes ſo far from him as to become viſible.

Def. XVIII. The *Latitude* of a place upon the ſurface of the earth, is its diſtance from the earth's equator; it is meaſured by the arc of the geographical meridian of the place intercepted between the place and the equator: latitude is either northern or ſouthern.

Def. XIX. *Parallels of Latitude*, are circles on the ſurface of the earth drawn parallel to the equator.

PROP. II. A degree in the equator is to a degree in any parallel of latitude, as radius to the coſine of latitude.

Let EPQ be a geographical meridian, EQ the equator, and FB a parallel of latitude. The circumference EQ is to the circumference FB, and any part of EQ, to any ſimilar part Plate 9. Fig. 5.

part of FB, as CQ, or CB, the radius of EQ, to AB the radius of FB; and AB is the coſine of the arc BQ, which is the latitude of the parallel FB. Therefore a degree in EQ is to a degree in FB, as radius to the coſine of latitude.

DEF. XX. The *Longitude* of a place, is the diſtance between the meridian of that place, and the meridian of ſome other place, taken at pleaſure, and called the firſt meridian; it is meaſured by the arc in the equator intercepted between theſe two meridians. Longitude is either eaſtern or weſtern, and is meaſured 180 degrees each way.

PROP. III. The altitude of one pole, and the depreſſion of the other, at any place on the earth's ſurface, is equal to the latitude of that place.

Plate 9. Fig. 2. Let R be a place upon the earth's ſurface; Z, N, its zenith and nadir; P, S, the poles of the heavens, and F, *s*, the poles of the earth; EE, the celeſtial equator, *ee*, the terreſtrial, and HO the horizon. The latitude of the place is *e*R, or the equal arc EZ; and PO is the elevation of one pole, and HS the depreſſion of the other. Becauſe ZO is the diſtance of the zenith from the horizon, it is an arc of 90 degrees; and becauſe EP is the diſtance of the pole from the equator, it is alſo an arc of 90 degrees: ZO and EP are therefore equal. Take from each of theſe the common arc ZP, and the remainders EZ and PO are equal. But HS and PO are equal, becauſe they ſubtend the equal angles HTS, PTO: therefore the elevation of one pole PO, and the depreſſion of the other HS, are equal to the latitude of the place EZ.

COR. Hence the circumference of the earth may be meaſured, by meaſuring the length on the ſurface of the earth paſſed over in a line which lies north and ſouth, while the pole gains one degree of elevation, and multiplying this length by 360. A degree of latitude contains $69\frac{1}{4}$ Engliſh miles, whence 24930 miles is the meaſure of the circumference of the earth, and the radius 3985; but, as will be ſhewn hereafter, the earth is a ſpheriod, whoſe polar diameter, is to the equatorial as 229 to 230.

PROP. IV. The elevation of the equator at any place is equal to the complement of its latitude.

Plate 9. Fig. 2. Becauſe ZO is equal to EP (each being an arc of 90 degrees) EZ is equal to PO, that is, (by Prop. III.) to the latitude of the place. But EH, the elevation of the equator, is the complement of EZ, it is therefore equal to the complement of the latitude of the place.

PROP. V. The earth revolving daily round its axis from weſt to eaſt, the heavenly bodies will appear to a ſpectator on the earth to revolve in the ſame time from eaſt to weſt.

Let RCBF be the earth, T its centre, HTO the rational horizon to a ſpectator at R whoſe zenith is Z; let a ſtar appear in the horizon at H. The earth revolving from weſt to eaſt, that is, in the order of the letters R, C, B, F, in a fourth part of one revolution, the ſpectator will be carried from R to C : conſequently, his horizon will become ZN, and the ſtar which appeared in his horizon at H when he was at R, will now appear nearly in the zenith. When another fourth part of the revolution is completed, the ſpectator will be at B, and N being now his zenith, and HO his horizon, the ſtar will be ſet with reſpect to him, and will not riſe till he is again in the ſtation R, that is, till the earth has completed one revolution. Thus whilſt the earth has turned once round upon its axis from weſt to eaſt, all the heavenly bodies in the concave ſphere of the heavens will appear to have turned round from eaſt to weſt.

PROP. VI. The alternate ſucceſſion of day and night is the effect of the revolution of the earth round its axis.

For, all the heavenly bodies appearing (Prop. V.) to move from eaſt to weſt, while the earth revolves from weſt to eaſt, the ſun will appear, in each revolution, to riſe above the horizon in the eaſt, and after deſcribing a portion of a circle, to ſet in the weſt, and will continue below the horizon, till by the revolution of the earth it again appears in the eaſt ; and thus day and night will be alternately produced.

Schol. The time of noon is found, by obſerving the inſtant when the centre of the ſun is cut by the perpendicular hair in a meridian teleſcope, as deſcribed Def. XII. Schol. 2, or by a ſun-dial.

SECT. II.—*Of the* ANNUAL MOTION *of the* EARTH *round the* SUN.

PROP. VII. The earth revolving round the ſun in 365 days, 6 hours, 56 minutes, 4 ſeconds, the ſun appears to revolve round the earth in the ſame time, but in the contrary direction.

Let S repreſent the ſun, BAC the orbit of the earth, and FGHE the ſtarry concave. Whilſt the earth is moving from A through B to C, it is manifeſt that, to a ſpectator on the earth, the ſun muſt appear to move in the contrary direction from E through F to G, in the great circle of the heavens formed by the plane of the earth's orbit. In like manner, while the earth is paſſing from C to A, the ſun will appear to paſs from G to E. Plate 9. Fig. 8.

Schol. 1. It is manifeſt that the circle in which the ſun appears to move, is the ſame, in which the earth would appear to move to a ſpectator in the ſun. Hence the apparent place of the ſun being found, the true place of the earth in its orbit is known.

Schol. 2. The orbit in which the earth revolves round the ſun is not a circle but an ellipſe, having the ſun in one of its foci. For the computations of the ſun's place, upon this ſuppoſition, allowing for the diſturbing forces of the planets, are found to agree with obſervations. See Prop. XXXIII.

Def. XXI. The circle which the ſun appears to deſcribe annually in the concave ſphere of the heavens, is called the *Ecliptic.*

Def. XXII. A portion of the heavens, about 16 degrees in breadth, through the middle of which paſſes the ecliptic, is called the *Zodiac.*

Schol. Within this zone lie the orbits of all the planets.

Def. XXIII. The ſtars in the Zodiac are divided into 12 *Signs*: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius, Piſces. Figures, repreſenting theſe ſigns, are drawn upon the celeſtial globe, in that portion of its ſpherical ſurface, which correſponds to the portion of the concave ſphere of the heavens, in which the ſtars belonging to each ſign are reſpectively placed. Plate 9. Fig. 8.

PROP. VIII. The axis of the earth in every part of the earth's revolution about the ſun, makes, with the plane of its orbit, that is, of the ecliptic, an angle of $66\frac{1}{2}$ degrees.

Plate 9. Fig. 7. Let BA repreſent the plane of the ecliptic or earth's orbit, ſeen edgeways; S the ſun; and P*p* produced the axis of the equator. If the earth be at S, its axis is not perpendicular to the plane of the ecliptic, but makes an angle with it, PSA, about 66° 30′. In any other part of its orbit, as at M, or X, the axis of the earth is ſtill inclined to the plane of the ecliptic in the ſame angle.

Plate 9. Fig. 6. Cor. 1. The axis, in any one part of the orbit, is in a poſition parallel to that in which it was at any other part of the orbit. Suppoſing the line FG to repreſent the ſituation of the axis of the earth when at DFG, and to be parallel to the line HI; then when the earth is at *dfg*, or any other part of its orbit, its axis *fg* will ſtill be parallel to the ſame line HI, therefore, *fg* is parallel to FG.

Cor. 2. The planes of the equator and ecliptic, make with each other an angle of $23\frac{1}{2}$ degrees nearly.

Schol. The obliquity of the ecliptic is not permanent, but is continually diminiſhing, by the ecliptic approaching nearer to a paralleliſm with the equator, at the rate of about $\frac{1}{2}$ a ſecond in a year, or from 50″ to 55″ in 100 years. The inclination at this time is 23° 28′ 5″ nearly. The diminution of the obliquity of the ecliptic to the equator is owing to the action of the planets upon the earth, eſpecially the planets Venus and Jupiter. The whole variation, it is ſaid, can never exceed one degree, when it will again increaſe.

The obliquity of the ecliptic may be thus found. Obſerve with a good inſtrument, very accurately divided, the meridian altitude of the ſun's centre, on the days of the ſummer and winter ſolſtice, then the difference of the two, will be the diſtance between the tropics; the half of which will be the obliquity ſought.

By the ſame method, the declination of the ſun for every day in the year may be found, and a table conſtructed. See Prop. XVIII.

Def. XXIV. The ecliptic being divided into twelve equal parts, each of theſe parts is called a *Sign*; and the names of the ſigns in the ecliptic are the ſame with thoſe in the zodiac, but do not exactly correſpond with them.

Def. XXV. The two points in which the ecliptic cuts the equator, are called the *Equinoctial Points*: the vernal equinox is at the firſt degree of Aries in the ecliptic; the autumnal, at the firſt of Libra.

Schol. The moment of time in which the ſun enters the equator, may be found by obſervation, the latitude of the place of the obſerver being known. For in the equinoctial day, or near it, with an inſtrument exactly divided into degrees, minutes, and parts of minutes, take the meridian altitude of the ſun: if it be equal to the altitude of the equator, or to the complement of the latitude, the ſun is then in the equator; but if it is not equal, mark the difference, which will be the declination of the ſun. The next day, again obſerve the meridian altitude of the ſun, and gather from thence his declination. If theſe two declinations be of different kinds, as the one ſouth and the other north, the equinox happens ſome time between the two obſervations; if they be both of the ſame ſort, the ſun has either not entered the equinoctial, or has paſt it. And from theſe two obſervations of the ſun's declination, the moment of the equinox is thus inveſtigated.

Plate 9. Fig. 9.

Let CAB be a portion of the ecliptic, EAQ an arc of the equator, and let their interſection be in A. Let CE be the declination of the ſun at the time of the firſt obſervation, OD his declination in the ſecond obſervation; the arc CO will be the motion of the ſun in the ecliptic for one day. In the ſpherical triangle AEC, right-angled at E, we have the angle A which the equator and the ecliptic make, as alſo CE the declination of the ſun, known by obſervation, by which may be found the arc CA. And in the ſame manner in the triangle AOD the ſide AO is found; and thence the arc CO, which is the ſum or difference of the arcs CA, AO. Therefore as CO is to CA, ſo is 24 hours to the time between the firſt obſervation, and the moment of the ingreſs of the ſun to the equinox.

Def. XXVI. The points of the ecliptic which are at the greateſt diſtance from the equator, are called the *Solſtices*; and the circles which paſs through theſe points parallel to the equator, are called the *Tropics*: the ſummer ſolſtice is at the firſt of Cancer, the winter ſolſtice at the firſt of Capricorn: the northern tropic is called the tropic of Cancer, the ſouthern, of Capricorn.

Cor. The ſun is once in the year at each of the tropics, and twice at the equator.

Def. XXVII. Circles which paſs through the poles at right angles to the equator, or any other great circle, are called *Secondaries* to that circle:

the ſecondary which paſſes through the equinoctial points, is called the *Equinoctial Colure.*

Def. XXVIII. That pole which is neareſt the tropic of Cancer, is called the *North Pole*; that which is neareſt the tropic of Capricorn, is called the *South Pole.*

Def. XXIX. An imaginary line paſſing through the centre of the ecliptic, and perpendicular to the plane of it, is the *Axis of the Ecliptic*: its extremities are the *Poles of the Ecliptic*, and all circles, paſſing through theſe poles, and perpendicular to the ecliptic, are its ſecondaries.

Cor. The axis of the ecliptic makes an angle of $23\frac{1}{2}$ degrees nearly with that of the equator. Compare Prop. VIII. Cor. 2. and Schol.

Def. XXX. The *Polar Circles* are deſcribed by the revolution of the poles of the ecliptic about the poles of the equator; that which is next to the north pole, is called the *Arctic* circle; the oppoſite, the *Antarctic* circle.

Def. XXXI. The *Declination* of any heavenly body, is its diſtance from the equator; this is either northern or ſouthern. The degrees of declination of any body are reckoned upon a ſecondary of the equator paſſing through that body.

Def. XXXII. The *Right Aſcenſion* of any heavenly body, is its diſtance from the firſt of Aries reckoned upon the equator: this is meaſured, by obſerving the arc which is intercepted between Aries and a ſecondary to the equator paſſing through the ſun or ſtar.

Def. XXXIII. The *Latitude* of any heavenly body is its diſtance from the ecliptic; and the degrees of latitude are reckoned on a ſecondary of the ecliptic paſſing through the body.

Def. XXXIV. The *Longitude* of any heavenly body is its diſtance from the firſt of Aries; and is meaſured on the ecliptic by the arc intercepted between the firſt of Aries and the ſecondary of the ecliptic which paſſes through the body: the longitude increaſes, as the body recedes from Aries, through the whole revolution, till it reaches 360°, or comes again to Aries.

Def. XXXV. Two bodies are ſaid to be in *Conjunction* with each other, when they have the ſame longitude, or are in the ſame ſecondary of the ecliptic on the ſame ſide of the heavens, though their latitude be different: they are ſaid to be in *Oppoſition*, when their longitudes differ half a circle, or they are on oppoſite ſides of the heavens.

PROP. IX. The axis of the heavens is perpendicular to the planes of all the circles which the heavenly bodies deſcribe in their apparent diurnal motions.

For the heavenly bodies, from the revolution of the earth round its axis, appear to move from eaſt to weſt in circles perpendicular to the axis.

Cor. 1. The planes of all theſe circles are parallel to the equator.
Cor. 2. The axis paſſes through the centres of the circles.

Def. XXXVI. The celeſtial ſphere is called *right*, *oblique*, or *parallel*, as the celeſtial equator is at right angles, oblique, or parallel to the horizon.

PROP. X. In all places on the equator, the poles lie in the horizon, and all the circles of daily motion make right angles with the horizon.

For theſe places (by Def. XVIII.) having no latitude, the poles (by Prop. III.) are neither elevated above nor depreſſed below the horizon; and ſince the equator is 90 degrees from the poles, it is at right angles to the horizon, and alſo all circles parallel to it.

PROP. XI. Thoſe who live at the equator are in a right ſphere; and, conſequently, their days and nights are always equal.

The great circle of the celeſtial equator and its parallels (by laſt Prop.) make right angles with the horizons of all places in the earth's equator; therefore (by Def. XXXVI.) the inhabitants of thoſe places live in a right ſphere. Hence, becauſe the celeſtial axis PTS is in the plane of their horizon, and that this axis is at right angles to the plane of the equator, and (by Prop. X.) paſſes through its centre and through that of all circles parallel to the equator, the plane of the horizon alſo paſſes through the centres of theſe circles; and conſequently divides the equator and its parallels into two equal parts. One half of theſe circles will therefore always be above the horizon, and the other half below it. But each of the heavenly bodies in their daily motion deſcribes ſome one of thoſe circles, and the diurnal motion of the earth is uniform; therefore any heavenly body will, in this ſituation, be juſt as long above the horizon as below it. And becauſe this will be the caſe with reſpect to the ſun, as well as any other body, in whatever part of the heavens he is ſeen, the days and nights at the equator will always be of equal length. Plate 9. Fig. 10.

PROP. XII. At the poles of the earth, one celeſtial pole is in the zenith, and the other in the nadir; the equator coincides with the horizon, and all the circles of daily motion are parallel to the horizon.

For the latitude of the poles is 90 degrees from the equator, and the circles of daily motion are parallel to the equator.

PROP. XIII. Thoſe who live at either pole are in a parallel ſphere; they ſee the heavenly bodies carried round them in circles parallel to the horizon, and their day and their night, continues each half year.

Plate 9. Fig. 10.

An inhabitant at P has the equator EQ in the horizon, and all its parallel circles alſo parallel to the horizon. Therefore each of the heavenly bodies, in its apparent daily motion, being in ſome one of theſe circles, muſt deſcribe a path parallel to the horizon: ſo that thoſe which are above the horizon will never ſet by this motion, and thoſe which are below it will never riſe. The ſun, therefore, in this ſituation, will not riſe or ſet by the diurnal motion of the earth. But from the annual motion of the earth, the ſun daily changes its apparent place in the heavens, till it has deſcribed the circle of the ecliptic CL; one half of which is above the horizon, and the other half below it, becauſe theſe circles have a common centre T, the centre of the earth. Therefore, for one half of the year the ſun will be in ſome part of CT, that half of the ecliptic which is above the horizon, and will daily revolve in circles above the horizon; and for the other half, it will be in ſome part of TL, and will perform its daily revolutions in circles below the horizon.

PROP. XIV. In any place between the poles and the equator, one celeſtial pole will be elevated, and the other depreſſed, at an angle leſs than a right angle; and the celeſtial equator will make an angle leſs than a right angle with the horizon.

For, ſince the place is not in the equator, it has ſome latitude; and ſince it is not at either of the poles, its latitude is leſs than 90 degrees: whence (by Prop. III.) the poles are elevated, or depreſſed, in an angle leſs than a right angle; and conſequently the equator, which is perpendicular to the axis, makes an angle leſs than 90 degrees with the horizon.

PROP. XV. Thoſe who live on any part of the ſurface of the earth between the equator and either pole, are in an oblique ſphere, and have all the circles of daily motion oblique to their horizon.

Plate 9. Fig. 10.

Let HO be the horizon of a place which lies between the earth's equator and either of its poles; the celeſtial equator EQ, and all its parallel circles, will be oblique to the horizon, and therefore each of the heavenly bodies, being in ſome one of theſe circles, will appear to move in a path oblique to the horizon.

PROP. XVI. When the ſun, in his annual apparent courſe, is in the points in which the ecliptic cuts the equator, the day and night will be of the ſame length at all places on the ſurface of the earth: but, when the ſun is in any other part of the ecliptic, the days will be longer as the ſun's declination towards the elevated pole increaſes, and ſhorter as its declination towards the depreſſed pole increaſes.

The plane of the horizon HO, of any place, paſſing through T, the centre of the ſphere, and alſo through the centre of the equator, divides the equator CL into two equal parts, one half above, and the other half below the horizon. When therefore the ſun has no declination, or is in the equator, it will appear in its daily revolution to deſcribe the equator CL, and, therefore, during one half of the revolution, it will be above the horizon, and, during the other half, below it. Plate 9. Fig. 10.

But ſuppoſe the ſun to have its declination towards P, the elevated pole, equal to E*m*: its diurnal apparent revolution will be in the circle *mm*, the centre of which is in a part of the axis above the horizon; whence the plane of the horizon does not paſs through the centre, and conſequently the circle *mm* is divided into two unequal parts, the greater above the horizon, and the leſs below it. Therefore the ſun, deſcribing the circle *mm*, with an uniform velocity, in its apparent diurnal revolution, will be longer in deſcribing the part above the horizon, than the part below it. And this difference manifeſtly increaſes, as the circle of the ſun's apparent diurnal motion recedes from the equator, that is, as the ſun's declination towards P increaſes. In like manner, it may be ſhewn, that the days will be ſhorter, as the ſun's declination towards the depreſſed pole increaſes.

Or thus: Let AB repreſent the plane of the ecliptic ſeen edgeways; S the ſun in the focus of the orbit; MO, KL, XY, the earth in different parts of its orbit. If FI, the axis of the ecliptic BA, were alſo the axis of the earth, that is, if the planes of the equator and ecliptic were coincident, it is manifeſt that the ſun, the apparent annual motion of which is in the plane of the ecliptic, would at all times of the year appear to move in the circle of the equator, and to be equally diſtant from the poles, and conſequently could produce, by its apparent motion, no varieties in the length of days and nights. But the earth's axis being inclined to the plane of its orbit, as P*p*, when the earth is at MO, the pole P will be towards the ſun, and the pole *p* turned from it, and the reverſe when the earth is arrived at XY. When the earth is in the middle ſtation between B and A, in either part of its orbit, both the poles will be in the circle illuminated as at KL. Plate 9. Fig. 7.

In the poſition MO, ſince the ſun muſt always illuminate one half of the globe, the light will paſs beyond the pole P as far as F, and will extend towards the pole *p* no farther than I. Conſequently, in the diurnal revolution of the earth round its axis, while the earth remains in this poſition, all the parts of the globe between F and G will be illuminated, and all the parts between I and H will be dark. Farther, in this poſition greater portions of thoſe parallels which lie between the equator and the circle FG, will at any inſtant be in the illuminated, than in the dark, hemiſphere; and, on the contrary, greater portions of thoſe which lie between the circle HI and the equator, will at any inſtant be in the dark, than in the enlightened, hemiſphere. Conſequently, any given place on the ſide of the equator towards P, will, in one diurnal revolution, be longer in the light than in the dark, and the reverſe on the ſide towards *p*. The difference between the length of day-light and night, will decreaſe on either ſide of the equator, as we approach towards it ; and at the equator, the illuminated and dark portions of the circle being always equal, the days and nights will be of equal length. The contrary to all this will take place in the ſituation XY. Continual variations will take place, while the earth paſſes from MO to KL, and from KL to XY. But in the ſituation KL, the illumination extending exactly to both poles, all the parallel circles are half illuminated, and half dark : conſequently, any place upon the globe will, in a diurnal revolution, have equal portions of light and darkneſs ; that is, day and night will be every where of equal length. This muſt happen twice in every annual revolution.

Cor. 1. All bodies which are on the ſame ſide of the equator with the ſpectator, continue longer above the horizon than below it, and vice verſa.

Cor. 2. As the orbits of the moon and planets are inclined to the equator, a variation of the times of their continuance above and below the horizon will take place.

Schol. 1. When the ſun is very near either of the tropics, the days do not appear of different lengths, for the circles of apparent diurnal motion are ſo near to each other, that they cannot be ſenſibly diſtinguiſhed.

Schol. 2. The different degrees of heat at different ſeaſons of the year are owing partly to the different lengths of the days, and partly to the different degrees of obliquity with which the rays fall upon the atmoſphere at different altitudes of the ſun.

* PROP. XVII. When the ſun, or any other heavenly body, is in the equator, it riſes in the eaſt, and ſets in the weſt.

For it then riſes and ſets in the points in which the equator cuts the horizon ; that is, becauſe the equator is at right angles to the meridian, which paſſes through the north and ſouth points, in the points of eaſt and weſt.

Cor. In north latitude, thoſe bodies which have north declination, riſe between the eaſt and north ; thoſe which have ſouth declination, riſe between the eaſt and ſouth.

PROP. XVIII. When the declination of the ſun is towards the elevated pole, its meridian altitude is equal to its declination added to the elevation of the celeſtial equator : when its declination is towards the depreſſed pole, its meridian altitude is equal to its declination ſubtracted from the elevation of the equator.

Plate 9. Fig. 10. Let HO be the horizon, T the earth, P and S the celeſtial poles, D the zenith, N the nadir, EQ the equator. If the ſun be at C, having its declination towards the elevated pole P, when it arrives at the meridian PS, its meridian altitude CH is equal to the ſum of CE its declination, and EH the elevation of the equator. If the ſun be at I, having its declination towards the depreſſed pole S ; when it arrives at the meridian, its altitude IH is equal to the difference of EH the elevation of the equator, and EI the ſun's declination, as appears from the figure.

PROP. XIX. When the declination of a heavenly body towards the elevated pole, is equal to the latitude of any place, the body will paſs through the zenith of that place : and when its declination towards the depreſſed pole is equal to the latitude, it will paſs through the nadir.

Any ſtar or planet which paſſes through D, the zenith, in its apparent diurnal revolution muſt deſcribe the circle DO ; whence its diſtance from the equator or declination will be ED. But ED is the diſtance of the zenith from the equator, which, becauſe the elevation of the equator is equal to the complement of latitude, (Prop. IV.) is equal to the latitude. In like manner the reverſe may be proved. Plate 9. Fig. 10.

PROP. XX. A heavenly body ſeen from any place, will never ſet from the diurnal motion of the earth, if the complement of its declination towards the elevated pole be equal to, or leſs than, the latitude of the place : and it will never riſe, if the complement of its declination towards the depreſſed pole be equal to, or leſs than, the latitude.

Let PD, which is the complement of declination of a body at D, and alſo the diſtance of the body at D from the pole, be equal to PO, the elevation of the pole, or (by Prop. III.) the latitude ; it is manifeſt, that the body at its loweſt depreſſion will be no farther from the pole than the horizon is ; that is, will never be below it. In like manner the reverſe may be ſhewn. A parallel of latitude, as DO, at a diſtance from the elevated pole equal to the latitude of the place, is called *the circle of perpetual apparition* ; and a parallel, as HN, at the ſame diſtance from the other pole, *the circle of perpetual occultation.* Plate 9. Fig. 10.

SCHOL. The latitude of a place may be found, by obſerving the greateſt and leaſt altitude of a fixed ſtar that never ſets.

Let A be a ſtar near the north pole, which in its daily motion deſcribes the circle AB without ſetting. A quadrant being placed in the plane of the meridian, or along the meridian line, obſerve its altitude when it is at A, and afterwards when at B ; the difference of theſe altitudes is AB. And, ſince the ſtar, in its revolution about the pole, is always at equal diſtances from it, if AB be biſected in P, this point will be the pole, and conſequently PO will be the elevation of the pole. But, ſince the lengths of the arcs AO, BO, have been found by obſervations, their difference AB, and the half of this difference, AP, or BP, is known : and PO is equal to BP+BO, or to AO—PA. Whence the elevation of the pole, that is, the latitude, is equal to the ſum of the leaſt altitude added to half the difference of the greateſt and leaſt altitude, or it is equal to the remainder ariſing from ſubtracting half the difference of the greateſt and leaſt altitudes from the greateſt altitude. Plate 9. Fig. 10.

Or, the latitude may be found from the ſun's meridian altitude and declination. If the ſun's meridian altitude, found by a quadrant, be CH, this altitude is equal to the ſun's declination CE, added to the elevation of the equator EH. Therefore, if CE, the declination towards the elevated pole, be taken from the meridian altitude, the remainder EH will be the elevation of the equator. But ſince the elevation of the equator is the complement of latitude, the latitude is the complement of the elevation of the equator. This elevation, therefore, being found, the latitude of the place is known.

Or, the latitude of a place is equal to the ſun's meridian zenith diſtance, added to his declination, when he paſſes the meridian between the zenith and the equator.

Ex. 1. To find the latitude from an obſervation of the ſun's altitude, Aug. 7, 1776, at the Obſervatory at Cambridge.

Apparent meridian altitude of the ſun's lower limb -	53°	46′	8″
Sun's apparent ſemi-diameter, from the Ephemeris -	0	15	50
Apparent altitude of the ſun's centre - - -	54	1	58
Deduct for refraction - - - - - -	0	0	41
Altitude of the ſun's centre - - - - -	54	1	17
Zenith diſtance of the ſun's centre is found by ſubtracting the laſt altitude from 90° - - -	35	58	43
Add the ſun's declination - - - - -	16	13	57
Latitude of the place - -	52	12	40

Ex. 2. Dec. 1, 1793. The obſerved meridian altitude of Sirius was 59° 50′; required the latitude.

Obſerved altitude - - - -	59°	50′ S.
Therefore, Zenith diſtance - - - -	30	10 N.
Declination of Sirius - - -	16	27 S.
Conſequently, the latitude required - - -	13	13 N.

DEF. XXXVII. The two tropics and two polar circles upon the ſurface of the earth, divide it into five parts, called *Zones*: the torrid zone lies between the two tropics; the temperate zones between the tropics and polar circles; and the frigid zones between the polar circles and the poles.

PROP. XXI. At any place in the torrid zone the ſun is vertical twice every year.

The ſun in paſſing from the equator to the tropic of Cancer, 23¼ degrees from the equator, has every northern declination from 0 to 23¼: and every place between the equator and the tropic of Cancer has ſome northern latitude between 0 and 23¼: therefore, in ſome part of its courſe from the equator to the tropic of Cancer, the ſun muſt have a declination equal to the latitude of every place between the equator and the tropic: whence it muſt be once in the zenith of every ſuch place in its courſe towards the tropic of Cancer. For the ſame reaſon it muſt be once in the zenith of every ſuch place in its courſe from the tropic to the equator. The like may be ſhewn on the ſouthern ſide of the equator.

PROP. XXII. The ſun is vertical once every year at the places which lie in the tropics.

For the ſun's declination is then 23¼ degrees, equal to the latitude of the tropics.

PROP. XXIII. At the polar circles, the longeſt day, and the longeſt night, is 24 hours.

When the ſun is in the tropic of Cancer, the complement of its declination towards the elevated pole is 66½ degrees, equal to the latitude of the arctic polar circle: on this day, therefore, (by Prop. XX.) the ſun will not ſet. When the ſun is in the tropic of Capricorn, the complement of its declination towards the depreſſed pole will be 66½ degrees, equal to the latitude of the arctic pole; whence the ſun will not riſe during that day. The ſame may be ſhewn with reſpect to the antarctic circle.

PROP. XXIV. The longeſt day, and the longeſt night, are each of them more than 24 hours within the frigid zone.

For, while the ſun's complement of declination towards the elevated pole is leſs than the latitude of the place, the ſun will not ſet; while the complement of declination towards the depreſſed pole is leſs than the latitude of the place, it will not riſe: but this muſt be the caſe with reſpect to every place within the frigid zones, in ſome part of the ſun's courſe towards the tropics.

PROP. XXV. The ſun is never vertical to any place in either of the temperate zones.

For the latitude of all places in the temperate zone, is greater than any declination of the ſun.

PROP. XXVI. The longeſt day, and the longeſt night, in any part of the temperate zones, are leſs than 24 hours; and the days and nights will be longer, the nearer the place is to the polar circles.

For the complement of the ſun's declination can never be leſs than, or equal to, the latitude of any place in the temperate zones; whence the ſun will riſe and ſet every day within theſe zones. But the farther any place is removed from the equator, the nearer the latitude approaches to an equality with the complement of the ſun's greateſt declination, when the day is 24 hours; that is, at the polar circles.

PROP. XXVII. At different places, the hour of the day differs in proportion to the difference of longitude; 15 degrees of longitude making the difference of one hour in time; 15′, one minute of time, 15″, one ſecond of time: and it is ſeen at any given place ſooner than at places which lie to the weſt of it, and later than at places which lie to the eaſt of it.

The ſun in its daily apparent motion, which is from eaſt to weſt, muſt arrive at the meridian of any given place, as London, ſooner than it will arrive at the meridian of any place which lies to the weſt of London, and later than at the meridian of any place to the eaſt of London: that is, ſince it is noon at any place when the ſun is in its meridian, it will be noon at London ſooner than at places weſt, and later than at places eaſt, of it.

For example, if any place lies 15 degrees eaſt of London, that is, has 15 degrees of eaſtern longitude from London taken as the firſt meridian, the ſun will be one hour ſooner at its meridian than at the meridian of London; for, ſince the ſun every day appears to make a complete revolution from any meridian to the ſame, in 24 hours, it will in every hour deſcribe a 24th part of the circle, that is, 15°. And ſince a minute of a circle is a 60th part of a degree, and a ſecond of a circle a 60th part of a minute, and 15′ the 60th part of 15°, and 15″ the 60th part of 15′, the ſun will move at the rate of 15′ in every 60th part of an hour, and 15″ in every 60th part of a minute, that is, in every minute or ſecond of time. Conſequently, it will be noon one minute or one ſecond ſooner at a place which is 15′ or 15″ eaſt of London, than at London.

PROP. XXVIII. The difference of longitude at two places may be found by obſerving, at the ſame time from both places, ſome inſtantaneous appearance in the heavens.

If the eclipſe of Jupiter's innermoſt ſatellite, on the inſtant of its immerſion into the ſhadow of Jupiter, be obſerved by two perſons at different places, it will be ſeen by both at the ſame inſtant. But if this inſtant be half an hour, for example, ſooner at one place than at the other, becauſe the places differ half an hour in their reckoning of time, their difference of longitude (by Prop. XXVII.) is 7°. 30′.

Schol. From tables of eclipſes correctly calculated for any place, the longitude of any place may be found by one obſerver. But ſuch obſervations can only be made with certainty by land, on account of the motion of a ſhip at ſea. In order to determine accurately the longitude at ſea, it is neceſſary to have a clock which ſhall not be ſenſibly affected by difference of climate, difference of gravity at different places, or the motion of the ſhip. Such a clock ſet for the meridian of London would conſtantly ſhew the hour of the day at London, which it is eaſy to compare with the hour of the day where the ſhip is, found by obſervations on the ſun or ſtars.

PROP. XXIX. Thoſe who live in oppoſite ſemicircles of the ſame meridian, but in the ſame circle of latitude, have oppoſite hours of the day, but the ſame ſeaſons.

Being both on the ſame ſide of the equator and at the ſame diſtance from it, when the ſun's declination makes it ſummer or winter in one of the places, it will be the ſame at the other: but becauſe they are diſtant from each 180 degrees of longitude, when it is noon at one place it will be midnight at the other: theſe are called *Periœci.*

PROP. XXX. Thoſe who live in oppoſite circles of latitude, but in the ſame ſemicircle of the meridian, have oppoſite ſeaſons of the year, but the ſame hour of the day.

When the ſun has declination towards the north pole, it will be ſummer to thoſe who live in the northern circle of latitude, and winter to thoſe who live in the ſouthern circle of latitude. But, having the ſame longitude, their hours of the day will be the ſame: theſe are called *Antœci*.

PROP. XXXI. Thoſe who live in oppoſite circles of latitude and oppoſite ſemicircles of the meridian, have both oppoſite ſeaſons of the year, and oppoſite hours of the day.

Becauſe they are in oppoſite latitudes, they will have oppoſite ſeaſons; and becauſe they are in oppoſite ſemicircles of the meridian, they will have noon when it is midnight at the other: theſe are called *Antipodes*.

Def. XXXVIII. Twelve ſecondaries to the celeſtial equator being conceived to be drawn at equal diſtances from each other, that is, dividing the equator into 24 equal parts, and the meridian of any place being made one of theſe ſecondaries, they are called *Hour-Circles* of that place. Compare Prop. XXVII.

PROP. XXXII. If the celeſtial ſphere had an opake axis, the ſhadow of the axis would always be oppoſite to the ſun; and when the ſun was on one ſide of any hour-circle, the ſhadow of the axis would fall upon the oppoſite ſide of the ſame hour-circle.

For all the hour-circles being ſecondaries to the equator, paſs through the poles, and the celeſtial axis is in the plane of every hour-circle. And the ſhadow of any opake body, being oppoſite to the ſun, is in the plane with the ſun. Therefore in whatever hour-circle the ſun is, the ſhadow of the ſuppoſed opake axis would be in the plane of that circle and oppoſite to the ſun, that is, while the ſun is in one ſemicircle of any hour-circle, the ſhadow of the axis would fall upon the oppoſite ſemicircle.

Cor. Hence as the ſun performs its apparent courſe from eaſt to weſt, the ſhadow of the ſuppoſed axis would move from weſt to eaſt.

Schol. The gnomon of a ſun-dial repreſents the ſuppoſed axis, and hence its ſhadow is a meaſure of time.

To conſtruct an HORIZONTAL DIAL.

In every ſun-dial the gnomon, when fixed, is parallel to the earth's axis. Now when the ſun is in the meridian of any place, the 12 o'clock hour-circle is perpendicular to the plane of the horizon, and the arc from the pole to this plane is equal to the latitude of the place; and the one o'clock hour-circle makes an angle at the pole with it of 15°, and forms the hypothenuſe of a right angled triangle to the above perpendicular, and the baſe is the arc meaſuring the angle between 12 and 1 o'clock; therefore we have, by Spherical Trigonometry, Rad : Sin. L : : tan. 15° : tan. of the hour-angle between 12 and 1 o'clock. If inſtead of 15, we ſubſtitute 30, 45, &c. we get the angles between 12 and 2, 3, &c. o'clock; the ſame may be done for the half hours or other diviſions.

Note. The rational and ſenſible horizons are, in this caſe, ſuppoſed coincident, which, on account of the ſun's great diſtance, will not occaſion any ſenſible error.

To conſtruct a VERTICAL SOUTH-DIAL.

In doing this we muſt conceive a plane paſſing through the centre of the earth perpendicular both to the horizon and meridian; and on the ſouth ſide, lines muſt be drawn from the centre to the points where the hour-circles cut that plane. In finding theſe points, we ſay, as Rad : co. s. Lat : : tan. 15° : tan. of the hour-angle between 12 and 1 o'clock. For the arc of the meridian, from the pole to the plane, is equal to the complement of latitude. The other hour-angles, &c. muſt be obtained in the ſame way as in the laſt.

On the ſubject of Dialing, ſee Ferguſon's Lectures.

PROP. XXXIII. The orbit in which the earth revolves about the ſun, is elliptical.

It is known from obſervation, that the apparent motion of the ſun, that is, the real motion of the earth, in the ecliptic, is not uniform. But by the univerſal law of bodies revolving about a centre, if its orbit were circular, its velocity muſt be uniform; ſince (Book II. Prop. LXXII.) it muſt deſcribe equal areas in equal times. Whereas if its orbit be an ellipſe, and the ſun be placed in one of the *foci*, the ſame law will require (ſee Book II. Prop. LXVIII.) that its velocity ſhould not be uniform, but that in paſſing through its greateſt diſtance C, to its leaſt diſtance A, it ſhould be accelerated, and in paſſing from the leaſt diſtance A to the greateſt C, it ſhould be retarded. Since then the motion of the earth is in fact thus retarded and accelerated in different parts of its orbit, it is manifeſt, that its orbit is elliptical. Plate 9. Fig 8.

DEF. XXXIX. The greateſt diſtance of the earth or any other planet from the ſun, is called its *Aphelion*; its neareſt diſtance, its *Perihelion*; the longer axis of the ellipſe is called the *Linea Apſidum*, the aphelion is alſo called the *Summa Apſis*, and perihelion the *Ima Apſis*.

DEF. XL. The *Excentricity* of the earth, or any planet, is the diſtance between the ſun and the centre of the elliptical orbit.

PROP. XXXIV. The ſun is eight days longer in performing its apparent courſe through the ſix northern ſigns, than through the ſix ſouthern ſigns.

Let

Plate 9. Fig. 8.

Let ABCD be the orbit of the earth, S the ſun, and EFGH the ecliptic. While the earth moves in its orbit from B through C to D, the ſun appears to move in the ecliptic from F through G to H, paſſing through the ſix northern ſigns; and while the earth paſſes from D through A to B, the ſun appears to move from H to F, through the ſix ſouthern ſigns. Now the line HF biſects the circle EFGH, but divides the ellipſe ABCD unequally. And, while the ſun appears to paſs through the northern ſigns, the earth paſſes through more than half of its orbit; and while the ſun appears to paſs through the ſouthern ſigns, the earth paſſes through leſs than half of its orbit. Therefore, if the velocity of the motion of the earth were uniform, the ſun muſt appear to be longer in paſſing through the ſix northern than the ſix ſouthern ſigns. But whilſt the earth is paſſing through the greater part of its orbit BCD, it is farther from the ſun, and conſequently moves ſlower than in the leſſer part DAB. On both theſe accounts, the ſun's apparent motion is ſlower in the northern ſigns than the ſouthern: the difference is found by obſervation to be about eight days.

PROP. XXXV. The apparent diameter of the ſun is greater in winter than ſummer.

It is found by obſervation, that the diameter of the ſun in winter is 32 minutes, 47 ſeconds; in ſummer, 31′. 40″. And his mean apparent diameter is 32′. 12″. according to Sir I. Newton, in his theory of the Moon.

Cor. Hence it appears, that the earth, at the winter ſolſtice, or Capricorn, is in its perihelion.

Schol. 1. The difference between ſummer and winter in the degrees of heat, is owing chiefly to the different heights to which the ſun riſes above the horizon, and the different lengths of the days. When the ſun riſes higheſt, in ſummer, its rays fall leſs obliquely, and conſequently, more of them fall on the earth's ſurface than in winter; and when the days are long, and the nights ſhort, the earth and air are more heated in the day than they are cooled in the night, and the reverſe.

Schol. 2. The doctrine of the Sphere having been explained in the preceding propoſitions, ſome of the more uſeful Problems to be performed on the Terreſtrial and Celeſtial Globes are here ſubjoined.

Problem I. *To find the latitude of any place.* Bring the place to the graduated ſide of the fixed braſs meridian; the degree under which it is found, is its latitude. All places under the ſame degree are in the ſame latitude. Thus the latitude of London is 51° ½ north, that of the Cape of Good Hope 34° ſouth.

Prob. II. *To find the longitude of any place.* Bring the place to the fixed meridian; the diſtance of this meridian from the firſt meridian, meaſured on the equator, is the longitude of the place. The longitude of Boſton in New England is 70° ½ weſt, or 4 hours, 42 minutes in time. That of Rome 12° ½ eaſt, or 50 minutes in time.

Prob. III. *To rectify either globe to the latitude of any place, the zenith, and the ſun's place.* If the place be in the northern hemiſphere, raiſe the north pole above the horizon; but if the place be in the ſouthern hemiſphere, raiſe the ſouth pole. Then move the meridian up and down in the notches, till the degree of the place's latitude, counted upon the meridian, below the pole, cuts the horizon; and then the globe is adjuſted to the latitude of the place.

Having elevated the globe according to the latitude of the place, count the ſame number of degrees upon the meridian, from the equator towards the elevated pole, and that point will be the zenith or vertex of the place. To this point of the meridian ſcrew the quadrant of altitude, ſo that its graduated edge may be joined to the ſaid point; then is the globe rectified for the zenith.

Bring the ſun's place in the ecliptic to the meridian, and ſet the hour-index to 12 at noon; and then the globe will be rectified for the ſun's place.

Prob. IV. *To determine the difference of time in different places.* Find the longitude of each place, and reduce the difference into time, allowing an hour for every 15 degrees, and proportionally for leſſer parts; the difference of time will be found: if the place lies weſtward of another, it has its noon later than that other; if eaſtward, ſooner. The longitude of Rome is 12° ½ eaſt, that of Conſtantinople 29°, the difference is 17° ½, conſequently the difference of time between Rome and Conſtantinople is 1 hour 10 minutes.

Prob. V. *The latitude and longitude of any place being known, to find the place on the globe.* Bring the degree of the equator which expreſſes the given longitude to the fixed meridian, then find the given latitude on the meridian; under this point is the place ſought.

Prob. VI. *To find the diſtance between any two places, and their bearing, or relative ſituation with reſpect to the points of the compaſs.* Rectify the globe to the latitude of one of the places, and bring that place to the fixed meridian: Then fix the quadrant of altitude to the uppermoſt point of the meridian, and putting its lower end between the horizon and the globe, ſlide it along, till it paſſes through the other place: The number of degrees on the quadrant between the two places, will give their diſtance, allowing 69½ Engliſh miles for each degree; and the number of degrees upon the horizon between the meridian and the quadrant, will give the bearing of the ſecond place with reſpect to the firſt. Thus the bearing of the Lizard point from the iſland of Bermudas is nearly E.N.E.

Prob. VII. *To find the right aſcenſion and declination of the ſun, or any ſtar.* On the celeſtial globe find the day of the month under the ecliptic, againſt which is the ſun's place, or find his place by an *ephemeris;* bring that point under the meridian, and the degree which is over the point is the ſun's declination, and the degree of the equator then under the meridian will be the ſun's right aſcenſion. A ſtar's declination and right aſcenſion are found, by bringing the ſtar on the globe to the meridian, and proceeding as with reſpect to the ſun. The ſun's declination, April 19, is 11°. 14′. north, and his right aſcenſion 27°. 30′.—The right aſcenſion of Sirius is 99°. its declination 16°. 25′. ſouth.

Prob. VIII. *To find what ſtars paſs over, or near, the zenith of any place.* Having found the latitude of the place on the terreſtrial globe, all thoſe ſtars on the celeſtial globe, which paſs under the ſame degree of the meridian with the given latitude, become vertical at that place.

Prob. IX. *To find what ſtars never riſe, nor never ſet, in any given place.* The globe being rectified for the given place, thoſe ſtars which do not paſs under the wooden horizon, never ſet; thoſe which do not come above it, never riſe.

PROB. X. *To repreſent the appearance of the heavens at any time.* Rectify the globe to the latitude; bring the ſun's place in the ecliptic to the meridian, and ſet the horary index to the upper 12th hour; then turn the globe till the index points to the given hour. The north pole of the globe muſt be ſet to the north in the heavens, then will all the ſtars upon the globe correſpond to their places in the heavens, ſo that an eye at the centre of the globe would refer every ſtar upon its ſurface to the place of the ſtars in the heavens. By comparing, therefore, the ſtars in the heavens with their places on the globe, a perſon will eaſily get acquainted with all the ſtars.

Example. The ſituation of the ſtars at London on the 9th of February, at nine o'clock in the evening, is as follows: Sirius, or the dog-ſtar, is on the meridian, its altitude 22°.: Procyon, the little dog-ſtar, 16°. towards the eaſt, its altitude 43.: about 24°. above this laſt, and a little more towards the eaſt, are Caſtor and Pollux: S. 65° E. and 35°. in height, is Regulus, or Cor Leonis: exactly in the eaſt. and 22°. high, is Deneb in the Lion's-tail: 30°. from the eaſt towards the north, Arcturus is about 3°. above the horizon: directly over Arcturus and 31°. above the horizon, is Cor Caroli: in the north-eaſt are the ſtars in the extremity of the Great Bear's-tail.

Reckoning weſtward, we ſee the conſtellation Orion; the middle ſtar of the three in his belt is S. 20°. W. its altitude 35°. nine degrees below the belt, and a little more to the weſt is Rigel, the bright ſtar in his heel: above his belt, in a ſtraight line drawn from Rigel, between the middle and moſt northward in his belt, and 9°. above it, is the bright ſtar in his ſhoulder: S. 49°. W. and 45°.½ above the horizon is Aldebaran, the ſouthern eye of the Bull; a little to the weſt of Aldebaran, are the Hyades: the ſame altitude, and about S. 70°. W. are the Pleiades: in the W. by S. is Capella in Auriga, its altitude 73°. in the north-weſt, and 42°. high, is the conſtellation Caſſiopeia: and almoſt in the north, near the horizon is the conſtellation Cygnus.

PROB. XI. *The latitude of a place being given, to find the time of the ſun's riſing and ſetting, on any given day, at that place.* Having rectified the globe according to the latitude, bring the ſun's place in the ecliptic to the graduated edge of the meridian, and ſet the horary index to the upper 12. Then turn the globe to bring the ſun's place to the eaſtern part of the horizon, the index will point to the hour at which the ſun riſes; on the weſtern ſide, to the time of its ſetting. On the 5th of June, the ſun riſes at 3 h. 40 min. and ſets at 8 h. 20 min. The length of the day is 16 h. 40 min. that of the night 7 h. 20 min.

PROB. XII. *To find all the places on the globe to which the ſun will be vertical on a given day.* Bring the ſun's place to the fixed meridian, and obſerve the point of the ſun's declination; all the places which, in turning the globe round, paſs under that point, have the ſun vertical on the given day.

When the ſun's declination is equal to the latitude of any place, *then* the ſun will be vertical to the inhabitants of that place.

PROB. XIII. *To find the ſun's amplitude.* The globe being rectified for the latitude of the place, bring the ſun's place to the eaſtern ſide of the horizon; the arc of the horizon intercepted between that point and the eaſtern point, is the ſun's amplitude at riſing. Thus on the 24th of May, the ſun's amplitude at riſing is 36°. eaſt, and he ſets with 36°. weſtern amplitude.—The amplitude increaſes with the latitude of the place.

PROB. XIV. *To find the ſun's meridian altitude.* The globe being rectified for the latitude, zenith, and ſun's place, the number of degrees contained between the ſun's place and the zenith, is the diſtance of the ſun from the vertex at noon; the complement of which to 90 degrees, is the ſun's altitude. The meridian altitude of the ſun on the 17th of May, at London, is 57°. 55′.—The altitude being given, bring the quadrant of altitude to meet the ſun's place, and the interſection of the quadrant and horizon will ſhew the *azimuth.* Thus, on the 21ſt of Auguſt, at London, when the ſun's altitude is 36°. in the forenoon, the azimuth is 60°. from the ſouth.

PROB. XV. *To find the place of any heavenly body upon the globe, its longitude and latitude being given.* Place the firſt degree of the quadrant of altitude, upon that degree of the ecliptic which expreſſes the given longitude, and the 90th degree on the pole of the ecliptic; the point of the globe which is under that degree of the quadrant which expreſſes the given latitude, is the place of the body: For the quadrant repreſents a ſecondary of the ecliptic, an arc of which between the body and the ecliptic is its latitude, and the arc of the ecliptic between the ſecondary and the firſt degree of Aries its longitude.

PROB. XVI. *To find the place of any heavenly body upon the globe, its right aſcenſion and declination being given.* Bring that point of the equator which expreſſes the given right aſcenſion to the meridian; the place ſought is under that degree in the meridian, north or ſouth, which expreſſes the given declination.

PROB. XVII. *To find all thoſe places where it is noon, at any given hour of the day, at any given place.* Bring the given place to the braſs meridian, and ſet the index to the uppermoſt 12; then turn the globe till the index points to the given hour, and it will be noon to all the places under the meridian. When it is 4h. 50min. in the afternoon at Paris, it is noon at New Britain, St. Domingo, Terra Firma, Peru, Chili, and Terra del Fuego. As the diurnal motion of the earth is from *weſt to eaſt,* it is plain that all places which are to the eaſt of any meridian, muſt neceſſarily paſs by the ſun before a meridian which is to the weſt can arrive at it.

PROB. XVIII. *The hour being given at any place, to tell what hour it is in any other part of the world.* Bring the place where the time is given, under the meridian; ſet the hour index to the given time, and turn the globe till the other place comes under the meridian, and the horary index will point to the hour required. Thus, when it is nine in the morning at London, it is half paſt four in the afternoon at Canton, in China; when it is three in the afternoon at London, it is 18 minutes paſs ten in the forenoon at Boſton, in America.

PROB. XIX. *To find the antœci, the periœci, and the antipodes, of any place.* Bring the given place to the meridian, then, in the oppoſite hemiſphere, and in the ſame degree of latitude with the given place will be the *antœci.* The given place remaining under the meridian, ſet the index to 12, and turn the globe till the other 12 is under the index; then the *periœci* will be under the ſame degree of latitude with the given place, and the *antipodes* of the given place will now be under the ſame point of the braſen meridian where the *antœci* ſtood before.

PROB. XX. *To find the two days on which the ſun is in the zenith of any given place between the tropics.* That parallel of declination which paſſes through the given place, will cut the ecliptic line upon the globe in two points, which denote the ſun's place; againſt which, on the horizon, will be found the days required. The ſun is vertical at Barbadoes, April 24, and Auguſt 18.

PROB. XXI. *The time and place being given, to find all thoſe places where the ſun is riſing, ſetting, culminating; and alſo where it is day-light, twilight, or dark-night.* Find the place where the ſun is vertical at the given hour, rectify for the latitude of that place, and bring it to the meridian. Then all the places that are in the *weſt* ſemicircle of the horizon, have the ſun *riſing*; thoſe in the *eaſt* ſemicircle, have it *ſetting*; thoſe under the meridian above the horizon, have it *culminating*; and all places above the horizon, have the ſun ſo many degrees above the horizon as the places themſelves are. Thoſe places that are below the horizon, but within 18°. of it, have *twilight*; thoſe lower than 18°. have dark-night; and to thoſe under the meridian, it is midnight.

PROB. XXII. *To find the place of the moon, or any planet, for any given time.* Take the nautical almanac, or White's Ephemeris, and againſt the given day of the month, will be found the degree and minute of the ſign which the moon or planet poſſeſſes at *noon*. The degree thus found, and marked in the ecliptic on the globe, you may proceed to find the declination, right aſcenſion, latitude, longitude, altitude, amplitude, azimuth, riſing, ſetting, &c.

PROB. XXIII. *To find the planets which are above the horizon at ſun-ſet, upon any given day and latitude.* Find the ſun's place for the given day, bring it to the meridian, ſet the index hour to 12, and elevate the pole for the given latitude, then bring the ſun's place to the weſtern ſemicircle of the horizon, and obſerve what ſigns are in that part of the ecliptic above the horizon, then look to the ephemeris for the day, and it will be ſeen what planets are in thoſe ſigns, for ſuch will be viſible on the evening of that day.

PROB. XXIV. *To find whether Venus be a morning or an evening ſtar.* Rectify the globe for the latitude and ſun's place; find the ſituation of Venus by an ephemeris, and ſtick there a ſmall black patch; bring the ſun's place to the edge of the eaſtern horizon. If Venus be *in antecedentia*, that is, for inſtance, in Taurus when the ſun is in Gemini, ſhe will be a morning ſtar.

But, if Venus be *in conſequentia*, that is, for example, in Gemini when the ſun is in Taurus, ſhe will ſet after the ſun, and be an evening ſtar.

PROB. XXV. *To find all the places to which a lunar eclipſe will be viſible at any inſtant.* Find the place to which the ſun is vertical at the given time, and bring that place to the zenith, and the eclipſe will be viſible to all the hemiſphere below the horizon, becauſe the moon is oppoſite to the ſun.

Schol. 1. Since lunar eclipſes continue, in general, for a conſiderable time together, they may be ſeen in more places than in one hemiſphere of the earth; for, by the earth's rotation about its axis, during the time of the eclipſe, the moon will riſe to ſeveral places after its commencement.

Schol. 2. We cannot by a globe only, determine the places to which a ſolar eclipſe is viſible, becauſe that eclipſe does not happen to the whole hemiſphere next the ſun, nor does it happen at the ſame time to thoſe places where it is viſible. Calculations are therefore neceſſary.

SECT. III.—*Of* Twilight.

PROP. XXXVI. The atmoſphere above the horizon is enlightened by the rays of the ſun, when the ſun itſelf is below the horizon.

Plate 9. Fig. 11. Let ADL be the ſurface of the earth; CBM, the ſurface of the atmoſphere; A, any place upon the earth; PABN, the ſenſible horizon. When the ſun is at G, any point below the horizon, it cannot be directly ſeen by a ſpectator at A. But, becauſe rays from the ſun at G can paſs to the part of the atmoſphere above the ſenſible horizon of the place A, this part of the atmoſphere will be illuminated before the ſun riſes, or after it ſets, and will become viſible by reflection to the ſpectator at A; that is, twilight will be produced.

Cor. The atmoſphere is the cauſe of the apparent elevation of the heavenly bodies above their true places.

PROP. XXXVII. When the evening twilight ends, or the morning twilight begins, a ray of the ſun reflected from the higheſt part of the atmoſphere deſcribes, after reflection, a line which is the plane of the ſenſible horizon.

Plate 9. Fig. 11. As the ſun is depreſſed, the extreme ray of light from the ſun gradually recedes from C towards B, till at laſt it touches the horizon at B, from whence it is reflected in the direction BA, the plane of the horizon.

PROP. XXXVIII. When the evening twilight ends, or the morning twilight begins, a line drawn from the ſun to the top of the atmoſphere is a tangent to the ſurface of the earth.

Plate 9. Fig. 11. From E, the common centre of the earth and atmoſphere, draw a ſtraight line to B, the point in the top of the atmoſphere, from which (by laſt Prop.) the ſun's ray is reflected to A when the evening twilight ends, or the morning twilight begins. Becauſe this line EB is perpendicular to the reflecting ſurface of the atmoſphere, ABE is the angle of reflection. And AB, the reflected ray, being in the plane of the ſenſible horizon, deſcribes a tangent

to the ſurface of the earth. At the ſame time, the ſun muſt be at ſome point S, ſuch that the incident ray SB ſhall make the angle of incidence SBE, equal to ABE the angle of reflection. Draw AE; and from E, the centre to the incident ray SB, draw ED, making the angle DEB equal to the angle AEB. Hence (El. I. 26.) the ſide ED is equal to the ſide AE, that is, ED, is a ſemidiameter of the earth; alſo, the angle EDB is equal to the angle EAB, or is a right angle: whence (El. III. 16. Cor.) SB is a tangent to the circle; that is, when the reflected ray deſcribes a line in the plane of the ſenſible horizon, the line deſcribed by the incident ray falling upon the top of the atmoſphere, is a tangent to the ſurface of the earth.

PROP. XXXIX. If the time of the beginning of the morning, or end of the evening twilight is known, the height of the atmoſphere may be determined.

It is found, that at the beginning of the morning, or the end of the evening twilight, the ſun is 18 degrees below the horizon. If, therefore, S is the place of the ſun at that time, and ABN the horizon, the angle SBN is 18°. But at that time (by Prop. XXXVIII.) if B is the top of the atmoſphere, the incident ray SB, and the reflected ray BA, touch the earth at D and A. And, becauſe (El. III. 18.) in the quadrilateral figure ABDE, the angles ABD, AED, are together equal to the two right angles, or ABD is the ſupplement to two right angles of AED, and that ABD is alſo (El. I. 13.) the ſupplement to two right angles of SBN, AED, the angle contained by the ſemi-diameters AE, DE, is equal to SBN, the angle of the ſun's depreſſion: whence AED, at the time ſuppoſed, is an angle of 18°. But on account of the refraction of the ſun's rays in coming into the atmoſphere, his apparent place is, at the ſuppoſed time, 30′ higher than its true place. Allowance muſt therefore be made for this refraction in meaſuring the angle SBN, made by SB the line of direction in which the ſun's ray appears to come, and BN a line in the plane of the ſenſible horizon. Conſequently, the angle SBN, and its equal AED, is only 17°. 30′. and its half AEB is 8°. 45′.

Now, in the triangle AEB, HB, the height of the atmoſphere, is the difference between HE the radius, and BE the ſecant. Therefore as HE is to HB, the difference between the radius and ſecant of an angle of 8°. 45′. ſo is the number of miles in HE, a ſemi-diameter of the earth, to the number of miles in HB, the height of the atmoſphere. That is, as (Radius) 10.0000 : (Sec.) 8°. 45′. : : 3970 (Sem. Dia. of the earth) : EB; or by log. 10.00000 : 10.00508 : : 3.59879 : 3.60388 = 4016 miles; conſequently BH = 4016 — 3970 = 46 miles.

SCHOL. 1. The atmoſphere refracts the ſun's rays in ſuch a manner, as to bring him into ſight, every clear day, before he riſes in the horizon, and to keep him in view for ſome minutes, after he is really ſet below it. The effect of this refraction is about ſix minutes every day at a mean rate.

From the ſame cauſe, the heavenly bodies appear higher than they really are, ſo that to bring the apparent altitudes to the true ones, the quantity of refraction muſt be ſubtracted. The higher they riſe the leſs are the rays refracted, and when the heavenly bodies are in the zenith, they ſuffer no refraction, according to the principles of Optics already demonſtrated. Tables of refractions have been calculated by various aſtronomers, as Sir I. Newton, Mr. Thomas Simpſon, Dr. Bradley, Mr. Mayer, &c The following ſpecimen is taken from Dr. Bradley's table, which is eſteemed the moſt correct, and chiefly uſed by aſtronomers. For the method of calculating theſe tables, ſee Mr. Simpſon's Diſſert. p. 46, 4to. Gregory's Aſtron. Vol. I. Pr. 66. and Vince's Aſtron. Vol. I. 4to. ch. 7.

Mean Aſtronomical Refractions in Altitude.

App. Alt.	Refraction.		App. Alt.	Refraction.		App. Alt.	Refraction.		App Alt.	Refraction.		App Alt.	Refraction.	App. Alt.	Refraction.
°	′	″	°	′	″	°	′	″	°	′	″	°	″	°	″
0	33	0	11	4	47	23	2	14	35	1	21	48	51	78	12
1	24	29	12	4	23	24	2	7	36	1	18	50	48	80	10
2	18	35	13	4	3	25	2	2	37	1	16	52	44	82	8
3	14	36	14	3	45	26	1	56	38	1	13	55	40	85	5
4	11	51	15	3	30	27	1	51	39	1	10	58	35	88	2
5	9	54	16	3	17	28	1	47	40	1	8	60	33	89	1
6	8	28	17	3	4	29	1	42	41	1	5	62	30	90	0
7	7	20	18	2	54	30	1	38	42	1	3	65	26		
8	6	29	19	2	45	31	1	35	43	1	1	68	23		
9	5	48	20	2	35	32	1	31	44	0	59	70	21		
10	5	15	21	2	27	33	1	28	45		57	72	18		
			22	2	20	34	1	24				75	15		

SCHOL. 2. Dr Keill, in his Lectures on Aſtronomy, obſerves, that it is entirely owing to the atmoſphere that the heavens appear bright in the day time. For without it, only that part of the heavens would be luminous in which the ſun is placed: and if we could live without air, and ſhould turn our backs to the ſun, the whole heavens would appear as dark as in the night. In this caſe alſo we ſhould have no twilight, but a ſudden tranſition from the brighteſt ſun-ſhine to dark night immediately upon the ſetting of the ſun, which would be extremely inconvenient, if not fatal to the eyes of mortals. See Keill's Aſtron. Lect. xx.

PROP. XL. The twilight is longeſt in a parallel ſphere, and ſhorteſt in a right ſphere: and in an oblique ſphere, the nearer the ſphere approaches to parallel, the longer is the twilight.

In a parallel ſphere, the twilight will continue till the ſun's declination toward the depreſſed pole is 18°. but in this ſphere his declination is never more than 23½ degrees; whence the twilight will only ceaſe, whilſt the ſun's declination is increaſing from 18°. to 23½ degrees, and decreaſing again till in its decreaſe it becomes 18°. The twilight is here cauſed by the annual motion of the earth. In a right ſphere, the ſun appears to be carried, by the daily motion of the earth, in circles perpendicular to the horizon; whence it is carried directly downwards by the whole daily motion, and will arrive at 18°. below the horizon, the ſooneſt poſſible: whereas, in an oblique ſphere, its path is oblique to the plane of the horizon, and therefore will be longer before it is deſcended 18 degrees below the horizon: and the difference of the time of twilight will increaſe with the degree of obliquity. As the ſun ſets more obliquely at ſome parts of the year than others, the twilight varies in its duration.

SECT. IV.—*Of the* EQUATION *of* TIME.

PROP. XLI. The time in which the ſun completes one apparent diurnal revolution, is greater than that in which the earth revolves round its axis.

Plate 9. Fig. 2. If the earth turns round its centre T in the direction RCB, and at the beginning of one revolution the ſun was ſeen at Z, in the meridian ; from its apparent annual motion it will, after the diurnal revolution of the earth is completed, be ſeen advanced in its orbit towards E. The earth, therefore, muſt perform more than one revolution, and the ſpectator at R, after returning to the ſtation from which he ſet out, muſt advance forwards to *e*, before the ſun will be again in the meridian.

PROP. XLII. The obliquity of the ecliptic to the equator would cauſe the daily increments of the ſun's right aſcenſion to be unequal, although the ſun's motion in the ecliptic were uniform.

Plate Fig. 13. Let E be the firſt degree of Aries, EQ an arc of 90°. of the equator, EC the ſame of the ecliptic, and CQ an arc of the ſolſtitial colure, between Cancer and the equator. At E the ſun has neither longitude nor right aſcenſion : theſe may therefore be conſidered as equal, when the ſun ſets out from Aries. At C, the longitude is equal to the right aſcenſion ; for both EC and EQ are by ſuppoſition 90 degrees of great circles of the ſame ſphere. But if the ſun be any where between the firſt of Aries and the firſt of Cancer, as at S, the longitude will be *greater* than the right aſcenſion ER. For SR being an arc to the ſecondary of the equator paſſing through the ſun, ES is the longitude, and ER the right aſcenſion : but ES is greater than ER, becauſe the angle at R is a right angle, but the angle at S an acute angle. Now, if the ſun be ſuppoſed to move uniformly in the ecliptic, or to deſcribe equal arcs in equal times, the daily increments of longitude will be equal to one another ; and conſequently, ſince at the two extremes E and C the longitude and right aſcenſion are equal, and the longitude is ſuppoſed to increaſe uniformly, if the right aſcenſion alſo increaſed uniformly, they would at all times be equal. But at S, R, or any other points in the ſame ſecondary, between the firſt of Aries and Cancer, the longitude is *greater* than the right aſcenſion ; the daily increments of right aſcenſion are therefore unequal.

Plate 9. Fig. 14. The longitude and right aſcenſion are equal when the ſun is at C and at E, the former being 90°. the latter 180°. from Aries both on the ecliptic and equator. But between C and E, the longitude is *leſs* than the right aſcenſion ; becauſe ES, oppoſite to the right angle R, is greater than ER oppoſite to the acute angle S, and conſequently the point S is nearer Aries than the point R. But, the ſun being ſuppoſed to move uniformly, or to increaſe its longitude equally every day, if the right aſcenſion alſo increaſed equally every day, ſince the longitude and right aſcenſion are equal at C and E, they would be always equal. But at S, or any where between Cancer and Libra, the longitude is leſs than the right aſcenſion : conſequently, the daily increments of right aſcenſion are not equal. In like manner it may be ſhewn, that in the third quarter the ſun's longitude is greater than its right aſcenſion, and in the fourth, leſs.

PROP. XLIII. If the plane of the ecliptic coincided with that of the equator, the daily increments of the ſun's right aſcenſion would nevertheleſs be unequal.

Becauſe (by Prop. XXXIV.) the apparent annual motion of the ſun is not uniform, it would in ſome days deſcribe a longer arc than in others : that is, ſince its right aſcenſion and longitude would in this caſe be the ſame, the daily increments of its right aſcenſion would be unequal.

Def. XLI. A *Natural Day* is the time the ſun takes in paſſing from the meridian of any place, till it comes round to the ſame meridian again.

PROP. XLIV. Any place upon the earth's ſurface deſcribes more than a circle round the earth's axis in a natural day : and the arc which it deſcribes more than a circle in any day, is the ſun's increment of right aſcenſion for that day.

While the earth revolves round its axis, any place upon the earth's ſurface deſcribes a circle : but (by Prop. XLIII.) while the ſun completes its apparent diurnal revolution, any place on the earth's ſurface will move through one circle and an arc of a ſecond ; therefore any ſuch place deſcribes more than a circle (Def. XLI.) in a natural day.

Plate 9. Fig. 13. And ſince both a meridian, and a ſecondary of the equator, paſſing through the poles, are perpendicular to the equator, (Def. VI. and X.) if the ſun at S be in SR the meridian of any given place, it is alſo in a ſecondary of the equator paſſing through that place. In like manner, if the ſun be at T, and TV be after a natural day the ſituation of the meridian of the given place, the ſun will be in TV, which will be both the meridian of the place, and a ſecondary of the equator. Whence, RV being part of the equator, ſince ER was the ſun's right aſcenſion when it was at S, and EV is its right aſcenſion when it is arrived at T, RV muſt be the increment of the ſun's right aſcenſion for the natural day in which it is advanced from S to T. And, becauſe SR, TV, are both perpendicular to the equator, and any place, in one diurnal revolution of the earth deſcribes a circle parallel to the equator, RV taken in this circle will always be the ſame arc with RV in the equator, and therefore will be equal to the ſun's daily increment of right aſcenſion.

PROP. XLV. The natural days are not equal to one another.

For any natural day is the time in which the earth performs one revolution round its axis, and ſuch a portion of a ſecond as is equal to the ſun's increment of right aſcenſion for that day ; but the ſun's daily increments of right aſcenſion are unequal ; (by Prop. XLII. and XLIII.) therefore the additional portion of the ſecond revolution will ſometimes be greater and ſometimes leſs, and conſequently the times in which the natural days are completed will be unequal.

COR. Hence ariſes the difference between a ſun-dial and a clock, as meaſures of time: the former meaſuring the length of the natural day, the latter dividing time into equal portions, of 12 hours each, the clock will be before the dial, when the natural day is more than 24 hours, (the mean length of the natural day) and after it, when the natural day is leſs than 24 hours; and they will be together, only when the natural day is exactly 24 hours.

DEF. XLII. The *Equation*, is the difference between the mean length of the natural day (or 24 hours) and the length of any ſingle day meaſured by the ſun's motion, or between *mean time* and *apparent time*.

COR. The hour of the day by apparent time being known, in order to determine what is then the true mean time, the equation is to be *added* to the apparent time, when the day by the clock is ſhorter than the day by the ſun-dial, that is, when mean time *precedes* apparent time; and the equation is to be *ſubtracted* from the apparent time when the day by the clock is longer than the day by the ſun-dial, that is, when mean time *follows* apparent time. If, for example, the natural day is 24 hours, 1 minute, the day by the clock being 24 hours in length, it will be 12 by the clock, one minute before it is 12 by the dial, or mean time precedes apparent time one minute; therefore it will be 1 minute paſt 12 by the clock, when it is exactly 12 by the ſun; whence one minute, which is the equation, muſt be *added* to the apparent time, to give the true mean time, that is, the clock muſt be ſet at 1 minute paſt 12. The reverſe of this, when mean time follows apparent time, or the day by the clock is longer than the day by the ſun, is obvious.

PROP. XLVI. If the ſun were to move uniformly round the equator in the ſame time in which it appears to deſcribe the ecliptic, its apparent daily motion would be a meaſure of mean time.

For the natural days in that caſe being liable to no variation, either from the declivity of the ſun's orbit, or the irregularity of its motion, muſt be equal.

PROP. XLVII. The portion of time which paſſes between the arrival of the ſun in the ecliptic to the meridian of any place, and its ſuppoſed arrival at the ſame meridian, if it were to move uniformly in the equator, is the equation.

For (by the laſt Prop.) it would be noon by mean time at any place, when the ſun, if it moved uniformly in the equator, was arrived at the meridian of that place; and it is noon at the ſame place by apparent time, when the ſun in the ecliptic arrives at the ſame meridian: therefore the difference between theſe two arrivals, is the difference between mean time and apparent time, or the equation.

PROP. XLVIII. In the time which paſſes between the arrival of the ſun in the ecliptic to the meridian of any place, and its ſuppoſed arrival at the ſame meridian, if it were to move uniformly in the equator, an arc of the equator paſſes under the meridian, which is equal to the difference between the right aſcenſion of the ſun, as it moves in the ecliptic, and the right aſcenſion which the ſun would have, if it moved uniformly in the equator.

Plate 9. Fig. 13. Let the ſun be at S; and let EC be the ecliptic, EQ the equator, and E the firſt of Aries: then if a ſecondary of the equator paſſes through the ſun, SR, being at right angles to EQ, is an arc of that ſecondary, and (by Def. XXXII.) ER is the ſun's right aſcenſion, and the point R is the point in which the right aſcenſion ends; which being in the ſecondary of which SR is a part, that is, the ſecondary paſſing through the ſun, arrives at the meridian at the ſame time with the ſun. If the ſun were to move uniformly in the equator, and were arrived at P, EP would be its right aſcenſion, and conſequently P would be the point in which its right aſcenſion would end, which point P muſt arrive at the meridian at the ſame time with the ſun, becauſe the ſun is ſuppoſed to be in that point. Therefore RP, the diſtance of the two points R and P, is an arc of the equator, (paſſing under the meridian in the time ſpecified in the Propoſition) which is equal to the difference between the real and ſuppoſed right aſcenſions of the ſun, when he arrives at the meridian by his real motion in the ecliptic, and when he arrives at the ſame meridian by an uniform motion in the equator.

PROP. XLIX. The right aſcenſion of the ſun, if it were to move uniformly in the equator, would at any time be equal to the longitude which it would have at that time if it were to move uniformly in the ecliptic, or to its mean longitude.

For, on this ſuppoſition, the ſun, deſcribing the equator with an uniform velocity in the ſame time in which it actually deſcribes the ecliptic, its velocity would be the ſame with the mean velocity in the ecliptic. Conſequently, the diſtance of the ſun from the firſt of Aries in the equator would at any time be the ſame with its diſtance from the ſame point in the ecliptic, if it were to move uniformly therein with its mean velocity: that is, its right aſcenſion in the equator would always be equal to its mean longitude in the ecliptic.

PROP. L. An arc of the equator, equal to the difference between the ſun's right aſcenſion and its middle longitude, at any given time and place, converted into time, is the equation.

It has been ſhewn (Prop. XLVIII.) that in the portion of time which paſſes between the arrival of the ſun in the ecliptic to the meridian of any place, and its ſuppoſed arrival at the ſame meridian if it were to move uniformly in the equator, an arc of the equator paſſes under

der the meridian, which is equal to the difference of the right aſcenſion of the ſun as it moves in the ecliptic, and the right aſcenſion which it would have if it moved uniformly in the equator. And it has been proved, (Prop. XLVII.) that this portion of time is the equation, and (Prop. XLIX.) that the right aſcenſion which the ſun, at any given time and place, would have if it moved uniformly in the equator, is equal to its mean longitude in the ecliptic. Therefore, in the equation, an arc of the equator paſſes under the meridian, equal to the difference of the right aſcenſion of the ſun in the ecliptic, and its mean longitude. Conſequently, if this arc be converted into time, that is, if for 15 degrees be taken an hour, for 15′ one minute of time, for 15″ one ſecond of time, the equation of time will be found.

PROP. LI. If the ſun's mean longitude be greater than its right aſcenſion, mean time *follows* apparent time; if its mean longitude be leſs than its right aſcenſion, mean time *precedes* apparent.

If the right aſcenſion of the ſun, as before ſuppoſed in the equator, EP, that is, (by Prop. XLIX.) its mean longitude, be greater than the ſun's real right aſcenſion ER, the ſuppoſed place of the ſun in the equator P, will be to the eaſt of the point R, where the ſun's real right aſcenſion ends. Therefore when this point R, at apparent noon, is come to the meridian, the point P will not be arrived at the meridian; and mean noon will be later than apparent noon. Therefore, when the ſun's middle longitude is greater than its right aſcenſion, mean time *follows* apparent. In like manner the reverſe may be proved. Plate 9. Fig. 13.

Cor. Hence in the *former* caſe the equation is to be *ſubtracted* from the apparent time found by the dial, and in the *latter*, to be *added* to it, in order to obtain the mean time.

Schol. It appears from the foregoing Propoſitions, that the difference between mean and apparent time, depends upon two cauſes: (1) the obliquity of the ecliptic with reſpect to the equator: and (2) the unequal motion of the earth in an elliptical orbit. The obliquity of the ecliptic to the equator would make the ſun and clocks agree on four days of the year, viz. when the ſun enters Aries, Cancer, Libra, and Capricorn. But the other cauſe which ariſes from his unequal motion in his orbit would make the ſun and clocks agree only twice a year, that is, when he is in perigee, and apogee; conſequently, when theſe two points fall in the beginnings of Cancer and Capricorn, or of Aries and Libra, they will concur in making the ſun and clocks agree in thoſe points. But the apogee, at preſent, is in the ninth degree of Cancer, and the perigee in the ninth degree of Capricorn; and, therefore, the ſun and clocks cannot be equal at the beginnings theſe ſigns, nor, indeed, at any time of the year, except when the ſwiftneſs and ſlowneſs of the equation reſulting from one of the cauſes, juſt balances the ſlowneſs or ſwiftneſs ariſing from the other, which happens about the 15th of April, the 15th of June, the 31ſt of Auguſt, and the 24th of December; at all other times the ſun is too faſt or too ſlow for equal time by a certain number of minutes and ſeconds, which at the greateſt is 16′. 14″. and happens about the firſt of November: every other day throughout the year having a certain quantity of this difference belonging to it, which, however, is not exactly the ſame every year, but only every fourth year, for which reaſon, it is neceſſary, where great accuracy is required, to have four tables of this equation, viz. one for each of the four years in the period of leap years. The following conciſe table, adapted to the ſecond year after leap year, will always be found within about a minute of the truth, and therefore ſufficiently accurate for common clocks and watches.

TABLE for the Equation of Time.

Months.	Days.	Equation in Minutes.	Months.	Days.	Equation in Minutes.	Months.	Days.	Equation in Minutes.	Months.	Days.	Equation in Minutes.
Jan.	1	4+	Apr.	1	4+	Aug.	9	5+	Oct.	27	16—
	3	5		4	3		15	4	Nov.	15	15
	5	6		7	2		20	3		20	14
	7	7		11	1		24	2		24	13
	9	8		15	0		28	1		27	12
	12	9		*	——		31	0		30	11
	15	10		19	1—		*	——	Dec.	2	10
	18	11		24	2	Sept.	3	1—		5	9
	21	12		30	3		6	2		7	8
	25	13	May	13	4		9	3		9	7
	31	14		29	3		12	4		11	6
Feb.	10	15	June	5	2		15	5		13	5
	21	14		10	1		18	6		16	4
	27	13		15	0		21	7		18	3
Mar.	4	12		*	——		24	8		20	2
	8	11		20	1+		27	9		22	1
	12	10		25	2		30	10		24	0
	15	9		29	3	Oct.	3	11		*	——
	19	8	July	5	4		6	12		26	1+
	22	7		11	5		10	13		28	2
	25	6		28	6		14	14		30	3
	28	5					19	15			

Thoſe columns that are marked +, ſhew that the clock or watch is faſter than the ſun; and thoſe marked —, that it is ſlower. See Ferguſon's Aſtronomy, ch. 13. Phil. Tranſ. vol. 54.

CHAP. III.

Of the INFERIOR PLANETS, MERCURY *and* VENUS.

Def. XLIII. The *Elongation* of any planet is its apparent diſtance from the ſun.

Def. XLIV. The *Nodes*, of the orbit of a planet, are the two points in which the orbit cuts the plane of the ecliptic: and a right line drawn from one node to the other, is the *Line of the Nodes*.

Def. XLV. The *Limits* of the orbit of a planet, are two points in the middle between the two nodes.

DEF. XLVI. An inferior planet is in its *inferior* conjunction, when it is nearer the earth than the ſun is, and in its *ſuperior* conjunction, when it is farther than the ſun is from the earth, and both in the ſame ſecondary.

Let A be the place of the earth in its orbit ABO, E the place of Venus in its orbit EHG, S the ſun, and FD an arc of a circle in the heavens. Venus will be in its inferior conjunction when it is at E, and at its ſuperior when it is at G, and both are in the ſame ſecondary. Plate 9. Fig. 12.

PROP. LII. An inferior planet is at its greateſt elongation, when a line drawn from the earth through the planet, is a tangent to the orbit of the planet.

When the planet is at E, being in conjunction with the ſun, it has no elongation. As it moves from E towards X its elongation increaſes, till at X, when AF is a tangent to the orbit of Venus, its apparent place is F, and its elongation FQ, which is the greateſt elongation it can have; for in paſſing from X to G its elongation decreaſes, till at G it becomes nothing. This will be true in elliptical as well as circular orbits. Plate 9. Fig. 12.

COR. If the orbits of theſe planets were circular, the diſtance of each, from the ſun, would be to the earth's diſtance, as the ſine of its greateſt elongation to the radius.

The orbits are not circles, but ellipſes, having the ſun in one focus; for on this ſuppoſition their computed places are always found to agree with their obſerved places.

PROP. LIII. The inferior planets are never in oppoſition to the ſun.

For in oppoſition the earth is between the ſun and the planets, which can never happen when the orbit of the planet is nearer to the ſun than that of the earth.

DEF. XLVII. A planet is in *Quadrature*, when it is 90 degrees in the celeſtial ſphere diſtant from the ſun.

PROP. LIV. The inferior planets are never in quadrature.

The greateſt angle of elongation is that contained by AQ, drawn from the earth through the ſun, and AF a tangent to the orbit of the planet. Now if QAF were a right angle, AF would be (El. III. 18.) a tangent to the earth's orbit; but AF is a tangent to an orbit leſs than that of the earth; it therefore makes an angle with AQ leſs than a right angle, that is, QF the greateſt elongation is leſs than 90 degrees. Plate 9. Fig. 12.

COR. Hence the inferior planets never appear far from the ſun.

PROP. LV. While Venus is moving from the ſuperior conjunction to the inferior, it ſets after the ſun; while it is moving from the inferior conjunction to the ſuperior, it riſes before the ſun.

Plate 9. Fig. 12. Whilſt Venus is moving from G, its ſuperior conjunction, through P, to E its inferior conjunction, being in the eaſtern part of its orbit, the ſun will be weſtward of Venus; therefore Venus, if far enough from the ſun, will be ſeen in the weſt after the ſun is gone down, whence it is then called the evening ſtar. But on the weſtern ſide of its orbit, the ſun being eaſtward of it, Venus will ſet before the ſun, and conſequently riſe before it, whence it is then called the morning ſtar.

PROP. LVI. The greateſt elongation of an inferior planet on one ſide of the ſun is not always equal to that on the other.

For ſince the planet moves in an elliptical orbit, at the time of its greateſt elongation on one ſide it may be in its aphelion; and at its greateſt elongation on the other ſide, it may be in ſome part nearer the ſun: hence its real diſtance from the ſun at its elongations being unequal, its apparent diſtances will be ſo likewiſe.

Cor. The greateſt elongation of Venus being found greater than that of Mercury, the latter muſt be nearer the ſun than the former.

PROP. LVII. The apparent velocity of the inferior planets is greateſt at the conjunctions.

Since the plane of the orbit of Venus is oblique to that of the earth, thoſe parts of this orbit which are viewed by a ſpectator on the earth directly, would appear longer than other equal parts viewed obliquely; whence its motions, if uniform, muſt appear unequal. If the orbit EPGH of Venus be ſeen obliquely by an eye placed at A, the parts about E and G, or near the conjunctions, will be ſeen directly, for AE is perpendicular to a tangent at E; but the parts about X and P will be ſeen obliquely: whence the Propoſition is manifeſt. Plate 9. Fig. 12.

Schol. The time when an inferior planet will come again into a given ſituation with reſpect to the ſun and the earth, may be thus found. Whilſt Venus performs one revolution, the earth, whoſe periodical time is longer than that of Venus, will not have completed its revolution. Before Venus and the earth can be again in the inferior conjunction, Venus muſt, therefore, beſides its entire revolution, deſcribe an arc equal to that which the earth has paſſed over: conſequently, the number of degrees paſſed over by each, or their angular motions, in the ſame time, will be reciprocally as their periodical times; that is, as the periodical time of the earth is to the periodical time of Venus, ſo is the angular motion of Venus (which is equal to four right angles added to the angular motion of the earth between two inferior conjunctions) to the angular motion of the earth in the ſame time: whence (El. V. 17.) as the difference between the periodical times of the earth and Venus, is to the periodical time of Venus, ſo are four right angles, or 360°. to the number of degrees over which the earth paſſes in her orbit from one inferior conjunction to another. This is only true upon the ſuppoſition that the planets moved in circular orbits, in which caſe the following general rule would apply to the finding the time from conjunction to conjunction, or from oppoſition to oppoſition of any two planets. "Multiply their periodic times together, and divide the product by their difference, and you have the time ſought." For let $P =$ the periodic time of the earth, $p =$ that of the planet (ſuppoſe an inferior) $t =$ time required: then $P : 1$ day $:: 360° : \frac{360°}{P}$, the angle deſcribed by the earth in 1 day: for the ſame reaſon $\frac{360°}{p}$ is the angle deſcribed by the planet in 1 day: hence $\frac{360°}{p} - \frac{360°}{P}$ is the daily angular velocity of the planet *from* the earth. Now if they ſet out from conjunction, they will return into conjunction again, after the planet has gained 360°; hence $\frac{360°}{p} - \frac{360}{P} : 360° :: 1$ day $: t = \frac{Pp}{P-p}$. For a ſuperior planet $t = \frac{pP}{p-P}$.

DEF. XLVIII. The apparent motion of a planet, if ſeen from the earth, is called its *Geocentric Motion*; if ſeen from the ſun, its *Heliocentric Motion.*

PROP. LVIII. When the inferior planets are paſſing from their greateſt elongation, through their ſuperior conjunction, to their greateſt elongation on the other ſide, their geocentric motion is direct, or they appear to move from weſt to eaſt.

When Venus is at X, it appears to a ſpectator at A to be in the point F of the concave ſphere of the heavens: when it has moved forwards in its orbit to H, G, N, P, it will appear ſucceſſively in the ſeveral points F, Q, R, D. But the motion from F to D is from weſt to eaſt; for whilſt the ſun and earth are on the ſame ſide of the planet, it muſt appear to move in the ſame direction, whether it be viewed from the earth or the ſun, becauſe the ſpectator at either ſtation views the concave ſide of the planet's orbit: but from the ſun it is always ſeen to move from weſt to eaſt; therefore its apparent geocentric motion in this ſituation is direct, or *in conſequentia.* Plate 9. Fig. 12.

PROP. LIX. While the inferior planets are moving from the greateſt elongation on one ſide, to the greateſt elongation on the other, through their inferior conjunction, their geocentric motion is retrograde, or from eaſt to weſt.

While the planet is in this ſituation, the convex ſide of its orbit is towards a ſpectator on the earth, but its concave ſide towards a ſpectator at the ſun: hence the former will ſee the planet move in a direction contrary to that in which it will appear to the latter to move.

Thus,

Thus, when the planet is at P, it will appear in the heavens at D; and as it paſſes through E to X, it appears to move from D through R, Q, P, to F: but the motion from D to F is from eaſt to weſt; therefore the apparent motion of the planet in this part of its orbit is retrograde, or *in antecedentia*.

PROP. LX. When the inferior planets are at their greateſt elongation, they appear ſtationary.

At either of the greateſt elongations P, of the planet Venus, AD is a tangent to the orbit; which ſo nearly coincides with a ſmall arc of the curve, that a ſpectator's eye placed at the earth could not diſtinguiſh the tangent from this part of the curve: conſequently, a planet, while it is in this part of the curve, will appear to lie in the tangent AD, that is, to be ſtationary in the point D.

Schol. 1. In the preceding Propoſitions, the earth's motion in its orbit is not regarded, becauſe the only effect of this motion on the appearances above deſcribed, is, that they take place in different parts of the heavens; whereas, without this motion the inferior planets would always be direct or retrograde in their motion, or be ſtationary, in the ſame parts of the heavens.

Plate 9. Fig. 12. If ABO be the orbit of the earth, and EHG that of Venus; when the earth is at A, the ſun's geocentric place is Q, and Venus, in order to be in conjunction, muſt be in the line AQ at E or G. If the earth ſtood ſtill at A, and Venus were in her inferior conjunction at E, the ſuperior conjunction would always be at G, and her greateſt elongations at X and P. But if the earth be advanced in her orbit from A to *a*, the ſun's geocentric place is now at *m*, and Venus, in order to be in conjunction, muſt now be in one of the points in which her orbit cuts the line *am*. Thus the places of the conjunctions, and conſequently of the greateſt elongations *af*, *ad*, are continually carried round the ecliptic in the order of the ſigns.

Schol. 2. As ſeen from the ſun the motion of theſe, and indeed of all the planets, appears to be conſtantly direct: the appearance of the ſtations and retrogradations is entirely owing to the ſituation and motion of the earth. And from theſe cauſes the apparent motion of Mercury and Venus appears to us to be in looped curves. In order to repreſent the apparent paths of theſe planets, Ferguſon removed from an Orrery the balls repreſenting the Sun, Mercury, and Venus, and put in their places black-lead pencils, with the points upward, and fixed a circular piece of paſte-board, ſo that it remained conſtantly in the ſame poſition, during the motion of the machine, with its centre reſting on the terreſtrial ball. Then, the paſte-board being gently preſſed on the pencils, and the pencils repreſenting Mercury and Venus, together with the terreſtrial ball being made to revolve about the pencil in the ſun's place, by turning the winch of the machine, the three pencils deſcribed on the paſte-board a diagram, of which Plate 1. Fig. 14. is a copy, reduced to a leſs ſize. Plate 1. Fig. 14. The ſolar pencil marked the dotted circle of months, which repreſents the ſun's apparent motion in the ecliptic, and is the ſame every year. The pencil in Mercury's place traced the curve, having 23 loops and croſſing itſelf the ſame number of times, which repreſents that planet's apparent motion for 7 years. The pencil in the place of Venus deſcribed the curve, having 5 loops, which repreſents the apparent path of Venus for 8 years;

after which time this planet moves again very nearly in the ſame apparent path. The ecliptic encloſes theſe curves, in the figure, and dotted lines are drawn from the earth to the ecliptic, to ſhew the apparent or geocentric motion of Mercury for one year, which is eaſily traced from A, where it commences, through B, C, &c. At B Mercury is ſtationary, and the dotted line, drawn through B, ſhews its place in the ecliptic; from B to C it is retrograde; at C ſtationary; from C it proceeds directly. And, univerſally, on each ſide of the loops the planets appear to be ſtationary; in the part of the loops neareſt to the earth, retrograde; and in the reſt of the path, direct.

PROP. LXI. The heliocentric latitude of an inferior planet is the greateſt when the planet is in one of its limits.

For the planet is then (Def. XLV.) at its greateſt diſtance from the ecliptic, and therefore will have the greateſt latitude, as ſeen from the ſun.

PROP. LXII. The geocentric latitude of an inferior planet is directly as its heliocentric latitude, and inverſely as its diſtance from the earth.

The apparent length of a line drawn from the planet to the plane of the ecliptic, that is, its geocentric latitude, is (by Book VI. Prop. LXIX. LXX.) directly as its real length, and inverſely as the diſtance of the ſpectator's eye. But the real length of a line drawn from the planet to the plane of the ecliptic, is its heliocentric latitude; and the ſpectator's eye is at the earth; whence the propoſition is manifeſt.

Cor. When Venus is in its inferior conjunction, its heliocentric latitude is leſs than its geocentric; for it is then farther from the ſun than from the earth. The contrary takes place with reſpect to Mercury.

PROP. LXIII. The ſun enlightens only one half of a planet, and only one half of a planet is viſible, at once.

This is ſufficiently manifeſt from the ſpherical form of the planets, and the rectilinear motion of light.

Schol. Venus and Mercury, in paſſing from the inferior to the ſuperior conjunction, are obſerved to have all the phaſes of the moon from new to full.

Def. XLIX. The hemiſphere of a planet which is towards the earth is called its *Diſc*, becauſe it appears like a plane circle.

PROP. LXIV. The inferior planets are inviſible in their inferior conjunction; their whole diſc is illuminated, when they are in their ſuperior conjunction; and they are more or leſs illuminated, as they are nearer or farther from their ſuperior conjunction.

When Venus, or Mercury, is in its ſuperior conjunction, the whole of its enlightened hemiſphere is towards the earth, and its entire diſc is viſible; as it paſſes towards its inferior conjunction, its enlightened hemiſphere turns, by degrees, from the earth, till, at the inferior conjunction, it is wholly turned from the earth, and the planet becomes inviſible.

PROP. LXV. If an inferior planet is in one of its nodes at the time of its inferior conjunction, it will appear as a ſpot in the diſc of the ſun.

When the planet is in the nodes, it will be in the plane of the ecliptic; and if at the ſame time it be in its inferior conjunction, it will neither appear above nor below the ſun, as it does when in conjunction in other parts of its orbit, but on the ſun's diſc.

CHAP. IV.

Of the SUPERIOR PLANETS, MARS, JUPITER, SATURN, *and the* HERSCHEL.

Def. L. The ſuperior planets are Mars, Jupiter, Saturn, and the Herſchel.

PROP. LXVI. The ſuperior planets are ſometimes in conjunction with the ſun, ſometimes in oppoſition, and ſometimes in quadrature.

This is known from obſervation.

Plate 10. Fig. 1. Let S be the ſun; QPO a part of the orbit of Jupiter; P the planet; *adg*, or *nkg*, the earth's orbit. When the earth is at *d*, the ſun at S, and the planet at P, the planet is in conjunction: when the earth is at *k*, the ſun at S, and the planet at P, the planet is in oppoſition: when the earth is at *n* or *g*, and the planet at P, the planet will be in quadrature.

Cor. Therefore the ſuperior planets move in orbits farther diſtant from the ſun, than the orbit of the earth.

PROP. LXVII. The apparent diameter of a ſuperior planet is greateſt when the planet is in oppoſition.

For, when the planet is in conjunction, its diſtance from the earth is greater than when it is in oppoſition, by the diameter of the earth's orbit.

PROP. LXVIII. If a ſuperior planet were at reſt, its apparent geocentric motion in any given time would be proportional to the angle ſubtended by the arc deſcribed by the earth in that time, and ſeen from the planet.

When the earth is at *a*, the planet P appears in the right line *a*PA, and among the ſtars in the heavens at A: when the earth is at *b*, the planet appears at B. Therefore while the earth moves from *a* to *b*, the planet appears to move from A to B. But this arc AB is proportional to the angle APB, that is, to the oppoſite angle *a*P*b*, which is the angle which the arc *ab* would ſubtend to an eye placed at the planet P. Plate 10. Fig. 1.

PROP. LXIX. The geocentric velocity of a ſuperior planet is greateſt at its conjunction and oppoſition.

Arcs of a given length near the points *d* and *k*, when the planet is ſeen from the earth in conjunction or oppoſition, would appear greater than arcs in any other part of the earth's orbit, viewed from the planet P, becauſe the former are ſeen directly, the latter obliquely: conſequently, theſe arcs would ſubtend greater angles; whence the apparent velocity of the planet, as viewed from the earth, is greater at the conjunction or oppoſition, than at any other time.

PROP. LXX. When a line drawn from a ſuperior planet to the earth is a tangent to the earth's orbit, the planet appears ſtationary.

While the earth is near *a* or *g*, points in its orbit lying in a line which, drawn from the planet, is a tangent to that orbit, the arc which it then deſcribes ſo nearly coincides with the tangent line, that the earth for ſome time ſeems to move directly from or towards the planet in the line of the tangent; whence the planet muſt, during that time, appear to be in that line, or to be ſtationary.

PROP. LXXI. When a ſuperior planet is paſſing from one of its ſtationary ſituations to the other through the conjunction, its geocentric motion is direct; when through the oppoſition, retrograde.

While the earth is moving from *a* through *d* to *g*, the ſun and planet being both on the ſame ſide of the earth, the motion of the earth will produce an apparent motion in the ſun and planet the ſame way, and both will appear to move from A towards G. But while the earth moves from *g* to *n* through *k*, the ſun and planet being on contrary ſides of the earth, the motion of the earth in its orbit will produce an apparent motion of the ſun and planet in contrary directions, that is, whilſt the ſun appears to move from weſt to eaſt, the planet will appear to move from eaſt to weſt in the direction G, H, I, &c. Plate 10. Fig. 1.

Schol. The progreſs, regreſs, and ſtations of the ſuperior planets, muſt take place notwithſtanding the motion of the earth, ſince this only changes the place of the conjunctions and oppoſitions, on which theſe depend, as was ſhewn, Prop. LX. Schol.

PROP. LXXII. Mars ſometimes appears round, ſometimes gibbous; Jupiter, Saturn, and the Herſchel, always appear round.

When Mars is either in oppoſition or conjunction, his whole illuminated hemiſphere is towards the earth, but when it is in quadrature, ſome part of his illuminated diſc is turned from the earth. The ſame muſt happen in the revolutions of Jupiter, Saturn, and the Herſchel, about the ſun; but their great diſtance renders the difference between the perfect and imperfect illumination of their diſc imperceptible.

SCHOL. The following particulars reſpecting Mars are given by Dr. Herſchel, after long and accurate obſervations.

The axis of Mars is inclined to the ecliptic 59°. 42′.

The node of the axis is in 17°. 47′. of Piſces.

The obliquity of the ecliptic on the globe of Mars is 28°. 42′.

The point Aries on the *martial* ecliptic anſwers to our 19°. 28′. of Sagitarius.

The figure of Mars is that of an oblate ſpheroid, whoſe equatorial diameter is to the polar one as 1355 to 1272, or as 16 to 15 nearly.

The equatorial diameter of Mars, reduced to the mean diſtance of the earth from the ſun is 9″. 8‴.

And that planet has a conſiderable, but moderate atmoſphere, ſo that its inhabitants, probably, enjoy a ſituation, in many reſpects, ſimilar to ours.

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CHAP. V.

Of the MOON.

SECT. I.—*Of the* VARIATIONS *in the* APPEARANCE *of the* MOON.

DEF. LI. When the moon is at its greateſt diſtance from the earth in its orbit, which is elliptical, or at its higher apſis, it is ſaid to be in its *Apogee;* when at its leaſt diſtance, or lower apſis, in its *Perigee.*

DEF. LII. When the moon is in conjunction with the ſun, it is *New Moon;* when in oppoſition, it is *Full Moon:* its conjunction and oppoſition are called by the common name of its *Syzygies.*

DEF. LIII. A *Periodical Month* is the time in which the moon deſcribes its orbit; a *Synodical Month* is the time which paſſes between one new moon and the next.

PROP. LXXIII. A ſynodical month is longer than a periodical month.

Becauſe the moon moves in the ſame direction with the ſun, while the moon performs one revolution in its orbit, the ſun, by its apparent annual motion, is advanced in the ecliptic;

tic; conſequently, the moon muſt paſs beyond the point in which it has completed its revolution before it comes again into conjunction with the ſun.

Plate 10. Fig. 8.

Let S be the ſun, BA a part of the earth's orbit, *md*, MD, the diameter of the moon's orbit when the earth is at B, or A. While the earth is at A, if the moon be at D, it will be in conjunction, and if the earth continue in the ſame place, after one revolution in its orbit, it would be again in conjunction: but if, during the revolution of the moon, the earth is removed to B, the moon at the end of the revolution will be at *d*, a point which is not between the earth and ſun; it muſt therefore move on from *d* to *e* before it will be in conjunction.

PROP. LXXIV. The moon, at its conjunction, is inviſible; at its oppoſition, its whole diſc is enlightened; at its quadratures, it is half enlightened; between the conjunction and quadrature, it is horned; and between the quadrature and oppoſition, it is gibbous.

Plate 10. Fig. 2.

Let QTL be a part of the earth's orbit, S the ſun, T the earth, ACEG the moon's orbit. When the moon is at E, or in conjunction, becauſe it is between the earth and ſun, its illuminated hemiſphere will be wholly turned from the earth, conſequently, its diſc will be dark. At A, being in oppoſition, its illuminated hemiſphere will be wholly towards the earth, and its whole diſc will be viſible. At C, or G, the apparent diſtance of the moon from the ſun will be 90 degrees; for a right line from C or G, will make a right angle with the line TS, in which the ſun appears: whence the moon at each of them is half way between the oppoſition and conjunction, that is, in the middle ſtate between the perfect illumination and the entire darkneſs of its diſc; conſequently, its diſc is half enlightened. In paſſing from C to E, and from E to G, its diſc will be leſs than half illuminated, or it will appear horned; in paſſing from G to A, and from A to C, its diſc will be more than half illuminated, or it will appear gibbous.

Cor. The earth muſt be a ſatellite to the moon, and ſubject to the ſame changes, but more than 13 times larger than the moon appears to us. At new moon to us, the earth appears full to her.

Def. LIV. A circle ſuppoſed to be drawn upon the ſurface of the moon, ſeparating the illuminated from the dark hemiſphere, is called the *Circle of Illumination*: a circle which ſeparates its viſible from its inviſible hemiſphere, is called the *Circle of the Diſc*.

PROP. LXXV. If the centres of the ſun, the earth, and the moon, are joined by ſtraight lines forming a triangle, the external angle of this triangle at the moon, is equal to the angle contained between the circle of illumination, and the circle of the diſc.

Plate 10 Fig. 3.

Let S be the ſun, T the earth, O the centre of the moon, and *mrnq* the moon's orbit. Let STO be the ſuppoſed triangle. Draw the line *rq* perpendicular to SO, and *nm* perpendicular to TOP. The angle SOP will be equal to the angle mOq, contained between rq, which repreſents the circle of illumination, and nm, which repreſents the circle of the diſc. Becauſe the angles SOr, TOm, are by conſtruction right angles, they are equal: and POr, qOT, are vertical angles, and therefore equal. Conſequently, the remaining angles POS, mOq, are equal.

Plate 10. Fig. 3. Schol. This Propoſition ſerves to explain all the different phaſes of the moon. For example; when the moon is moved from O to A, the line SO coincides with SA, and TO with TA; therefore TO, OS, lie in the ſame line, and the external angle POS is nothing: whence the two circles of illumination and of the diſc coincide; and becauſe the diſc is then turned from the ſun, it is wholly dark. When the moon is in quadrature, the line TOP will be a tangent to the moon's orbit; whence SOP will be a right angle, and the two circles will be at right angles to each other, and the diſc will appear half illuminated. If the angle POS be leſs or greater than a right angle, the circle of illumination will make an angle with that of the diſc, leſs or greater than a right angle: whence the illuminated part will appear horned or gibbous. Laſtly, when the moon is in oppoſition, the lines SO, ST, become one and the ſame line; whence the circles coincide, and the whole diſc is illuminated.

PROP. LXXVI. The horns of the moon, before the conjunction, are turned towards the eaſt; after it, towards the weſt.

The ſun, after the conjunction, ſetting before, that is, to the weſt of the moon, illuminates the weſt ſide of the moon's diſc; whence its horns, which are towards the dark part of the diſc, are towards the eaſt. The reverſe takes place at the oppoſition, when the moon is ſeen in the eaſt, before the ſun riſes.

PROP. LXXVII. When the moon is horned, its obſcure part is viſible by the reflection of the rays of the ſun from the earth.

Plate 10. Fig. 2. When the moon is at D or F, near the conjunction, the enlightened hemiſphere of the earth will be towards the moon, and reflect the rays of the ſun upon it.

PROP. LXXVIII. The moon always has nearly the ſame ſide towards the earth.

This is proved by obſervation.

Cor. 1. Hence, if the moon revolves about its axis, its periodical time muſt be equal to that of its revolution in its orbit round the earth. This is alſo found to be the caſe with the fifth ſatellite of Saturn as it regards its primary.

Cor. 2. Though the year is of the ſame abſolute length, both to the earth and moon, yet the number of days in each is very different; the former having $365\frac{1}{4}$ natural days, but the latter only about $12\frac{7}{19}$, every day and night in the moon being as long as $29\frac{1}{2}$ on the earth.

SCHOL. This propoſition would be true without any limitation, if the angular velocity of the moon about the earth were equal to the angular velocity about her axis, for then the ſame face muſt be always exactly turned towards the earth. But the angular velocity of the moon about the earth is *not* uniform, while that about its axis is uniform, and therefore the ſame face is not always turned to the earth. Since, however, the oppoſite face of the moon is never ſeen, the time of the moon's rotation about her axis muſt be equal to the *mean* time in her orbit. See Prop. LXXX. and LXXXI.

PROP. LXXIX. The moon appears to have two librations, one upon a line perpendicular to its axis, called its libration in latitude; the other upon its axis, called its libration in longitude.

This appears from obſervation, ſome ſmall portions of the ſurface of the moon being viſible in ſome parts, and inviſible in other parts, of its orbit : that is, in conſequence of her libration in latitude we ſometimes ſee one pole, and ſometimes the other. And by the libration in longitude, more of the weſtern limb is ſometimes ſeen, and at others, more of the eaſtern. The inclination of her axis to her orbit is 6°. 49′.

COR. The axis of the moon being almoſt perpendicular to the ecliptic, there will be but a ſmall difference in its ſeaſons.

SCHOL. The moon's axis being ſo nearly perpendicular to the ecliptic, that the ſmall obliquity of her orbit cannot cauſe the ſun ſenſibly to decline from the equator. Hence we learn, that the inhabitants of the moon, muſt deviſe ſome method, different from ours, for aſcertaining the length of their year. We know the length of our years by the return of the equinoxes ; but theſe are not diſcoverable in the moon, where the days and nights are equal.

The Lunarians may, perhaps, meaſure their year by obſerving when either of the poles of our earth begins to be enlightened, and the other to diſappear, which happens at our equinoxes ; they being conveniently ſituated for obſerving large tracts of land about the poles of the earth which, at preſent, are unknown to the moſt experienced of our navigators.

PROP. LXXX. The librations of the moon may be explained on the ſuppoſition that the moon has a revolution round its axis.

Plate 10.
Fig. 4.

Let IH repreſent the plan of the moon's orbit, E the earth, and CMD the moon ; and let AB be the axis round which the moon revolves, and A be called its north pole, and B its ſouth pole. CMD will in this ſituation be the viſible hemiſphere, and CD the plane of the diſc. By the libration in latitude the line AB appears to vibrate on the line IH, ſo that ſometimes the point A is viſible, and ſometimes the point B. This variation attends the moon's revolution in its orbit : in one half of the orbit, the pole A is always viſible, and in the other half the pole B. It muſt therefore ariſe from the inclination of the axis of the moon to the plane of its orbit. When the moon is at I, if the axis AB is inclined to IH the plane of the orbit, making with it the angle AIH to a ſpectator at E, the viſible hemiſphere will be CND, and the pole A will be within the diſc : but when the moon is at H, the viſible hemiſphere will be CMD; and, the axis AB being always parallel to itſelf, the pole B will be within the diſc.

Fig. 5.

Again, let the moon be at A, the earth at T, and a line *d c s*, paſſing through the centre of the moon, be its axis. If a circle, whoſe plane is perpendicular to T*c*, paſs through the line *d c s*, this circle will be that of the diſc, and *d b q s* will be the viſible hemiſphere. But the moon has an apparent motion, by which the diſc is changed, ſo that at one time *r b* is the circle of the diſc, and *r q b* the viſible hemiſphere, at another *f q* and *f c q*. In the former caſe, *s r* becomes viſible, and *d b* inviſible; in the latter caſe, *d f* becomes viſible, and *q s* inviſible. This libration in longitude ariſes from the elliptical form of the moon's orbit. If the moon has a rotation round its axis, it has been ſhewn that its revolution will be completed in the time of one revolution in its orbit; and becauſe the motion round the axis is uniform, one quarter of a revolution will be completed in one fourth part of the periodical time. But, the moon's orbit being ſuppoſed elliptical, and the earth placed at T one of the foci, the moon will move ſlower at A its apogee, than at P its perigee, and its velocity will continually increaſe in moving from A to P, and decreaſe in moving from P to A. If therefore when the moon is at A, *d b q s* is the viſible hemiſphere, after it has moved from A to B, through that quarter of its orbit, in which it moves with its leſs velocity, and conſequently takes up more than a quarter of its periodical time, *d s* will not be perpendicular to T*c*, but the point *s* will have turned from weſt to eaſt more than a quarter round the axis C: hence the point *s* will not be viſible when the moon is at B, and *b l* inſtead of *d c* will be the circle of the diſc. In paſſing from B to P, its exceſs of velocity will make up for its defect in paſſing from A to B; and at B it will have completed half its orbit in half its periodical time: but in half its periodical time, it will have revolved half round its axis, therefore at P, *d s* will again be perpendicular to T*c*, and *d x s* will again be the viſible hemiſphere. The reverſe will take place in paſſing from P to A through D, when *l b* will again be the circle of the diſc, and *s* will be within it.

Plate 10. Fig. 5.

PROP. LXXXI. The moon revolves about its axis in the ſame time in which it revolves about the earth.

Plate 10. Fig. 4.

Without ſuch a revolution, the phenomena of its librations could not happen. If the point A were viſible in one part of the moon's orbit, it would be always viſible, without a rotation about an axis oblique to the plane of the orbit, to produce an apparent motion of the point A, or the libration of latitude. If *d s* were perpendicular to *c*T, when the moon is at A, it would be ſo in every other part of the orbit; and therefore *d x s* would always be the illuminated hemiſphere, if there were not a revolution about the axis to produce, in the manner above explained, the libration of longitude. Theſe librations therefore prove the exiſtence of this revolution: and it has been ſhewn, that if there be ſuch a revolution, its periodical time is the ſame with that of the moon in its orbit.

Fig. 5.

Fig. 2.

Cor. Hence it is evident, that one half of the moon is never in darkneſs; the earth (Prop. LXXIV. Cor.) conſtantly affording it a ſtrong light, during the abſence of the ſun: but the other half has a fortnight's light and darkneſs by turns.

PROP. LXXXII. The orbit of the moon is an ellipſe.

It is only on this ſuppoſition that the libration of longitude can be explained: from this phenomenon, therefore, the elliptical form of the orbit may be inferred.

Schol. The moon revolves about the earth in an ellipſe, and with the earth revolves about the ſun. In Fig. 14, Plate 2, the broad curve line ABCDE repreſents as much of the earth's annual orbit as it deſcribes in 32 days, which are marked with numeral figures. The ſmall circles at *a*, *b*, *c*, *d*, *e*, ſhew the moon's orbit in proportion to that of the earth, and the narrow curve *a b c d e f* repreſents the moon's path in the heavens, or with reſpect to the ſun, for 32 days, reckoning from a new moon at *a*.

Plate 14. Fig. 2.

The earth being ſuppoſed to move from A, and the new moon at the ſame time from *a*; 7 days afterward the earth will be at B, and the moon in quadrature at *b*; when the earth is at C, the moon is full at *c*; when the earth is at D, the moon is in quadrature at *d*; and when the earth is at E, the moon is again new, or in conjunction at *e*.

From this figure it appears, that the moon's path is always concave toward the ſun. For, if we ſuppoſe the concavity of the earth's orbit to be meaſured by the length of a perpendicular C*g*, falling from the place of the earth at C, when the moon is full, on the right line *b g d*, connecting the places of the earth at the end of the firſt and third quarters of the moon, the length of the perpendicular will be about 640000 miles; when the moon is new, it is about 240000 miles nearer to the ſun than the earth is; therefore the length of a perpendicular from its place at that time on the ſame right line will be about 400000 miles, and ſhews the concavity of that part of the path.

PROP. LXXXIII. The moon's ſurface is irregular.

If the ſurface were every where regular, the limit between the enlightened and dark parts of the diſc being the circle of light, that is a perfect great circle of a ſphere, would be exactly defined when the moon is horned, half enlightened, or gibbous, contrary to obſervation.

Schol. The face of the moon, as ſeen through a teleſcope, appears diverſified with hills and valleys. This is proved by viewing her at any other time than when ſhe is full; for then there is no regular line bounding light and darkneſs; but the edge or border of the moon appears jagged: and even in the dark part near the borders of the lucid ſurface there are ſeen ſome ſmall ſpaces enlightened by the ſun's beams.

Beſides, it is moreover evident, that the ſpots in the moon taken for mountains and valleys, are really ſuch from their ſhadows. For in all ſituations of the moon, the elevated parts are conſtantly found to caſt a triangular ſhadow in a direction from the ſun; and the cavities are always dark on the ſide next to the ſun, and illuminated on the oppoſite ſide. Hence aſtronomers are enabled to find the height of the lunar mountains. Dr. Keil, in his Aſtronomical Lectures, has calculated the height of St. Catherine's-hill to be nine miles. Since, however, the loftieſt mountains upon the earth are but about three miles in height, it has been long conſidered as very improbable that thoſe of a planet ſo much inferior in ſize to the earth ſhould exceed in ſuch vaſt proportion the higheſt of our mountains.

By the obſervations of Dr. Herſchel, made in November, 1779, and the four following months, we learn, that the altitude of the lunar mountains has been very much exaggerated. His obſervations were made with great caution, by means of a Newtonian reflector, 6 feet 8 inches long, and with a magnifying power of 222 times, determined by experiment; and the method which he made uſe of to aſcertain the altitude of thoſe mountains, which, during that time, he had an opportunity of examining, ſeems liable to no objection. The rock ſituated near *Lacus niger*, was found to be about one mile in height, but none of the other mountains, which he meaſured, proved to be more than half that altitude; and Dr. Herſchel concludes, that with a very few exceptions, the generality of the lunar mountains do not exceed half a mile in their perpendicular elevation. See Keil's Aſtron. Lect. x. Phil. Tranſ. Vol. lxx.

To Dr. Herſchel alſo we are indebted for an account of ſeveral burning volcanoes, which he ſaw at different times in the moon. In the 77th vol. of the Phil. Tranſ. he ſays, April 19, 10 hours, 36 minutes ſidereal time, "I perceive three volcanoes in different places of the dark part of the new moon. Two of them are nearly extinct; or, otherwiſe, in a ſtate of going to break out, which perhaps may be decided next lunation. The third ſhows an actual eruption of fire, or luminous matter." On the next night, Dr. Herſchel ſaw the volcano burn with greater violence than on the preceding evening. He conſidered the eruption to reſemble a ſmall piece of burning charcoal when it is covered by a thin coat of white aſhes, which frequently adhere to it, when it has been ſome time ignited, and it had a degree of brightneſs about as ſtrong as that with which ſuch a coal would be ſeen to glow in faint day light. It is not yet determined whether there is an atmoſphere belonging to the moon. The queſtion is fully diſcuſſed in the Encyclo. Brit. Vol. II. p. 457, &c. See alſo Phil. Tranſ. for the year 1792.

SECT. II.—*Of* Eclipses.

Def. LV. An *Eclipſe of the Moon* happens when the earth, paſſing between the ſun and moon, caſts its ſhadow on the moon.

PROP. LXXXIV. The moon can only be eclipſed at the full, or in oppoſition.

For it is only when the moon is in oppoſition that it can come within the ſhadow of the earth, which muſt always be on that ſide of the earth which is from the ſun.

PROP. LXXXV. The moon can only be eclipſed when, at the full, it is in or near one of its nodes.

The earth being in the plane of the ecliptic, the centre of its ſhadow is always in that plane; if therefore the moon be in its nodes, that is, in the plane of the ecliptic, the ſhadow of the earth will fall upon it: alſo, ſince this ſhadow is of conſiderable breadth, it is partly above and partly below the plane of the ecliptic; if therefore the moon in oppoſition be ſo near one of its nodes, that its latitude is leſs than half the breadth of the ſhadow, it will be eclipſed. But, becauſe the plane of the moon's orbit makes an angle of more than 5 degrees with the plane of the ecliptic, it will frequently have too much latitude at its oppoſition to come within the ſhadow of the earth.

PROP. LXXXVI. The ſun is larger than the earth, and the ſhadow of the earth is a cone, the baſe of which is on the ſurface of the earth.

If the earth were larger than, or equal to, the ſun, it is manifeſt, that its ſhadow would either perpetually enlarge, or be always of the ſame dimenſion; but in this caſe, the ſuperior planets would ſometimes come within it, and be eclipſed, which never happens. Therefore the ſun is larger than the earth, and produces a ſhadow from the earth of a conical form, which does not extend to the orbit of Mars.

PROP. LXXXVII. The moon is eclipſed by a ſection of the earth's ſhadow.

Let CDE be the earth, CME the cone of its ſhadow, and FH a part of the moon's orbit paſſing through the ſhadow; it is manifeſt that as the moon deſcribes this part of its orbit, it paſſes through the circular ſection FGHL. Plate 10. Fig. 7.

DEF. LVI. The moon's *Horizontal Parallax*, is the angle which a ſemidiameter of the earth would ſubtend, if it were viewed directly from the moon.

LEMMA.

Half the angle of the cone of the earth's ſhadow may be taken as equal to the angle of the ſun's apparent ſemidiameter.

Let AFBG be the ſun, HED the earth, HMD or BMA the angle of the cone of the earth's ſhadow, and CMD half this angle. SA, a ſemidiameter of the ſun, drawn from its centre to the point of contact of the tangent AM, is perpendicular (El. III. 16.) to AM, and is therefore ſeen directly from D; and it ſubtends the angle SDA; it muſt therefore appear large in proportion to the magnitude of this angle. But in the triangle SDM, the external angle SDA is equal to the two angles CSD, CMD; of which, CSD, the angle in which the ſemidiameter of the earth is viewed from the ſun, is ſo ſmall, that without any ſenſible error it may be reckoned as nothing, and SDA be ſaid to be equal to SMD. Plate 10. Fig. 6.

PROP. LXXXVIII. The ſemidiameter of the ſection of the earth's ſhadow which eclipſes the moon, is equal to the difference between the horizontal parallax of the moon, and the ſun's apparent ſemidiameter.

CT, being a ſemidiameter of the earth drawn from the point of contact of the tangent CM, is perpendicular to CM. CT will therefore be ſeen directly, from the point F in the moon's orbit, ſubtending the angle CFT, which is (Def. LVI.) the moon's horizontal parallax. FMG is the ſemiangle of the cone of the earth's ſhadow, equal to the angle of the ſun's apparent ſemidiameter, becauſe MC produced would be a tangent to the circle of the ſun's diſc. FG is the ſemidiameter of the ſection FGHL of the earth's ſhadow through which the moon paſſes in an eclipſe; and FTG the angle which this ſemidiameter will ſubtend when it falls upon the moon, and is viewed from the earth. Now the angle CFT is equal to the two angles FMG, FTG, (El. I. 32.) conſequently, FTG is equal to Plate 10. Fig. 7.

CFT—FMG; but by the preceding Lemma, FMT may be taken for the angle of the ſun's apparent ſemidiameter; and CFT is the moon's horizontal parallax: whence the Propoſition is manifeſt.

Cor. Hence the ſection of the earth's ſhadow, by which the moon is eclipſed, is broader than the moon; for the ſemidiameter of the ſhadow, by this Prop. is $61' - 16' = 45'$. and the diameter of the moon is about $31'$.

Plate 10. Fig. 10. Def. LVII. An eclipſe of the moon is *partial*, when only a part of its diſc is within the ſhadow of the earth; it is *total*, when all its diſc is within the ſhadow; and it is *central*, when the centre of the earth's ſhadow falls upon the centre of the moon's diſc.

PROP. LXXXIX. The moon, at the full, will not be eclipſed, if its latitude is equal to, or greater than, the ſum of its own ſemidiameter, added to the ſemidiameter of the earth's ſhadow.

Plate 10. Fig. 10. Becauſe a circle or ellipſe appears as a right line when the eye is in the ſame plane with it, let OO repreſent the plane of the ecliptic, RR the plane of the moon's orbit, and N the node. At the full moon, if the earth's ſhadow be at A, the moon F muſt be in the ſame part of the heavens, becauſe it is in oppoſition. But becauſe only one half of the ſhadow of the earth, or about $45'$. is on the ſame ſide of the ecliptic with the moon, and only one half of the moon's breadth, or about $16'$. is on the ſide of its orbit towards the earth's ſhadow; if the centre of the moon is more than $61'$. from the centre of the ſhadow, the moon will paſs clear of the ſhadow, and will not be eclipſed.

Fig 9. Let GE be an arc of the moon's orbit, AB an arc of the ecliptic, and Cc an arc in a ſecondary of the ecliptic equal to the moon's latitude. If this arc be equal to, or greater than, Ct and cl, the ſum of the ſemidiameters of the earth's ſhadow, and of the moon, it is manifeſt that the ſhadow cannot paſs over any part of the moon's diſc.

PROP. XC. The moon, at the full, will be partially eclipſed, if its latitude is leſs than the ſum, but greater than the difference, of its own ſemidiameter and that of the earth's ſhadow.

Plate 10. Fig. 9. If Cc the latitude of the moon be ſuppoſed leſs than Ct, cl, the ſum of the ſemidiameters of the ſhadow and the moon, l the lower edge of the moon will be below t the upper edge of the ſhadow; whence the ſide of the moon towards the ecliptic will be eclipſed. But, becauſe Cc the moon's latitude, added to co its ſemidiameter, is greater than Ct the ſemidiameter of the earth's ſhadow, the upper edge of the moon o, cannot come within the ſhadow; whence the eclipſe will be partial. And becauſe in this caſe the moon's latitude, together with its ſemidiameter, is greater than the ſemidiameter of the ſhadow, the moon's latitude is greater than the difference of the ſemidiameters of the ſhadow and the moon: or, becauſe $Cc+co$ is greater than Ct, Cc is greater than $Ct-co$.

PROP. XCI. The moon will, at the full, be totally eclipſed, if its latitude is leſs than the difference between its own ſemidiameter, and that of the earth's ſhadow.

If $Cc+co$ be leſs than Ct, that is, if Cc be leſs than $Ct-cl$, the upper edge of the moon may come within the ſhadow of the earth. Fig. 9.

PROP. XCII. The moon is centrally eclipſed, only when, in oppoſition, it is in one of its nodes.

The node N, being the common interſection of the moon's orbit, and the plane of the ecliptic, is in both. Therefore when the moon is in the node, its centre is in the plane of the ecliptic, in which is the centre of the earth's ſhadow, and conſequently at the full theſe centres coincide.

Schol. 1. Both the moon and the ſhadow moving from eaſt to weſt, the moon would always be in eclipſe while it was at, or near, its nodes, if it moved with the ſame velocity as the earth: but becauſe it moves much faſter than the ſhadow of the earth, it ſoon paſſes from its oppoſition out of the ſhadow.

Schol. 2. The ſemiangle of the cone of the earth's ſhadow being known, the length of the ſhadow may be found.

The ſemiangle CMT being equal to the ſun's apparent ſemidiameter, or about 16′. and CT a ſemidiameter of the earth being perpendicular to the tangent CM, if TM be radius, CT is the ſine of the angle CMT. Therefore as the ſine of an angle of 16′. is to radius, ſo is CT to TM, the height of the cone; that is, as 1 to 217. Whence the length of the ſhadow is about 217 ſemidiameters of the earth. Plate 10. Fig. 7.

PROP. XCIII. The moon in a total eclipſe, is not wholly inviſible.

This is known by obſervation; and the phenomenon is produced by the reflection of rays of light falling upon the earth's atmoſphere, towards the ſhadow, and conſequently towards the moon in the ſhadow.

Def. LVIII. An *Eclipſe of the Sun* happens when the moon, paſſing between the ſun and the earth, intercepts the ſun's light.

PROP. XCIV. The ſun can only be eclipſed at the new moon.

For it is only when the moon is in conjunction, that it can paſs directly between the ſun and the earth.

PROP. XCV. The ſun can only be eclipſed, when the moon, at its conjunction, is in or near one of its nodes.

For unleſs the moon is in or near one of its nodes, it cannot appear in or near the ſame plane with the ſun; without which it cannot appear to us to paſs over the diſc of the ſun. At every other part of its orbit, it will have ſo much northern or ſouthern latitude, as to appear either above or below the ſun. If the moon be *in* one of its nodes, having no latitude, it will cover the whole diſc of the ſun, and produce a *total eclipſe*, except when its apparent diameter is leſs than that of the ſun: if it be *near* one of its nodes, having a ſmall degree of latitude, it will only paſs over a part of the ſun's diſc, or the eclipſe will be partial.

PROP. XCVI. In a total eclipſe of the ſun, the ſhadow of the moon falls upon that part of the earth where the eclipſe is ſeen.

Plate 10. Fig. 11. Let SL be the ſun, TR the moon, VM a part of the ſurface of the earth. A ſpectator placed any where between V and M, will not ſee any part of the ſun, becauſe the moon will intercept all the rays of light which come to him directly from the ſun; and it is manifeſt, that, in this ſituation, the moon, being an opaque body, will caſt its ſhadow upon VM, the part of the earth where the eclipſe is total.

PROP. XCVII. The ſhadow of the moon is a cone, terminated in a point.

Becauſe (by Prop. LXXXVIII. Cor.) the diameter of the moon is leſs than the diameter of the earth's ſhadow, where it eclipſes the moon, and this diameter (by Prop. LXXXVI.) is leſs than the diameter of the earth, the diameter of the moon is much leſs than that of the ſun: conſequently, its ſhadow will be a cone, terminated in a point. The tangents LAR, SAT, terminate in A; and only the rays that would paſs within theſe tangents are intercepted by the moon; therefore RTA is the form of the moon's ſhadow.

LEMMA. *Half the angle of the cone of the moon's ſhadow is equal to the angle of the apparent ſemidiameter of the ſun.*

Plate 10. Fig. 6. Let FBGA be the ſun; HED the moon; the cone HMD the moon's ſhadow: CMD the ſemiangle of this cone; SA the ſemidiameter of the ſun, and SDA the angle which this ſemidiameter would ſubtend, if viewed from D. It may be proved, as in Prop. LXXXVIII. *Lemma*, that CMD is equal to SDA; for the diſtance of the moon from the ſun is ſo nearly equal to that of the earth from the ſun, that the apparent ſemidiameter of the ſun, as ſeen from the earth or moon, may be conſidered as equal.

PROP. XCVIII. A ſemidiameter of the moon's ſhadow, where it falls upon the earth, is equal to the difference between the apparent ſemidiameters of the moon and ſun.

Plate 10. Fig. 7. Let CDE repreſent the moon, CME the cone of its ſhadow; FG the ſemidiameter of the moon's ſhadow where it falls upon the earth in a ſolar eclipſe; CT the ſemidiameter of the moon, CFT its angle viewed from F, and FTG the angle of the apparent ſemidiameter of the moon's ſhadow viewed from T.

In the triangle TFM, the external angle TFC (El. I. 32.) is equal to the two angles FTG, FMG. Therefore FTG is equal to the difference between TFC and FMG: and FMG is (by the preceding Lemma) equal to the angle of the ſun's apparent ſemidiameter, and TFC is the angle of the moon's apparent ſemidiameter: whence the Propoſition is manifeſt.

PROP. XCIX. In a partial eclipſe of the ſun, a *penumbra*, or imperfect ſhadow of the moon, falls upon that part of the earth where the partial eclipſe is ſeen.

A ſpectator at N, or P, might ſee the whole ſun; for a ray paſſing from the moſt remote ſide of the ſun, S or L, would not be intercepted by the moon. But at any point in NM, VP, the ſpaces between the moon's ſhadow and the points N, P, the ſpectator would only ſee a part of the ſun: thus at G, or D, he would ſee that half of the ſun which lies without the tangents DRC, GTC: conſequently, in all places between the points N, M, and P, V, there will be leſs light from the ſun, than if it were not at all eclipſed. This deficiency of light is called the moon's *penumbra*. Plate 10. Fig. 11.

PROP. C. The moon's penumbra is an increaſing cone, and its darkneſs increaſes towards the ſhadow of the moon.

While a ſpectator is in the ſpace between N and M, or P and V, he is in the penumbra; but at the points N, P, paſſes out of it. Therefore the tangents NS, PL, are the limits of the penumbra. If tangents be ſuppoſed drawn round the ſpherical ſurfaces of the ſun and moon, they will form two cones, having their common vertex at F, and increaſing, the one towards the points L, S, the other towards the points N, P. And as the ſpectator moves from N towards M, or from P towards V, a greater and greater portion of the ſun continually becomes inviſible to him; whence the penumbra increaſes in darkneſs towards M and V. Plate 10. Fig. 11.

PROP. CI. The ſemiangle of the moon's penumbra is equal to the angle of the ſun's apparent ſemidiameter.

Let SL be the ſun, and TR the moon. By Prop. C. the tangents SFN, LFP, terminate the cone of the penumbra. CE, drawn from the centre of the ſun to that of the moon, biſects TFR, the angle of this cone; whence EFT is its ſemiangle. LC being the ſemidiameter of the ſun, LTC is the angle under which this ſemidiameter would appear from the moon T. I ſay, TFE is equal to LTC. For, in the triangle TCF, the external angle EFT is equal to the two internal and oppoſite angles FTC, FCT, that is, to the two angles LTC, TCE. Therefore EFT is equal to LTC, TCE. But TCE, being the angle which the moon's ſemidiameter would ſubtend, if viewed from the ſun, is ſo ſmall that it may be neglected; whence EFT may be conſidered as equal to LTC. Plate 10. Fig. 11.

PROP. CII. The ſemidiameter of the moon's penumbra, in that part through which the earth paſſes in an eclipſe of the ſun, is equal to the ſum of the apparent ſemidiameters of the ſun and moon.

Plate 10. Fig. 12.

Let CD be the moon, CDAB its penumbra, and CMD the angle of the cone of the penumbra: and let AEBF be the ſection of the penumbra through which the earth paſſes in an eclipſe of the ſun, AB its diameter, and AT its ſemidiameter. MLT drawn from the vertex through the centre of the moon, will biſect the angle CMD. Therefore (by Prop. CI.) CML the ſemiangle of the penumbra is equal to the ſun's apparent ſemidiameter. And CL is the moon's apparent ſemidiameter, as ſeen from the earth A, ſubtending the angle CAL; and AT the ſemidiameter of the penumbra, ſeen from L, ſubtends the angle ALT. Now the angle ALT is equal to the two angles CML, CAL; whence the truth of the Propoſition is manifeſt.

Def. LIX. The diſc of the earth, is that hemiſphere of the earth, which is ſeen, as a circle, from the moon.

PROP. CIII. At the new moon the whole diſc of the earth is enlightened.

For, the moon being then between the ſun and the earth, the earth viewed from the moon, will appear in oppoſition, and conſequently its enlightened hemiſphere will be towards the moon.

PROP. CIV. The ſemidiameter of the earth's diſc, is equal to the moon's horizontal parallax.

The moon's horizontal parallax is the apparent ſemidiameter of the earth, as viewed from the moon, that is, it is equal to the ſemidiameter of the diſc, ſince the diſc is a hemiſphere of the earth viewed from the moon.

PROP. CV. If the latitude of the moon, when new, is equal to, or greater than, the ſum of the ſemidiameter of the penumbra and the moon's horizontal parallax, there will be no eclipſe of the ſun; if leſs, there will be an eclipſe, either partial, or total.

Plate 10. Fig. 9.

Let ACB be a part of the ecliptic; let GNE be the plane of the moon's orbit; let N be the node, DL the earth's diſc, and *o l* a ſection of the moon's penumbra. Becauſe the centre of the moon's penumbra is always in a right line paſſing through the centres of the ſun and the moon, the diſtance of *c*, the centre of the moon's penumbra, from the plane of the ecliptic muſt always be the ſame with the diſtance of the centre of the moon from the ſame plane, that is, with the latitude of the moon. And becauſe the centre of the earth, or the earth's diſc C, is in the ecliptic, C*c* the diſtance between the centres of the penumbra and of the earth's diſc, is always equal to the latitude of the moon. Now, if C*c*, or the latitude of the moon, be equal to, or greater than, *c l* the ſemidiameter of the penumbra, together with *t*C the ſemidiameter of the earth's diſc, or the moon's horizontal parallax, then no part of the penumbra will fall upon the diſc, that is, there will be no eclipſe. If C*c* the latitude of the moon be leſs than $tC + lc$, the edge of the penumbra will be nearer the ecliptic than the edge of the diſc, and there will be a partial eclipſe. And if Cc be leſs than Ct, the ſhadow of the moon will paſs over ſome part of the diſc of the earth, and where this happens, the eclipſe will be total,

PROP. CVI. If the moon, when new, is in one of its nodes, the eclipſe of the ſun will be central.

For then the centres of the earth, ſun, and moon, being all in the plane of the ecliptic, the centre of the moon will paſs between the ſun's centre, and that of the earth.

Schol. 1. The penumbra of the moon in a central eclipſe, will not cover the whole diſc of the earth.

The ſemidiameter of the moon's penumbra, being equal to the ſum of the apparent ſemidiameters of the ſun and moon, that is, about 16′. 23″. + 15′. 37″. or 32′. at the medium; its diameter is about 64′.; whereas the diameter of the earth's diſc is about 120′. whence the penumbra cannot cover the whole diſc.

Schol. 2. The height of the ſhadow of the moon is about 60½ ſemidiameters of the earth. The ſemiangles of the earth's ſhadow, and of the moon's ſhadow, being each equal to the ſun's apparent ſemidiameter, the angles are equal to one another, and theſe cones are ſimilar. Therefore as the ſemidiameter of the baſe of the earth's ſhadow (that is, of the earth) is to the ſemidiameter of the baſe of the moon's ſhadow, (that is, of the moon) ſo is the height of the earth's ſhadow to the height of the moon's ſhadow. Now the ſemidiameter of the earth is to that of the moon nearly as 100 to 28, and the height of the earth's ſhadow is about 217 ſemidiameters of the earth: whence the height of the moon's ſhadow is equal to about 60½ ſemidiameters of the earth; for 100 : 28 : : 217 : 60½ nearly.

Def. LX. An eclipſe of the ſun is ſaid to be *annular*, when at the time of the eclipſe, a ring of the ſun appears round the edges of the moon.

PROP. CVII. A central eclipſe of the ſun will be an annular one, if the diſtance of the moon from the earth at the time of the eclipſe be greater than its mean diſtance,

SL being the ſun, TR the moon, TAR the moon's ſhadow, and EA the height of this ſhadow, which is about 60½ ſemidiameters of the earth; if at the time of a central eclipſe PN is a part of the ſurface of the earth, thoſe who live in the parts PV, MN, being in the penumbra, will (by Prop. XCIX.) ſee a partial eclipſe; and thoſe who live between V and M, being in the ſhadow, will (by Prop. XCVI.) ſee a total eclipſe. But if the diſtance of the moon from the earth be equal to EA, or 60½ ſemidiameters of the earth, (which is the moon's mean diſtance) A will be the only point from which the eclipſe will appear total. And if the moon's diſtance be greater than EA, as EO, the ſhadow not reaching the earth, there will be no total eclipſe. Conſequently, though a ſpectator at O would ſee a central eclipſe, (becauſe the centres C, E, are in the ſame line with the point of viſion S) yet the eclipſe would not be total, Plate 10. Fig. 11.

total, becauſe the ſpectator is not in the ſhadow of the moon. Hence it muſt appear annular: for let ORX be a tangent to the moon drawn from the eye at O; and it will fall upon the ſun at X, and the part XL of the ſun will be viſible: in like manner parts of the ſun equal to XL will be viſible all round the moon, forming a ring.

Cor. Hence it appears, that in an annular eclipſe it is the penumbra of the moon which falls upon the earth.

Def. LXI. The *Lunar Ecliptic Limit*, is the leaſt diſtance that the moon can be at from one of its nodes, without being eclipſed at the time of oppoſition: the *Solar Ecliptic Limit*, is the leaſt diſtance the moon can be at from one of its nodes, without eclipſing the ſun at the time of conjunction.

PROP. CVIII. The ſolar ecliptic limit is greater than the lunar.

Plate 10. Fig. 9.

By Prop. CV. it is found, that 92′. is the leaſt latitude the moon can have, when new, without eclipſing the ſun. If, therefore, N*c* be the diſtance of the moon from the node, when its latitude, or *c*C, is 92′. in the triangle C*c*N, the angle *c*CN being a right angle, becauſe *c*C is perpendicular to the plane of the ecliptic; the angle *c*NC being about 5°. 30′. the inclination of the moon's orbit to the plane of the ecliptic; and the ſide *c*C being 92′. N*c* will be found by trigonometry to be about 16°. In the ſame manner, ſuppoſing C*c* the latitude of the moon to be 61′. according to Prop. LXXXIX. the length of the ſide N*c*, or the lunar ecliptic limit, will be found to be about 12°. Whence the truth of the Propoſition is manifeſt.

Cor. There are more eclipſes of the ſun, in a courſe of years, than of the moon; for, the ſun will be eclipſed, if, when the moon is new, it is within 16°. of one of the nodes, but the moon only when at the full it is within 12°. of one of the nodes; the ſun may be eclipſed while the moon is in 64 degrees of its orbit, but the moon only while it is in 48 degrees of its orbit.

Schol. Every eclipſe of the moon will be viſible, if the moon be above the horizon at the time of the eclipſe; becauſe that part of the moon on which the ſhadow of the earth falls, muſt appear obſcured wherever the diſc of the moon is viſible. But the ſun may be eclipſed, and yet the eclipſe be inviſible in places to which the ſun is above the horizon, becauſe there can be no eclipſe of the ſun, except in thoſe parts of the earth which are within the ſhadow or penumbra of the moon, and neither of theſe are large enough to cover the whole diſc. Hence, in any given place, more eclipſes of the moon than of the ſun will be ſeen in a courſe of years: for though there are more eclipſes of the ſun than of the moon, many of the former are not viſible at any one place while the ſun is above the horizon; but all the latter are viſible at the ſame place while the moon is above the horizon.*

PROP. CIX. When the moon is near the firſt of Aries, and is moving towards the tropic of Cancer, the time of its riſing will vary but little for ſeveral days together.

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* For the calculation and projection of lunar and ſolar eclipſes, ſee the Problems immediately preceding the Tables at the end of this book.

If the moon were to move in the equator, its motion in its orbit, by which it deſcribes a revolution, in reſpect of the ſun, in 29 days 12 hours, would carry it every day eaſtward from the ſun about 12° 11′. whence its time of riſing would vary daily about 50 minutes. But, becauſe the moon's orbit is oblique to the equator, nearly coinciding with the ecliptic, different parts of it make different angles with the horizon, as they riſe or ſet, thoſe parts which riſe with the ſmalleſt angles ſetting with the greateſt, and the reverſe. Now the leſs this angle is, the greater portion of the orbit riſes, in the ſame time. Conſequently, when the moon is in thoſe parts which riſe or ſet with the ſmalleſt angles, it riſes or ſets with the leaſt difference of time, and the reverſe. But in northern latitudes, the ſmalleſt angle of the ecliptic and horizon is made when Aries riſes and Libra ſets, and the greateſt when Libra riſes and Aries ſets; and therefore, when the moon riſes in Aries, it riſes with the leaſt difference of time. Now the moon is in conjunction in or near Aries, when the ſun is in or near Libra, that is, in the autumnal months; when, the moon riſing in Aries, whilſt the ſun is ſetting in Libra, the time of its riſing is obſerved to vary only two hours in 6 days in the latitude of London. This is called the harveſt moon.

Schol. This circumſtance takes place every month; but as it does not happen at the time of full moon, there is no notice taken of it. When the moon's right aſcenſion is equal to ſix ſigns, that is, when ſhe is in or about the beginning of Libra, there is the greateſt difference of the times of riſing, viz. about an hour and 15 minutes. Thoſe ſigns which riſe with the leaſt angle ſet with the greateſt, and the contrary; therefore, when there is the leaſt difference in the times of riſing, there is the greateſt in ſetting, and vice verſa.

The following table ſhews the daily mean difference of the moon's riſing and ſetting, on the parallel of London, for 28 days; in which time the moon finiſhes her period round the ecliptic, and gets 9 degrees into the ſame ſign from the beginning of which ſhe ſet out.

Days.	Signs.	Degrees.	Riſing Diff.		Setting Diff.		Days.	Signs.	Degrees.	Riſing Diff.		Setting Diff.	
			H.	M.	H.	M.				H.	M.	H.	M.
1	♋	13	1	5	0	50	15	♑	17	0	46	1	5
2		26	1	10	0	43	16	♒	1	0	40	1	8
3	♌	10	1	14	0	37	17		14	0	35	1	12
4		23	1	17	0	32	18		27	0	30	1	15
5	♍	6	1	16	0	28	19	♓	10	0	25	1	16
6		19	1	15	0	24	20		23	0	20	1	17
7	♎	2	1	15	0	20	21	♈	7	0	17	1	16
8		15	1	15	0	18	22		20	0	17	1	15
9		28	1	15	0	17	23	♉	3	0	20	1	15
10	♏	12	1	15	0	22	24		16	0	24	1	15
11		26	1	14	0	30	25		29	0	30	1	14
12	♐	8	1	13	0	39	26	♊	13	0	40	1	13
13		21	1	10	0	47	27		26	0	56	1	7
14	♑	4	1	4	0	56	28	♋	9	1	00	1	58

EXP. Let ſmall patches be placed on the ecliptic of a globe, as far from one another as the moon moves from any point of the ecliptic in 24 hours, that is, about $13\frac{1}{2}$ degrees; then, while the globe is turned round, obſerve the riſing and ſetting of the patches in the horizon; the hour index will ſhew the difference of time at which the moon riſes or ſets in different parts of its orbit.

CHAP. VI.

Of the SATELLITES *of* JUPITER, SATURN, *and the* HERSCHEL PLANET.

PROP. CX. Any ſatellite is at its greateſt elongation from its primary, when a line drawn from the earth through the ſatellite, is a tangent to the orbit of the ſatellite.

Plate 10. Fig. 14. Let FIE be a part of the orbit of the primary planet AXBT the earth's orbit, S the ſun, KGNL the orbit of a ſatellite. If the earth is at X, and the ſatellite at L or N, ſo that a line XL or XN drawn from the earth, is a tangent to the orbit KGNL, it may be ſhewn, as before concerning the planets, that L or N is the greateſt elongation of the ſatellite.

PROP. CXI. Any ſatellite appears in inferior conjunction with its primary, when the ſatellite is between the earth and the primary, and in ſuperior conjunction, when the primary is between the ſatellite and the earth.

If the earth is at X, and the planet at I, the outermoſt ſatellite will be in conjunction with its primary when they both appear in the ſame line MIV, and in its inferior conjunction at M, and its ſuperior at V.

PROP. CXII. The apparent motion of any ſatellite, as it paſſes from its greateſt elongation on one ſide of its primary through the ſuperior conjunction, to its greateſt elongation on the other ſide, is direct.

As the ſatellite paſſes from N, its greateſt elongation on one ſide, through V its ſuperior conjunction to L, its geocentric motion is from weſt to eaſt, or *in conſequentia,* as was ſhewn concerning the planets.

PROP. CXIII. The apparent motion of any ſatellite, as it paſſes from its greateſt elongation on one ſide of its primary through the inferior conjunction, to its greateſt elongation on the other ſide, is retrograde.

As the ſatellite paſſes from L through M to N, its geocentric motion will be from eaſt to weſt, or *in antecedentia,* as was proved concerning the planets.

COR. The ſatellites are ſometimes to the weſt and ſometimes to the eaſt of their primaries.

PROP. CXIV. The greateſt elongations of a ſatellite on each ſide are equal.

For by obſervation it is found, that the angles LXI, NXI, are equal with reſpect to all the ſatellites.

COR. Hence it appears that the orbits of the ſatellites are circular, or nearly ſo, having their primaries at the centre of their orbits.

SCHOL. The ſatellites move round the primary in orbits, that are nearly circular; but round the ſun in curves of a different kind. Let ABCDE, &c. to T, (Plate 15. Fig. 1.) be as much of Jupiter's orbit as is deſcribed by that planet in 18 days. Then the curves *a*, *b*, *c*, *d*, (according to Ferguſon) repreſent the paths of the four ſatellites. Plate 15. Fig. 1.

If we ſuppoſe Jupiter to move from A, the firſt ſatellite from *a*, the ſecond from *b*, the third from *c*, and the fourth from *d*; or each of the ſatellites from the point of conjunction with the ſun, as ſeen from the primary; then, at the end of one day, Jupiter will be at B, and the ſatellites at 1, in their reſpective courſes. At the end of the ſecond day Jupiter will be at C, and the ſatellites at 2 in the curves they reſpectively deſcribe, and ſo on; the capital letters ſhewing Jupiter's place in its path at the end of each day, the figure under each of them the number of the day, and the like figures on the paths of the ſatellites, their places at the ſame time. The firſt ſatellite appears to be ſtationary at + near C, as ſeen from the ſun; retrograde from + to 2, at 2 ſtationary again; thence direct till beyond 3; twice ſtationary and once retrograde between 3 and 4. This ſatellite interſects its own path every 42½ hours, making loops as in the diagram at 2, 3, 5, &c. ſoon after every conjunction. The ſecond croſſes its own path every 3 days 13 hours, as at 4, 7, 11, &c. making only 5 loops, and as many conjunctions while the firſt makes 10. The third ſatellite at the end of every 7 days 4 hours, makes an angle in conjunction with the ſun, as at 7, 14. The fourth ſatellite is always progreſſive, making neither loops nor angles; its firſt conjunction is at *e*, at the end of 16 days 18 hours. The firſt, ſecond and third are, according to the figure, nearly in the ſame relative ſituation, every ſeventh day.

The path of Saturn's firſt ſatellite about the ſun is looped, but not thoſe of the ſecond, third, fourth and fifth.

PROP. CXV. The ſatellites of Jupiter, Saturn, and the Herſchel, are eclipſed by their reſpective primaries.

The planet I being an opaque body, caſts a ſhadow IV, oppoſite to the ſun. Therefore, when one of its ſatellites in deſcribing the arc HOP comes to H, it will be eclipſed by falling into this ſhadow. If the earth is at B in its orbit, a ſpectator from the earth will loſe ſight of the ſatellite, when it is thus eclipſed at V; and then, as it emerges from the ſhadow, it becomes again viſible, till, at P, it paſſes behind its primary. If the earth be at X, the ſatellite will be eclipſed, and in occultation at the ſame time. Plate 10. Fig. 14.

PROP. CXVI. When one of the ſatellites paſſes between the ſun and its primary, it eclipſes the ſun.

A ſatellite at M will be between the ſun and its primary, and occaſion an eclipſe of the ſun on that part of the primary where the ſhadow of the ſatellite paſſes, which ſhadow will appear as a dark ſpot on the diſc of the planet to an inhabitant of the earth.

SCHOL. Dr. Herſchel, about ſeven years ago diſcovered, that the fifth ſatellite of Saturn is, in its rotation, ſubject to the ſame law that our moon obeys, that is, it turns round its axis in the ſame time in which it revolves about the planet. Hence, the doctor thought it natural to conclude, that all the ſecondary planets might be governed by the ſame laws to which thoſe are ſubject. This theory he thinks conſiderably confirmed by certain obſervations which he made on the ſatellites of Jupiter, and which he communicated to the Royal Society, June 1, 1797.

The following table will give the periodical times and diſtances of Jupiter's ſatellites, and the angles under which their orbits are ſeen from the earth, at its mean diſtance from Jupiter.

Satellites.	Days.	h.	min.	Diſt. in miles.	Angles of orbit.	
1	1	18	26.6	266.000	3′	55″
2	3	18	17.9	423.000	6	14
3	7	3	59.6	676.000	9	58
4	16	18	5.1	1.189.000	17	30

Theſe ſatellites of Jupiter are of great uſe in aſtronomy. (1.) In determining the diſtance of Jupiter from the earth. (2.) They afford a method of demonſtrating that the motion of light is progreſſive, and not inſtantaneous, as was once ſuppoſed. See Prop. CXVII. And (3.) The moſt conſiderable advantage is derived from the eclipſes of the ſatellites of Jupiter, in aſcertaining the longitude of different places on the earth.

EXP. Suppoſe two obſervers of an eclipſe, the one at London, the other at the Cape of Good Hope, the eclipſe will appear to both at the ſame moment of time; but being ſituated under different meridians, they count different hours, according to which their difference of longitude is found. Thus, if at the Cape of Good Hope, an emerſion of a ſatellite is obſerved at 10h. 46′ 45″ apparent time, and the ſame is ſeen at Greenwich at 9h. 33′ 12″, the difference of which times is 1h. 13′ 33″, the longitude of the cape *eaſt* of Greenwich in time, or 18° 23′ 15″.

Note. The third ſatellite is the largeſt of all: the firſt and fourth are nearly of the ſame ſize; the ſecond is the ſmalleſt.

Of the SATELLITES of SATURN.

Satellites.	Per. times.				Diſtance in miles.
1	1 day.	21 h.	18′	27″	170.000
2	2	17	41	22	217.000
3	4	12	25	12	303.000
4	15	22	41	13	704.000
5	79	7	48	—	2.050.000
6	1	8	53	8	135.000
7	—	22	37	22	107.000

Plate 15. Fig. 3.

Fig. 3. Plate 15, is a view of the proportional magnitudes of the orbits of the moon, Jupiters four ſatellites, and the five firſt of Saturn. M*m* repreſents the moon's orbit, the earth being ſuppoſed to be at E; J Jupiter, 1, 2, 3, 4, the orbits of the four ſatellites; Sat. Saturn, 1, 2, 3, 4, 5, the orbits of the five firſt ſatellites.

The 6th and 7th ſatellites were diſcovered by Dr. Herſchel, in the years 1787 and 1788. To prevent miſtakes, he calls them the 6th and 7th, though nearer to the planet than the other five. Dr. Herſchel obſerves, that Saturn has probably a conſiderable atmoſphere. It turns on an axis perpendicular to the ring, in 10 h. 16′ 0.44″. and is flattened at the poles, ſo that the equatorial diameter is to the polar as 11 to 10. Phil. Tranſ. Vol. 80, Part I. and II. and Vol. 84.

Of the SATELLITES of the HERSCHEL.

Dr. Herſchel has at different times diſcovered ſix ſatellites belonging to the new planet, two of which he deſcribed in the 77th and 78th Vols. of the Phil. Tranſ. where he ſays, "I confeſs that this ſcene appeared to me with additional beauty, as the little ſecondary "planets ſeemed to give a dignity to the primary one, which raiſes it into a more conſpic-"uous ſituation among the great bodies in our ſyſtem."

The following is the arrangement of the ſix ſatellites.

Satellites.		When diſcovered.		Periodical times.			
1	—	Jan. 18, 1790	—	5 d.	21 h.	25 m.	0″
2	—	Jan. 11, 1787	—	8	17	1	19
3	—	Mar. 26, 1794	—	10	23	4	0
4	—	Jan. 11, 1787	—	13	11	5	1½
5	—	Feb. 9, 1790	—	38	1	49	0
6	—	Feb. 28, 1794	—	107	16	40	0

"It will be hardly neceſſary," ſays Dr. Herſchel, "to add, that the accuracy of theſe pe-"riods depends entirely upon the truth of the aſſumed diſtances; ſome conſiderable differ-"ence, therefore, may be expected, when obſervations ſhall furniſh us with proper *data* for "more accurate determinations." See Phil. Tranſ. 1798.

PROP. CXVII. A ray of light is about 8 minutes in coming from the ſun to the earth.

Plate 10. Fig. 13.

Let A be the ſun, BECD the earth's orbit, F the planet Jupiter, and HNG the orbit of its inner ſatellite. Let FGH repreſent the ſhadow of Jupiter. While the ſatellite is between H and G, it is eclipſed; when it comes to H, it emerges, and becomes viſible to a ſpectator at B. From comparing the times of the apparent entrance and emerſion of the ſatellite, with tables calculated for the mean diſtances of the earth from the ſatellite, the viſible emerſion at the leaſt diſtance is found to happen about 8 minutes ſooner, and at the greateſt diſtance about 8 minutes later than by the tables: conſequently, the ray of light is about 16 minutes in paſſing through the earth's orbit, or 8 minutes in coming from the ſun to the earth.

COR. The diameter of the earth's orbit being 194,000,000 miles, the velocity of light will be $\frac{194,000,000}{16 \times 60} = 202,088$ miles in a ſecond.

PROP. CXVIII. Jupiter is ſurrounded by cloudy ſubſtances, ſubject to frequent changes in their ſituation and appearance, called his Belts. Saturn is encompaſſed with a Ring, whoſe greateſt apparent diameter, is to that of the planet as 9 to 4.

Plate 10. Fig. 15. Theſe are known from obſervation. The Belts of Jupiter are ſometimes of a regular form; ſometimes interrupted and broken; and ſometimes not at all to be ſeen. The plane of Saturn's ring is inclined to the plane of the ecliptic at an angle of 31 degrees; which appears like two arms to the planet, and which is only viſible when the ſun and the earth are both on the ſame ſide of its plane. On account of its inclination, it always appears oblique to the eye, and therefore elliptical: whence the part behind Saturn is inviſible, and the part before cannot be diſtinguiſhed from the planet. The ring being opaque, can only be viſible when the ſun's rays are reflected from its broad ſurface to the earth, that is, when the ſun and the earth are both on the ſame ſide of the plane of the ring.

The latter diſcoveries of Dr. Herſchel have ſhewn, that what was ſuppoſed to be a ſingle broad flat ring of Saturn, is divided into two parts, lying exactly in the ſame plane, and revolving about an axis perpendicular to that plane, in 10 h. 32′ 15″. The dimenſions of theſe concentric rings, and the ſpace between them, he ſtates to be as in the following table.

	Miles.
Inner diameter of the ſmaller ring - -	146,345
Outſide diameter of ditto - - -	184,393
Inner diameter of larger ring - - -	190,248
Outſide diameter of ditto - - - -	204,883
Breadth of the inner ring - - - -	20,000
Breadth of the outer ring - - - -	7,200
Breadth of the vacant ſpace - - - -	2,830

CHAP. VII.

Of COMETS.

PROP. CXIX. Comets are opaque and ſolid bodies.

A comet, at a given diſtance from the earth, ſhines much brighter when it is on the ſame ſide of the earth with the ſun, than when it is on the contrary ſide; from whence it appears that it owes its brightneſs to the ſun. The reſiſtance of the comet of 1680 to the action of the great heat, to which it was probably expoſed in its near approach to the ſun, furniſhes evidence in favour of its being a fixed and ſolid body.

PROP. CXX. The comets deſcribe very eccentric ellipſes about the ſun, placed in one of their *foci.*

They are obſerved to approach towards, and afterwards recede from, the ſun, and to deſcribe paths in the heavens, which agree with elliptic orbits; it is therefore moſt probable, that agreeably to the general analogy of nature, they move in ſuch orbits, and have the ſun in one of the *foci* of the ellipſe. The calculations framed upon this ſuppoſition, by which the returns of comets have been foretold, having, as far as obſervations have been made, been found to agree with the phenomena, ſtrongly confirm the truth of the Propoſition.

SCHOL. 1. Comets are often accompanied with a luminous train, called the tail, which is conjectured to be ſmoke riſing from the body in a line oppoſite to the ſun. The body of the comet is ſuppoſed to be ſurrounded by an atmoſphere; the ſun is alſo ſuppoſed to be ſurrounded by an ether, or a ſubtle fluid, extending to a great diſtance from the ſun, which may be conſidered as the ſolar atmoſphere. From the heat which the comet has acquired by approaching towards the ſun, and by the reflection of the ſun's rays from the ſolid body and atmoſphere of the comet, the parts of the ſolar atmoſphere where the comet paſſes are more heated, and conſequently more rarefied or ſpecifically lighter than elſewhere. The parts thus rarefied will be put into motion; and ſince there will be a conſtant ſucceſſion of freſh portions of the ſun's atmoſphere within that of the comet, there will be a perpetual ſtream of this rarefied matter. This ſtream will impel the particles of the comet's atmoſphere, and make them move along with it, thus producing the ſmoke which, reflecting the ſun's rays, forms the viſible tail. And this ſtream of rarefied ſolar atmoſphere will move thoſe parts of this atmoſphere which have the leaſt ſpecific gravity, that is, directly from the ſun.

SCHOL. 2. Of all the comets, the periods of only three are known with any degree of certainty. The firſt of theſe comets appeared in the years 1531, 1607, and 1682; and is expected to appear every 75th year. The ſecond of them appeared in 1532 and 1661, and was expected to return in 1789, and every 129th year afterwards. The third, having laſt appeared in 1680, and its period being no leſs than 575 years, cannot return until the year 2225. This comet, at its greateſt diſtance, is about 11 thouſand two hundred millions of miles from the ſun; and at its leaſt diſtance from the ſun's centre, which is 49,000 miles, is within leſs than a third part of the ſun's ſemidiameter from his ſurface. In that part of its orbit which is neareſt the ſun, it moves at the rate of 880,000 miles in an hour.

SCHOL. 3. Dr. Halley, who ſaw the comet which appeared in 1682, obſerves, "that there are many things which make me believe, that the comet which APIAN ſaw in the year 1531, was the ſame with that which KEPLER and LONGOMONTANUS more accurately deſcribed in the year 1607; and which I myſelf have ſeen return, and obſerved in the year 1682. All the elements agree, and nothing ſeems to contradict this opinion beſides the inequality of the periodic revolutions; which inequality is not ſo great, but that it may be owing to phyſical cauſes. For the motion of Saturn is ſo diſturbed by the reſt of the planets, eſpecially Jupiter, that the periodic time of that planet is uncertain for ſome whole days together. How much more, therefore, will a comet be ſubject to ſuch like errors, which riſes almoſt four times higher than Saturn, and whoſe velocity, though increaſed but very little, would be ſufficient to change its orbit from an ellipſe to a parabola. And I am the more confirmed in my opinion of its being the ſame; for, in the year 1456, in the ſummer-time, a comet was ſeen paſſing retrograde between the earth and ſun, much after the ſame manner; which, though nobody made obſervations upon it, yet, from its period, and manner of tranſit, I cannot think

different

different from thoſe I have juſt now mentioned. And ſince looking over the hiſtory of comets, I find, at an equal interval of time, a comet to have been ſeen about Eaſter in the year 1305, which is another double period of 151 years before the former. Hence I think I may venture to foretel that it will return again in the year 1758."

Dr. Halley computed the effect of Jupiter upon this comet in 1682, and found that it would increaſe its periodic time above a year; in conſequence of which, he predicted its return at the end of the year 1758, or the beginning of 1759. M. Clairaut computed the effects of both Saturn and Jupiter, and found that the former would retard its return in the laſt period 100 days, and the latter 511 days; and he determined the time when the comet would come to its perihelion to be on April 15, 1759; obſerving, that he might err a month, from neglecting ſmall quantities in the computation. The comet did paſs the perihelion on March 13, within 33 days of the time computed. Now, if Dr. Halley meant the time of its paſſing the perihelion, and we add 100 days for the action of Saturn, which he did not conſider, it will bring it very near to the time in which it paſſed the perihelion, and prove his computation of the effect of Jupiter to have been very accurate. But if he meant the time when the comet would firſt appear, his prediction was accurate, for it was firſt ſeen on December 14, 1758. Dr. Halley, therefore, had the glory, firſt to foretel the return of a comet, and the event anſwered, in a remarkable manner, his prediction. He farther obſerved, that the action of Jupiter, in the deſcent of the comet towards its perihelion in 1682, would tend to increaſe the inclination of its orbit; and accordingly the inclination in 1682 was found to be 22′ greater than in 1607.

Dr. Halley ſuſpected the comet in 1680, to have been the ſame which appeared in 1106, 531, and 44 years before Chriſt. He alſo conjectured, that the comet obſerved by Apian, in 1532, was the ſame as that obſerved by Hevelius in 1661; if ſo, its period was 129 years, and it ought to have returned in 1789, but it did not appear. M. Mechain having collected all the obſervations in 1532, and calculated the orbit again, found that it differed materially from that calculated by Dr. Halley, which renders it extremely doubtful whether this was the comet which appeared in 1661; and this doubt is increaſed by its not appearing in 1789.

Schol. 4. From the beginning of our era to this time, it is probable, according to the beſt accounts, that there have appeared about 500 comets. Before that time above 100 others are recorded to have been ſeen, but it is probable that not above half of them were comets. And when we conſider that many others may not have been perceived, from being too near the ſun;—from appearing in moon light;—from being in the other hemiſphere;—from being too ſmall to be perceived; or which may not have been recorded, we might imagine the whole number to be conſiderably greater; it is, however, highly probable, that of the comets which are recorded to have been ſeen, the ſame may have appeared ſeveral times, and therefore the number may be leſs than is ſtated. Miſs Caroline Herſchel, the ſiſter of Dr. Herſchel, has diſcovered ſeveral comets within the laſt 15 years, accounts of which are in the different volumes of the Philoſophical Tranſactions.

On the ſubject of Comets, ſee Mr. Vince's very excellent "*Complete Syſtem of Aſtronomy*," Vol. I. Quarto.

CHAP. VIII.

Of the Sun.

PROP. CXXI. The ſpots which appear upon the ſun's diſc, adhere to its ſurface.

If one of theſe ſpots appears upon the eaſtern limb or edge of the ſun's diſc, it moves from thence towards the weſtern edge, and arrives at the weſtern edge in about 13½ days. Here the ſpot diſappears; and in about 13½ days more, it is ſeen again upon the eaſtern edge; and ſo continues to go round completing its apparent revolution in 27 days; during one half of which time we ſee it on the diſc of the ſun, and during the other half it diſappears; which could not happen, if the ſpots did not adhere to the ſurface of the ſun. Let A be the centre of the ſun's diſc, C its eaſtern, and D its weſtern edge; HEG the orbit of an opaque body moving round it, and B the eye of the ſpectator at the earth. If two lines BD and BC are ſuppoſed to be drawn from the ſpectator's eye B, ſo as to touch the ſun at D and C, then DBC, the angle contained between theſe lines, is the angle under which the ſun's diameter appears to a ſpectator on the earth. EG is the only part of the ſuppoſed body's orbit that is within this angle DBC, and conſequently, if the body was in any other part of its orbit, except EG, it would not appear upon the ſun's diſc. But EG is leſs than half its orbit; and the body would not take up half the time of a revolution to deſcribe EG. Therefore ſuch a body would not be ſeen upon the ſun's diſc, as the ſpots are for half the time of a revolution. But if the orbit HEG is not greater than LDFC, or is cloſe to the ſun, that is, if the ſpot adheres to the ſun's ſurface, then half its orbit DFC, will be within the angle DBC, and conſequently, the ſpot will appear upon the ſun's diſc during one half of its revolution; but during the other half of its revolution, whilſt it deſcribes CLD, it will diſappear, becauſe then it will be behind the ſun, and ſo will be concealed from the earth, which agrees with the phenomena. Plate 10. Fig. 16.

PROP. CXXII. The ſun is a ſpherical body, which revolves upon its axis from weſt to eaſt.

The ſpots which appear in the ſun's diſc, adhere to its ſurface, (by Prop. CXXI.) and theſe ſpots revolve; therefore the ſun revolves round its axis.

Whatever ſide of the ſun is turned towards the earth in this rotation, it always appears to be a flat bright circle: but all the ſides of it could not appear in this manner unleſs it was a ſphere: therefore the ſun is a ſpherical body.

Schol. A real revolution of a ſpot, and conſequently of the ſun round its axis, is completed in 25 days, two days leſs than its apparent revolution, in conſequence of the earth's motion in its orbit in the ſame direction in which the ſpot moves.

PROP. CXXIII. The axis of the ſun is inclined to the plane of the ecliptic, Each ſpot upon the ſun muſt deſcribe a circle round the ſun, either coinciding with its equator, or parallel to it. If therefore the ſun's axis were perpendicular to the plane of the ecliptic, the plane of the ſun's equator would be in the plane of the ecliptic; and a ſpectator on the earth, whoſe eye is in the ecliptic, would ſee the ſpots deſcribing right lines, either in the ſun's equator, or parallel to it: but the ſpots are ſometimes ſeen to deſcribe lines oblique to the plane of the ecliptic: therefore the axis of the ſun is inclined to the plane of the ecliptic. This inclination is obſerved to be an angle of about 82½ degrees. When the ſun's equator croſſes the plane of the ecliptic, the ſpots appear to deſcribe right lines parallel to the ſun's equator.

Schol. The following particulars reſpecting the ſun are related by Sir I. Newton.

1. That the denſity of the ſun's heat, which is proportional to his light, is 7 times as great in Mercury as with us, and that water there would be all carried off in the ſhape of ſteam; for, he found, by experiments with the thermometer, that a heat 7 times greater than that of the ſun's beams in ſummer will ſerve to make water boil.

2. That the quantity of matter in the ſun is to that in Jupiter nearly as 1100 to 1, and that the diſtance of that planet from the ſun is in the ſame ratio to the ſun's ſemidiameter; conſequently, that the centre of gravity of the ſun and Jupiter is nearly in the ſuperficies of the ſun.

3. That the quantity of matter in the ſun is to that in Saturn as 2360 to 1, and that the diſtance of Saturn from the ſun is in a ratio but little leſs, than that of the ſun's ſemidiameter. And hence the common centre of gravity of Saturn and the ſun is a little within the ſun.

4. By the ſame method of calculation it will be found, that the common centre of gravity of all the planets cannot be more than the length of the ſolar diameter diſtant from the centre of the ſun.

5. The ſun's diameter is equal to 100 diameters of the earth, and therefore its magnitude muſt exceed that of the earth one million of times.

6. If 360 degrees (the whole ecliptic) be divided by the quantity of the ſolar year, it will give 59′. 8″. which therefore is the medium quantity of the ſun's apparent daily motion; hence his horary motion is equal to 2′. 27″. By this method the tables of the ſun's mean motion are conſtructed as found in aſtronomical books.

CHAP. IX.

Of the PARALLAXES, DISTANCES, *and* MAGNITUDES *of the* HEAVENLY BODIES.

Def. LXII. The *Parallax* of the heavenly bodies, is the change of their apparent ſituation with reſpect to each other, as the ſpectator views them from different ſtations.

Def. LXIII. The *Diurnal Parallax*, is the diſtance between the apparent place of a heavenly body, as viewed from the ſurface of the earth, and its apparent place, as viewed from the centre of the earth.

Let DAB be the earth, C its centre, A the ſtation of a ſpectator on the ſurface of the earth; and F, G, H, different places of the moon, or any other heavenly body: TO, NM, LI, are its different parallaxes, and THO, or AHC; MGN, or AGC, &c. angles of parallax. Plate 10. Fig. 17.

SCHOL. If a ſpectator in his firſt ſtation at A, ſees a planet at G, its apparent place in the heavens will be N; if now, by the diurnal rotation of the earth, he comes into the ſtation P, the planet will appear at M, which is the place in which it would have appeared if viewed from C the centre: thus, in all caſes, the parallax which ariſes from the diurnal motion is the ſame which would ariſe from a change of ſtation from the ſurface to the centre; for in either caſe, the change of the ſpectator's line of view is the ſame. Hence appears the propriety of the above definition of the *diurnal parallax.*

PROP. CXXIV. The parallax of any planet is always proportional to the angle which a ſemidiameter of the earth, drawn from the ſtation of the ſpectator upon the ſurface to the centre, would ſubtend, if viewed from the planet.

If the planet be at H, and the ſpectator at A, AHT will be his line of view; on changing the ſtation of the ſpectator from A to C, the line of view will become CHO: whence TO will be the parallax. But TO ſubtends and is proportional to THO, or (El. I. 15.) AHC, the angle which the earth's ſemidiameter would ſubtend, if viewed from the planet H. Plate 10. Fig. 17.

PROP. CXXV. The parallax of a planet depreſſes its apparent place, by the parallactic arc.

If the planet be viewed from C, its apparent place is O; if from A, its apparent place is T, farther from Z the vertex than O, by the parallactic arc TO. Plate 10. Fig. 17.

COR. When the altitude of a body is obſerved, it muſt be corrected by parallax and refraction, adding the former, and ſubtracting the latter, in order to get the true altitude, or the altitude above the rational horizon at the centre of the earth. See table of refractions, Schol. 1. Prop. XXXIX.

PROP. CXXVI. The diurnal parallax of any planet, at a given diſtance from the earth, is greateſt when the planet is in the horizon, and decreaſes as the altitude of the planet increaſes.

The parallax (by Prop. CXXIV.) is proportional to the angle which AC would ſubtend, if ſeen from the planet H; but this given line, viewed from the given diſtance of the planet, would continually diminiſh in its apparent magnitude (by Book VI. Prop. LXXIII.) as the degree of obliquity at which it is viewed increaſes, that is, as the planet advances from H towards E; therefore the parallax is greateſt in the horizon, and decreaſes as the planet approaches the vertex. The parallactic angle AGC is leſs than AHC, and AFC leſs than AGC.

PROP. CXXVII. To find the parallax of the moon, or any planet.

Plate 10. Fig. 18. Let HMO be an arc of the horizon ; APVM an arc in the meridian ; P the elevated pole ; V the vertex ; E the apparent place of the planet, as ſeen from the ſurface of the earth, and S its place, as ſeen from the centre : then ES is the diurnal parallax in the vertical circle VE. Before the planet comes to the meridian, obſerve its altitude, at E, above the horizon, whence the complement of its altitude, VE, will be known : at the ſame time obſerve its diſtance from the meridian, or its azimuth, EVM. After the planet has paſſed the meridian, obſerve when it has the ſame altitude as at the firſt obſervation, that is, when *e*V is equal to EV. Now, if E is the apparent place of the planet when at the time of the firſt obſervation it is viewed from the earth's ſurface, and S would be its place, at that time, if viewed from the centre ; and if *e* is its apparent place when viewed at the ſecond obſervation, from the ſurface, and *s* would be its place, at that time, if viewed from the centre ; the parallax ES is equal to the parallax *es*, ſince the altitude was the ſame at both obſervations, and conſequently SV is equal to *s*V. So that if PS is the ſecondary of the equator which paſſed through the planet at the firſt obſervation, and P*s* the ſecondary which paſſed through it at the ſecond obſervation, the planet between the times of the firſt and ſecond obſervation, muſt have deſcribed the arc S*s* in a circle of daily motion. From the time which has paſſed between the two obſervations, the arc S*s* (by Prop. XXVII.) may be found, and conſequently the angle SP*s*. Now, becauſe the angle EVM is known, PVS its ſupplement to two right angles is known : and, becauſe at the two obſervations the planet was at equal altitudes, that is, at equal diſtances from the meridian, the meridian biſects the angle SP*s*, which is known ; whence its half VPS is found. Alſo, if the latitude of the place be known, PV, the diſtance of the elevated pole from the vertex, or the complement of its diſtance from the horizon, that is, (by Prop. III.) the complement of latitude is known. Therefore in the ſpherical triangle PVS, two angles and one ſide are known ; whence the length of SV may be determined. Take SV from EV, which is already known, and SE, the planet's parallax, will be found. The moon's mean parallax has been found to be 57′. 11″.

Or thus : obſerve when the planet whoſe parallax is to be found, and any fixed ſtar in conjunction with it, croſs the meridian at the ſame inſtant : obſerve the ſame planet and ſtar after three hours, and remark how much ſooner the planet reaches a line placed perpendicularly in the teleſcope than the ſtar. As 24 hours is to this difference of time, ſo will 360 degrees be to the arc which ſubtends the angle of the parallax ; whence the arc and angle will be known.

PROP. CXXVIII. Any parallax of a planet being given, to find any other parallax.

Plate 10. Fig. 17. The parallactic angle AFC being given, it is required to find the angle AHC. Having meaſured the angle ZAL, let the angle ZAH, the apparent diſtance of the planet from the zenith, be alſo meaſured. Then, in the triangle CAF, the ſine of the angle CAF is to the ſine of the angle CFA, as the ſide CF is to the ſide AC. Again, in the triangle CAH, the ſine of the angle CAH is to the ſine of the angle CHA, as CH is to AC. But CH is equal to CF ;

CF; therefore the ſine of the angle CAF is to the ſine of the angle CFA, as the ſine of the angle CAH is to the ſine of the angle CHA: but the three firſt terms are known, therefore the fourth, namely, the angle CHA, may be found.

PROP. CXXIX. At a given altitude of different planets, their diurnal parallaxes are inverſely as their diſtances from the centre of the earth.

Plate 10. Fig. 17.

Let one planet be at f, where its altitude is fAp, and another at h, having an equal altitude hAp. If the planet f is viewed from A at the ſurface of the earth, the line of view is Afr, and r is its apparent place in the heavens: viewed from C, its apparent place would be t: whence, its parallax (by Prop. CXXV.) is rt. In the ſame manner it may be ſhewn, that rs, which is leſs than rt, is the parallax of the planet h. But (by Prop. CXXIV.) the parallax of each planet is proportional to the angle which AC would ſubtend, if viewed from the planet: and, ſince AC is given, and alſo the degree of obliquity at which it is viewed, the apparent length of AC, or the angle which AC would ſubtend, at either planet, would be (by Book VI. Prop. LXIX.) inverſely as the planet's diſtance from C. Therefore the parallaxes of theſe planets are inverſely as their diſtances from the centre of the earth.

PROP. CXXX. The diurnal parallax of a planet in a vertical circle produces a parallax of declination, and alſo, if the planet is not in the meridian, of right aſcenſion.

Plate 11. Fig. 1.

Let HQ be the horizon; EC an arc of the equator, which cuts the horizon at C; P the pole of the equator; Z the zenith; ZV a vertical circle; F, the apparent place of a planet in the vertical circle ZV, as viewed from the ſurface of the earth, and I its apparent place, as viewed from the centre: then (by Def. LXIII.) FI is the diurnal parallax in a vertical circle. When the apparent place is F, PFA is a ſecondary of the equator paſſing through it, and when it is I, PIB is the ſecondary which paſſes through it. Therefore AF is the declination of the planet when it appears at F, and BI its declination when it appears at I; the difference of which, DI, is the change of the apparent declination ariſing from the different ſtation of the ſpectator, at the ſurface or centre of the earth. When the apparent place is F, the diſtance of A from the firſt of Aries, is the right aſcenſion; when it is I, the diſtance of B from the firſt of Aries, is the right aſcenſion: for PFA and PIB are ſecondaries of the equator paſſing through the planet. The difference of right aſcenſion, therefore, produced by the parallax FI, is AB. If the planet is in the meridian PZH, and if L be its apparent place, as viewed from the ſurface, and N, as viewed from the centre of the earth, LN will be its diurnal parallax; LE its declination, as viewed from the ſurface; NE its declination, as viewed from the centre; and NL its parallax of declination. But, becauſe PZH is a ſecondary of the equator, in whatever part of this vertical circle the planet appears, its right aſcenſion will be the diſtance of the point E from the firſt of Aries, that is, the diurnal parallax, in this caſe, makes no parallax of right aſcenſion.

PROP. CXXXI. The diurnal parallax of a planet in a vertical circle produces a parallax of latitude, and alſo, if the vertical circle be not a ſecondary of the ecliptic, of longitude.

Plate 11. Fig. 1. Let HQ be the horizon; P the pole of the ecliptic; EC an arc of the ecliptic, which cuts the horizon at C; and ZV a vertical circle: and this Propoſition may be proved in the ſame manner as the laſt.

PROP. CXXXII. The ſemidiameter of the earth is to the diſtance of any planet from the centre of the earth, as the ſine of the planet's parallax is to the ſine of its apparent diſtance from the vertex.

Plate 10. Fig. 17. If a planet is at F, and the ſpectator at A, where the line of view is AFL, the planet will appear at L, and ZAL will be the angle of its apparent diſtance from the vertex Z. Let the parallax IL, or the angle AFC proportional (by Prop. CXXIV.) to IL, be found. In the plane triangle ACF, (the ſides being to one another as the ſines of the oppoſite angles) AC, the ſemidiameter of the earth, is to FC, the diſtance of the planet from the centre of the earth, as the ſine of the angle AFC the angle of the parallax, is to the ſine of the angle FAC, or of its ſupplement to two right angles ZAL, the angle of the planet's apparent diſtance from the vertex.

Cor. When the horizontal parallax is taken, the ſemidiameter of the earth AC is to HC, the diſtance of the planet, as the ſine of the horizontal parallax AHC is to the ſine of HAC or radius.

PROP. CXXXIII. To meaſure the diſtance of the moon from the earth.

Plate 10. Fig. 17. Let H be the moon in the ſenſible horizon obſerved by a ſpectator at A, and C the centre of the earth. In the triangle AHC, let the angle AHC, the moon's horizontal parallax, be found, by Prop. CXXVII. The angle HAC is a right angle, and AC the ſemidiameter of the earth is known to be 3985 miles. Hence, AC the ſine of AHC, 57′. 11″. is to 3985, as AH, taken as radius, to the number of miles in AH the moon's diſtance from the earth; the moon's mean diſtance is thus found to be 240,000 Engliſh miles.

Schol. According to Mr. de la Lande, the horizontal ſemidiameter of the moon, is to its horizontal parallax for the mean radius of the earth as 15′. is to 54′. 57″.4, or very nearly as 3 to 11; hence the ſemidiameter of the moon is $\frac{3}{11}$ of the radius of the earth. And as the magnitudes of ſpherical bodies are as the cubes of their radii, the magnitude of the moon is to that of the earth as 3^3 to 11^3, that is, as 1 : 49.

PROP. CXXXIV. To determine the relative diſtances of the inferior planets from the ſun.

Plate 9. Fig. 12. Let S be the ſun, EHG, the orbit of Venus, and LCM the orbit of Mercury. Let AXF be a tangent to the orbit of Venus, and let the elongation of Venus, that is, the angle XAS, be found by obſervation. Then as radius to the ſine of the angle XAS, ſo is AS to XS or ES. In like manner, if the elongation of Mercury, or the angle CAS, be obſerved; as radius to the ſine of CAS, ſo is AS to CS or LS. If AS, the ſun's diſtance from the earth, be ſuppoſed to be divided into 1000 equal parts, the diſtance of Mercury will in this manner be found to be 387, and that of Venus 723.

PROP.

PROP. CXXXV. To determine the relative diſtances of the ſuperior planets from the ſun.

Plate 10. Fig. 1.

Let S be the ſun, nkg the orbit of the earth, OPQ the orbit of Mars, and NKG a part of a great circle in the heavens, in which the planet appears to have a retrograde motion; let P be the place of Mars. Whilſt the earth is paſſing in its orbit from k to n, Mars will appear to move from K to N. The angle of retrogradation KPN is then known by obſervation. To this the vertical angle nPS is equal. In the triangle nSP, the angle at n is a right angle; the angle nPS is the angle of retrogradation which is known, whence the other angle nSP is known, and the ratio of the ſides of the triangle to each other is known: whence the ratio of Sn to SP is found. If the mean diſtance of the earth from the ſun be called 1000, that of Mars will be found to be 1523, that of Jupiter 5201, and that of Saturn 9538.

PROP. CXXXVI. To find the parallax of the ſun by the tranſit of Venus.

Plate 11. Fig. 2.

Let FIG be the earth, L Venus, and S the ſun. To an obſerver at I, Venus will appear juſt entered upon the ſun's diſc at C, and its apparent place in the heavens will be Q. But at the ſame inſtant to a ſpectator at G it will appear above the ſun in the right line GLN. The angle NLQ, or BLG, is the horizontal parallax of Venus; and the angle BCG, or its equal TCQ, is the ſun's horizontal parallax. Becauſe the plane of Venus's orbit is inclined to the plane of the ecliptic, this planet, in its inferior conjunction, will commonly paſs northward or ſouthward of the ſun; but, when the inferior conjunction happens at, or very near, one of the nodes, it will paſs over the ſun's diſc. If, at the time of this tranſit, a ſpectator at I, and another at G, (at the diſtance of 90 degrees, in longitude, from each other) obſerve the exact time of the total ingreſs of the planet, the difference, between the actual time of ingreſs, and the time at which the ingreſs would have happened without any parallax, being known, and (by Prop. XXVII.) converted into parts of a degree, the horizontal parallax of Venus, ILG, may be found. Or, if ſeveral different obſervations of the time of ingreſs be made at different places, after due allowance is made for difference of longitude and latitude at thoſe places, the parallax of Venus at that time, and thence its horizontal parallax, will be accurately diſcovered.

If two obſervations of this angle be taken at the ſame time on oppoſite meridians, the errors attending the obſervations may ſerve to correct each other. For, ſuppoſe MO to be a part of the orbit of Venus, and V, v, u, the planet in different ſituations. The time in which Venus will paſs through ſuch an arc V v of its orbit, as when viewed from the earth ſubtends an equal angle with the diameter or chord CD of the ſun being found, and alſo the arc V v; let a ſpectator on the earth's ſurface view the planet juſt entering within the ſun's diſc at C. If the earth remained at reſt, the ſpectator would ſee the planet paſſing over the diſc in the line CD, whilſt in its path it deſcribes V v: but becauſe he is, in the mean time, by the earth's diurnal revolution, carried from A towards P, at the inſtant when he ſees the planet paſſing off the ſun's diſc at D, the planet is advanced in its orbit to U. Conſequently, the tranſit will be to this obſerver as much longer than the computed time, as the heliocentric arc VU is longer than Vv. If another obſervation of the ſame kind be taken at the ſame time on the oppoſite meridian, where the ſpectator is carried in a direction contrary to the former, the duration of the tranſit will to him be as much ſhorter than the computed time, as in the other ſituation it was longer.

The horizontal parallax of Venus being thus found, the ſun's horizontal parallax, on the day of the tranſit, may be diſcovered by Prop. XXIX. For, as the ſun's proportional diſtance, at the tranſit, from the earth (taking 1000 for his mean diſtance) is to the proportional diſtance of Venus from the earth at that time, ſo is the horizontal parallax of Venus, ILG, to the horizontal parallax of the ſun, ICG, on the day of the tranſit. Whence the ſun's horizontal parallax, at the time of his mean diſtance from the earth, may be found: for (by Prop. XXIX.) as the ſun's mean diſtance from the earth, is to his proportional diſtance at the tranſit, ſo is his horizontal parallax at that time to his mean horizontal parallax.

In this manner the ſun's mean horizontal parallax has been found, from comparing the tranſits of Venus in 1761 and 1769, to be $8.65''$. or about $8\frac{2}{3}$ ſeconds. See Philoſophical Tranſactions, Vol. LXII. p. 611, and Ferguſon's Aſtronomy, Chap. XXIII.

SCHOL. The tranſits of Venus happen but very ſeldom: the firſt that ſeems to have been noticed was in the year 1639, by Mr. Horrox and his friend Mr. Crabtree. With a view of engaging the attention of future aſtronomers to the above method of determining the ſun's parallax, and thereby his real diſtance from the earth, Dr. Halley communicated a paper to the Royal Society in the year 1691, containing an account of the ſeveral years in which ſuch a tranſit would happen. He particularly mentioned thoſe which would be ſeen in 1761 and 1769, preſuming that on thoſe periods this important problem would be ſolved with great accuracy. No other tranſit will happen until the year 1874.

Except ſuch tranſits as theſe, Venus exhibits the ſame appearance to us regularly every eight years; her conjunctions, elongations, and times of riſing and ſetting, being nearly the ſame, on the ſame days as before.

PROP. CXXXVII. To find the diſtance of the ſun from the earth.

Plate 10. Fig. 17. In the triangle AHC, ſuppoſe H to be the ſun. As the ſine of $8\frac{2}{3}$ ſeconds, the horizontal parallax of the ſun AHC, is to radius, ſo is the ſemidiameter of the earth AC, which is found by menſuration to be 3985 Engliſh miles, to the number of ſemidiameters of the earth contained in the diſtance of the ſun from the earth. Hence the ſun's diſtance from the earth is found to be about 95,173,000 Engliſh miles; for by log. we have 5.621914 (ſine of $8''.65$) : 10.000000 : : 3.600428 (log. of 3985) : 95,173,000 miles.

PROP. CXXXVIII. To meaſure the diſtance of Mercury or Venus from the ſun.

Plate 11. Fig. 3. Let S be the ſun, E the earth, and M Mercury or Venus. Meaſure the angle SEM, and obſerve accurately the time when this meaſure is taken. When Mercury has made one revolution, and arrives at the ſame point M, the earth will be in ſome other part of its orbit, as R; meaſure at that time, the angle SRM, and obſerve the time when the meaſure is taken.

By theſe two obſervations the time in which the earth paſſes from E to R, is known: hence, as 1 year is to the time employed in paſſing from E to R, ſo are 360 degrees to the arc ER: whence the arc ER, and the angle ESR, are found. In the triangle ESR the ſides SE, SR, (the diſtance of the ſun from the earth) being known, and alſo the contained angle RSE, let the angles at the baſe SER, SRE, and the baſe RE, be found. Then from the known angle SER take away the angle SEM, which is alſo known, there will remain MER; and from the known angle SRE take away the known angle SRM, there will remain MRE. The two angles MER, MRE, being thus found, the third angle RME is alſo known; and the ſide RE is known. Wherefore, the ſine of the angle RME is to the ſide RE, as the ſine of the angle MRE is to the ſide ME, or as the ſine of the angle MER is to the ſide MR. In the triangle SRM, the ſides RS, RM, being thus found, the ſum of the two ſides RS, RM, is to their difference, as the tangent of half the ſum of the angles at the baſe RSM, RMS, is to the tangent of half their difference. To half the ſum add half the difference, and the greater angle at M is found; and from half the ſum take away half the difference, and the leſſer angle at S is found. Whence, the ſine of the angle at M is to the ſide RS, or the ſine of the angle at S is to the ſide RM, as the ſine of the angle at R is to the baſe SM, which is the diſtance required.

PROP. CXXXIX. To meaſure the diſtance of Mars from the ſun.

Plate 11.
Fig. 4.

Let S be the ſun, E the earth, and M Mars. Meaſure the angle SEM: When Mars has made one revolution, obſerve the place of the earth in its orbit R, and meaſure the angle SRM. Having found as before the arc ER, and the angle ESR, in the triangle ESR, in which the two ſides SE, SR, and the contained angle ESR, are known, let the angles at the baſe SER, SRE, and the baſe RE, be found. If from the angle SEM (which has been obſerved) be taken SER, there remains REM; and if from the angle SRM (which has been obſerved) be taken SRE, there remains ERM. Whence, in the triangle RME, the angles at R and E being found, the third angle is known: and the ſine of the angle at M is to the ſide RE, as the ſine of the angle at E is to the ſide RM. Wherefore, in the triangle SRM, the two ſides of which, RS, RM, and the contained angle at R, are known; whence, as in the preceding Propoſition, the two angles at the baſe S, M, and laſtly the baſe SM, which is the diſtance required, may be found.

PROP. CXL. To meaſure the diſtance of Jupiter or Saturn from the ſun by their ſatellites.

Plate 12.
Fig. 5.

Let S be the ſun, E the earth, and J Jupiter. Firſt, obſerve the inſtant in which the Satellite R diſappears behind Jupiter, and the inſtant in which it again appears: then, dividing the intermediate time into two equal parts; this will give the inſtant in which the earth E, Jupiter I, and the ſatellite R, are in one right line EID. Next, obſerve the inſtant in which the ſatellite diſappears behind the ſhadow of Jupiter, and the inſtant in which it again appears; and divide the time between theſe inſtants into two equal parts to find the inſtant in which the Satellite is in the midſt of the ſhadow, that is, in which the Sun, Jupiter and the Satellite form a right line SIR. Hence, the time taken up in paſſing from D to R is known: whence, the time of the entire revolution of the Satellite is to 360 degrees as the time employed in paſſing from D to R is to the arc DR. Thus the arc DR, and the angles RID, EIS, are found. Laſtly, having taken an obſervation of the angle IES, the other angle ESI is found; and the ſide ES, the earth's diſtance from the ſun, is known: whence, the ſine of the angle EIS is to the ſide ES, as the ſine of the angle IES is to the ſide IS, the diſtance required.

PROP.

PROP. CXLI. To meaſure the diſtance of any planet from the ſun.

Becauſe the real diſtances of the planets from the ſun are as their proportional diſtances; as the proportional diſtance of the earth from the ſun is to the proportional diſtance of any other planet from the ſun, ſo is the real diſtance of the earth from the ſun in miles, to the real diſtance of any other planet from the ſun in miles.

Hence are found the diſtances of the planets from the ſun in Engliſh miles. Mercury, 36,841,468; Venus, 68,891,486; Mars, 145,014,148; Jupiter, 494,990,976; Saturn, 907,956,130; and the Herſchel, 1800,000,000.

PROP. CXLII. The horizontal parallax of any planet being given; to find its diſtance from the earth.

Plate 10. Fig. 17. Let H be the planet, whoſe horizontal parallax AHC is known. The ſemidiameter of the earth AC being known, in the triangle CAH the ſine of the angle AHC is to the ſide AC, as the ſine of the angle HAC is to the ſide HC, the diſtance ſought.

PROP. CXLIII. The diſtance of any planet being given, to meaſure its real magnitude.

Plate 11. Fig. 6. Let A be the earth and C the centre of any planet; and let the diſtance CA be known. Suppoſe two right lines, AB, AD, drawn tangents to the planet, the angles CBA, CDA, are right angles; therefore the ſquare of AC is equal to the two ſquares of AB and BC together. The ſame ſquare of AC will alſo be equal to the two ſquares of AD and CD. And ſince the ſquare of the radius CB is equal to the ſquare of the radius CD, (on account of the ſpherical figure of the planets) the ſquare of the tangent AB is equal to the ſquare of the tangent AD, and the tangent AB to the tangent AD. Hence the two triangles ABC, ADC, are equal, and conſequently, the angles BAC, CAD, are equal. The angle BAD being meaſured by a micrometer, its half BAC is known; whence, in the triangle ABC, the ſine of the angle at B, which is a right angle, is to the ſide AC, as the ſine of the angle at A is to the ſide BC. The radius, and conſequently the diameter of the planet, being thus found; becauſe ſpheres are as the cubes of their diameters, its magnitude is known by finding the cube of its diameter.

Plate 14. Fig. 4. SCHOL. In Plate 14. Fig. 4, we have a view of the *proportional* magnitudes of the planets Mercury, Venus, the Earth and Moon, Mars, Jupiter and Saturn, according to Ferguſon; with the addition of Herſchel. In proportion to theſe figures of the planets, the ſun's diameter is about two feet.

PROP. CXLIV. To find the periodical time of a planet,

Becauſe, whilſt any planet is performing its revolution the earth is carried forward in its path, the planet, after one greateſt elongation, muſt not only complete a revolution, but likewiſe the whole angular ſpace which the earth deſcribed in that time, before it arrives again at the ſame elongation. Thus, before Venus can return to the ſame elongation, beſides performing an entire revolution in its orbit, (equal to 4 right angles) it muſt paſs through as much more angular ſpace, as the earth has done in the mean time. Hence, as the angular motion of Venus is to the angular motion of the earth in the time between the greateſt elongation and its return, ſo is the periodical time of the earth to the periodical time of Venus. In this manner the periodical times of all the planets may be found.

Or, obſerve when a planet is in any point of its orbit, and after any number of revolutions, obſerve when it comes to the ſame point again; then divide that interval of time by the number of revolutions, and you get the time of one revolution. The obſervations of ancient aſtronomers are here very uſeful; for as they have put down the places of the planets from their own obſervations, by comparing them with the places obſerved now, we take in a very great number of revolutions, and, therefore, if we divide the interval of time by the number of revolutions, ſhould a ſmall error be made in the whole time, it will affect ſo much leſs the time of one revolution. The periodical times of the planets will be found in the table at the end of the chapter.

Cor. Becauſe the ſquares of the periodical times of the planets were found by Kepler to be as the cubes of their diſtances, the periodical times of any two planets being known, and the comparative or real diſtance of one of them from the ſun being given, the diſtance of the other may from this proportion be found.

PROP. CXLV. To find the mean velocities of the planets.

The periodical time of a planet being known, and alſo its diameter, and conſequently its circumference, (for the diameter of a circle is to its circumference nearly as 113 to 355) its mean velocity, or the velocity with which it would move if its motion were uniform, may be thus found: As the whole periodical time of the planet is to an hour, ſo is the whole circumference of its orbit to the angular ſpace paſſed over in an hour. Thus it is found, that the mean horary velocity of the earth is 68,216.9 Engliſh miles. In like manner, the horary velocity of the other planets may eaſily be found.

Cor. By comparing this propoſition with Cor. Prop. CXVII. the velocity of light will be found to be the velocity of the earth in its orbit as 10632 to 1.

PROP. CXLVI. The planets revolve round their axes.

It is found by obſervation, that the earth revolves about its axis in 23h. 56′. 4″. mean ſolar time: Saturn in 12h. 13′. $\frac{1}{4}$: Jupiter in 9h. 56′.: Mars in 24h. 40′.: Venus in 23h. 20′. The ſun is found to revolve on his axis in 25d. 10h.

The time of Saturn's rotation is computed from the ratio of its diameters, which Dr. Herſchel makes to be about 11 to 10. The time of the rotation of the other planets, is not known; nor has it yet been determined whether they do revolve about their axes.

Schol. The following Table contains a ſynopſis of the diſtances, magnitudes, periods, &c. of the ſeveral planets, according to obſervations made down to the year 1784.

TABLE of PLANETARY MOTIONS, DISTANCES, &c.

Anno 1784.	Mercury.	Venus.	Earth.	Mars.	Jupiter.	Saturn.	Herſchel.
Greateſt Elongation of the inferior, and Parallax of ſuperior Planets.	° ′ 28 20	° ′ 47 48	* *	° ′ 47 24	° ′ 11 51	° ′ 6 29	° ′ 3 4$\frac{1}{4}$
Periodical Revolutions round the Sun.	d. h. m. 87 23 15$\frac{1}{2}$	d. h. m. 224 16 49$\frac{1}{4}$	d. h. m. 365 6 9$\frac{1}{4}$	d. h. m. 686 23 30$\frac{3}{4}$	d. h. m. 4332 8 51$\frac{1}{2}$	d. h. m. 10761 14 36$\frac{3}{4}$	d. h. 30445 18
Diurnal Rotations upon their Axes.	* * *	h. m. 23 22	h. m. s. 23 56 4	h. m. s. 24 39 22	h m. 9 56	h. m. 12 13$\frac{1}{2}$	* *
Inclinations of their Orbits to the Ecliptic.	° ′ 7 0	° ′ 3 23$\frac{3}{4}$	* *	° ′ 1 51	° ′ 1 19$\frac{1}{4}$	° ′ 2 30$\frac{3}{4}$	′ ″ 48 0
Place of their aſcending Node.	s ° ′ 1 15 46$\frac{3}{4}$	s ° ′ 2 14 44	* * *	s ° ′ 1 17 59	s ° ′ 3 8 50	s ° ′ 3 21 48$\frac{1}{4}$	s ° ′ 3 13 1
Place of the Aphelion, or Point fartheſt from the Sun.	s ° ′ 8 14 13	s ° ′ 10 9 38	s ° ′ 9 9 15$\frac{1}{4}$	s ° ′ 5 2 6$\frac{1}{4}$	s ° ′ 6 10 57$\frac{1}{2}$	s ° ′ 9 0 45$\frac{1}{2}$	s ° ′ 11 23 23
Greateſt apparent Diameters ſeen from the Earth.	″ 11	″ 58	* *	″ 25	″ 46	″ 20	″ 4 nearly
Diameters in Engliſh Miles, that of the Sun being 883217.	3222	7687	7964	4189	89170	79042	34217
Proportional mean Diſtances from the Sun.	38710	72333	100000	152369	520098	953937	1903421
Mean Diſtances from the Sun in ſemidiameters of the Earth.	9210	17210	23799	36262	123778	227028	453000
Mean Diſtances from the Sun in Engliſh Miles.	37 millions	68 millions	95 mill.	144 millions	490 millions	900 millions	1800 mill.
Eccentricities, or Diſtance of the Focus from the Centre.	7960	510	1680	14218	25277	53163	4759
Proportion of Light and Heat; that of the Earth being 100.	668	191	100	43	3.7	1.1	0.276
Proportion of Bulk; that of the Sun being 1380000.	$\frac{1}{15}$	$\frac{8}{9}$	1	$\frac{7}{24}$	1$\frac{2}{5}$	1000	90
Proportion of Denſity; that of the Sun being $\frac{1}{4}$.	2	1$\frac{1}{4}$	1	·7	.23	.02	*

BOOK

BOOK V. PART II.

Of the CAUSES *of the* CELESTIAL MOTIONS *and of other* PHENOMENA.

CHAP. I.

Of the CAUSE *of the* REVOLUTIONS *of the* HEAVENLY BODIES *in their* ORBITS.

PROP. CXLVII. The moon is retained in its orbit by a force which impels it towards the centre of the earth.

Since (by Book II. Prop. I.) the moon, or any other planet, being put into motion, would continually move on uniformly in a right line, there muſt be ſome force which draws it from its rectilineal path. Whatever this force is, ſince it is found by obſervation that the moon by a radius drawn to the earth's centre deſcribes equal areas in equal times, it follows (from Book II. Prop. LXXIII.) that it is impelled, by that force, towards the earth's centre. The earth indeed is not at reſt; but becauſe both the moon and earth revolve round the ſun, the motion of the moon with reſpect to the earth is the ſame as if the earth were at reſt.

PROP. CXLVIII. The force which retains the moon in its orbit is, at different diſtances from the earth, inverſely as the ſquares of thoſe diſtances.

The moon's orbit being an ellipſe, which has the earth in one of its foci, the force which retains it in its orbit, muſt (by Book II. Prop. LXXXI.) in different parts of the orbit be inverſely as the ſquares of the diſtances from the earth.

PROP. CXLIX. The moon is retained in its orbit by a force which carries it towards the earth with the ſame velocity with which a body acted upon by gravitation at the diſtance of the moon, would fall towards the earth.

Let AER be the earth, PLV a part of the moon's orbit, LC an arc which the moon deſcribes in its orbit in one minute of time. Since the moon deſcribes its whole orbit in 27 days 7 hours 43 minutes, that is, in 39343 minutes, the length of the arc LC, which the moon deſcribes in one minute, is the $\frac{1}{39343}$ part of 360°. or 33″. If the moon ſetting out Plate 11. Fig. 7.

from L, were not impelled towards the earth, it would move in the right line LB. Since therefore it moves in the arc LC, there muſt be a force impelling it towards the earth's centre which draws it from the tangent LB, ſo that, at the end of 1 minute, when it is arrived at C, it will have departed from the tangent as far as BC, or LD: or, becauſe the moon deſcribes the diagonal LC in 1 minute, it would in the ſame time, by the projectile force, deſcribe LB, and by the centripetal, LD. LD is then the ſpace through which the centripetal force makes the moon fall towards the earth in 1 minute; and LD is the verſed ſine of the arc LC, which is an arc of 33″. Therefore the force which impels the moon, would make it fall, in 1 minute of time, through the verſed ſine of an arc of 33″. Becauſe AER, the earth's circumference, is found to meaſure 123249600 Paris feet, its ſemidiameter AT will be about 19615800 ſuch feet. Since therefore the mean diſtance of the moon from the earth is found to be 60 ſemidiameters of the earth, AT multiplied by 60 will give the length of LT a ſemidiameter of the moon's orbit, namely, 1176948000 feet. And as the radius is to the verſed ſine of 33″. ſo is LT to LD, or nearly as 1176948000 to $15\frac{1}{12}$ Paris feet, which is nearly equal to $16\frac{1}{2}$ Engliſh feet, or 1 pole. Therefore, if the moon were to fall towards the earth, the centripetal force which impels it towards the earth would make it fall 1 pole in the firſt minute of its deſcent. But becauſe (by Prop. CXLVIII.) this centripetal force is inverſely as the ſquares of the diſtances, a body which is at the diſtance of the moon, or 60 ſemidiameters of the earth, will be attracted by a force as much leſs than that at the ſurface, as the ſquare of 60, or 3600, is greater than the ſquare of 1, or 1; that is, the force at the ſurface being 1, it will be to the force at the diſtance of the moon, as 1 to $\frac{1}{3600}$, and the velocities will have the ſame ratio. But a body at the ſurface of the earth falls through 1 pole in a ſecond of time, that is, (by Book II. Prop. XXVI.) through 3600 poles in a minute. Therefore, at the diſtance of the moon the body would fall through $\frac{1}{3600}$ part of this length, that is, through 1 pole in a minute. But it has been ſhewn that the moon, by its centripetal force, falls towards the earth 1 pole in a minute: therefore the moon is retained in its orbit by a force which moves it with the ſame velocity with which a body, acted upon by gravitation, and removed to the diſtance of the moon from the earth, would fall towards the earth.

PROP. CL. The moon is retained in its orbit by the force of gravitation.

The force which retains the moon in its orbit agrees with gravitation in its direction (by Prop. CXLVII.) and in its degree of force; (by Prop. CXLIX.) it may therefore be concluded to be the force of gravitation.

PROP. CLI. The primary planets are retained in their orbits by a force which impels them towards the ſun.

It is found by obſervation, that each of them, as they revolve in their reſpective orbits, deſcribe, by a radius drawn to the ſun, equal areas in equal times. Therefore (by Book II. Prop. LXXIII.) the force which retains them in their orbit, impels them towards the ſun.

PROP. CLII. The forces which retain the primary planets in their reſpective orbits are, at different diſtances from the ſun, inverſely as the ſquares of thoſe diſtances.

It appears from obſervation, that the ſquares of the periodical times of the planets are as the cubes of their mean diſtances from the ſun. For example, Saturn's periodical time being found to be to Jupiter's about as 30 to 12, and the diſtance of the former from the ſun, to that of the latter nearly as 9 to 5; the ſquares of the times are 900 and 144; which are to one another nearly as 729 to 125, the cubes of the diſtances. This proportion takes place in all the primary planets: hence (by Book II. Prop. LXXIX.) the force by which they are retained in their orbits is inverſely as the ſquares of the diſtances.

Schol. It has been remarked in the laſt chapter, that Kepler found by obſervation that the ſquares of the periodical times of all the primary planets are as the cubes of their mean diſtances from the ſun. Aſtronomers have ſince found, that the ſame law holds good in the ſecondaries with reſpect to their primaries. To Sir I. Newton we are indebted for an inveſtigation of this law on phyſical principles. He has demonſtrated, that, in the preſent ſtate of nature, ſuch a law was inevitable.

PROP. CLIII. The primary planets are retained in their orbits by the force of gravitation.

The moon having been ſhewn from the direction, and the law of its centripetal force, to be retained in its orbit by gravitation; ſince the primary planets are impelled towards the ſun (by Prop. CLI.) as the moon is towards the earth, and ſince their centripetal force acts with reſpect to the ſun by the ſame law, by which the force which retains the moon in its orbit acts with reſpect to the earth, namely, that this force is inverſely as the ſquare of the diſtance of the planet from the ſun; it may be concluded, as in the caſe of the moon, that they are retained in their orbits by the force of gravitation.

This follows likewiſe from their moving in elliptical orbits, ſince it has been proved (Book II. Prop. LXXXI.) that bodies revolving in ſuch orbits have their centripetal forces inverſely as the ſquares of their diſtances from the centre about which they revolve.

PROP. CLIV. The ſatellites of Jupiter and Saturn are retained in their reſpective orbits by the force of gravity.

They are obſerved to deſcribe equal areas round the reſpective primaries in equal times, and conſequently (by Book II. Prop. LXXIII.) are impelled towards them; and the forces which retain them in their orbits, are at different diſtances inverſely as the ſquares of thoſe diſtances, (by Book II. Prop. LXXXI.) becauſe it has been obſerved that the ſquares of their periodical times are as the cubes of their diſtances from their reſpective centres. Therefore the force which retains the ſatellites of Jupiter and Saturn in their orbits, act in the ſame manner, and by the ſame law, as the force which retains the moon in its orbit acts with reſpect to the earth. But all effects of the ſame ſort are, without proof of the contrary, to be conſidered as produced by the ſame cauſe. Therefore the power which retains the ſatellites in their orbits is gravitation.

PROP. CLV. The ſun and any planet revolve round a common centre of gravity, which remains at reſt.

Plate 11. Fig. *a*.

Let S be the ſun, and P any planet, mutually attracting each other. If neither of the two bodies revolved in any orbit, they would move towards each other, and would meet at C their common centre of gravity; and during the approach of theſe two bodies, C their common centre of gravity would be at reſt, by Book II. Prop. LI. But if the body P has a projectile force given to it in the direction P*t*, and if this projectile force and its gravitation towards S make it deſcribe an orbit round S, (by Book II. Prop. LXVIII.) ſuch a projectile force will prevent the body P from approaching to S, though it gravitates towards S. But if S has not as great a projectile force given to it at the ſame time in the oppoſite direction S*s*, then becauſe S continues to gravitate towards P, and there is no force which can prevent its approaching to P, it follows that S will approach to P, or as P revolves round S the mutual gravitation of theſe two bodies will diminiſh the diſtance SP. Now it appears from Book II. Prop. LI. that C the common centre of gravity always divides this diſtance SP in the inverſe ratio of S to P, or that SC is always as much leſs than PC, as the quantity of matter in S is greater than the quantity of matter in P: conſequently, ſince the quantity of matter in S and in P is always the ſame, SC and PC have always the ſame ratio to one another. But as S approaches to P, SC decreaſes. Therefore PC muſt decreaſe in the ſame ratio. But PC can decreaſe no otherwiſe than either by the approach of P to C, or by the approach of C to P. But the projectile force prevents P from approaching to C. Therefore C muſt approach to P. Thus it appears that, if P has a projectile force given to it, and is made to revolve, unleſs S has an equal projectile force given to it at the ſame time, the mutual gravitation of theſe two bodies towards one another will put C, which is their common centre of gravity, in motion; contrary to Book II. Prop. LI. Cor. Therefore as the planet P begins to move in the direction P*t*, the ſun S will likewiſe begin to move in the direction S*s*; and C their common centre of gravity will continue at reſt. And as they tend mutually towards each other, or towards C their common centre of gravity, their motions will not continue to be performed in right lines, but the planet P will revolve round C in an orbit of which PR is a part, and the ſun S will revolve round C in an orbit of which SQ is a part.

PROP. CLVI. The ſun and any planet, whilſt they mutually gravitate towards each other, deſcribe ſimilar figures round their common centre of gravity, and round each other.

Plate 11. Fig. *a*.

Let S be the ſun, P the earth, or any other planet, and C their common centre of gravity, about which (by the laſt Prop.) they revolve. To a ſpectator at P, who imagines the planet to be at reſt, the ſun will appear to revolve about P, and the reverſe at S. Becauſe

cauſe the common centre of gravity of the ſun S, and any planet P, is always in a right line drawn from the ſun to the planet, if the planet moves through any ſmall ſpace from P to p, the line pC continued muſt paſs through the ſun; and conſequently the ſun muſt have moved from S to s. Thus Pp, Ss, are arcs deſcribed by the planet and ſun in their reſpective orbits in the ſame time, and PCp, SCs, are areas deſcribed in the ſame time by the radii CS, CP. And becauſe the vertical angles at C are equal, and that SC is to PC as sC to pC, (for SP, sp, are both divided in C in the inverſe ratio of the quantities of matter in P and S) the areas PCp, SCs, are ſimilar. In like manner, any other parts of the two orbits deſcribed in the ſame time, may be ſhewn to be ſimilar; conſequently, the whole orbits are ſimilar.

Again, when P has completed its revolution round C, or $\frac{1}{2}$ or $\frac{1}{4}$ of its orbit, it will appear to a ſpectator at S, to whom S ſeems at reſt, to have completed its orbit, or $\frac{1}{2}$ or $\frac{1}{4}$ of its orbit round S. And univerſally, the angular motion of the planet P about C, in any given time, will be equal to its apparent angular motion about S, conſidered as at reſt by a ſpectator at S. If therefore the planet P in any given time has moved from P to p, in which (by the laſt Prop.) the ſun S has moved from S to s, the angle PCp, which is the meaſure of the planet's angular motion about C, will be equal to the apparent angular motion round S. Let St be taken equal to sp, and make the angle PSt equal to the angle PCp; P will by a radius drawn to S apparently deſcribe the area PSt, whilſt, by a radius drawn to C, it is deſcribing the area PCp. Now, becauſe (as was before ſhewn) SC is to PC, as sC to pC, (El. V. 18.) SC+CP, or SP is to PC, as sC+Cp, or sp is to pC; and sp is equal to St: therefore PS is to PC, as St is to pC. Conſequently, the two figures PCp, PSt, are ſimilar. In like manner it may be ſhewn, that any other part, deſcribed in any given time, of the orbit the planet appears to move in round the ſun conſidered as at reſt, will be ſimilar to other parts, deſcribed in the ſame time, of the orbit in which the planet moves round the common centre of gravity of the ſun and the planet: therefore the whole orbits are ſimilar. And ſince the orbits which the ſun, and the planet, deſcribe about their common centre have been proved to be ſimilar, it follows, that the orbit which any planet appears to deſcribe round the ſun conſidered as at reſt, is ſimilar to the orbit which the ſun in the mean time deſcribes round the common centre of gravity.

In like manner it might be proved, that the orbit which the ſun S appears to deſcribe round the planet P conſidered as at reſt, is ſimilar to either of the orbits which the planet and ſun deſcribe about the centre of gravity.

COR. If the ſun's apparent motion, ſeen from the earth, is an ellipſe, having the earth in one of its foci, the earth's apparent motion, ſeen from the ſun, will be in a ſimilar ellipſe, having the ſun in one of its foci: and if the ſun and earth mutually gravitate towards each other, they deſcribe ſimilar elliptic orbits about their common centre.

PROP. CLVII. The common centre of gravity of the ſun and all the planets is at reſt, and is the centre of the ſolar ſyſtem.

Since, from the mutual gravitation of the ſun and any one planet, they will revolve about their common centre (by Prop. CLV.) the ſame muſt hold good with reſpect to the ſun and all the planets. Conſequently, there muſt be ſome one point in the ſolar ſyſtem which is its centre of gravity, and is at reſt.

CHAP. II.

Of the LUNAR IRREGULARITIES.

PROP. CLVIII. The nearer the moon is to its ſyzygies, the greater is its velocity; and the nearer it is to its quadratures, the ſlower it moves.

Plate II. Fig. 8. Let S repreſent the ſun, T the earth, and LMNO the orbit of the moon; let the moon be in one of its quadratures at L, and let the lines LS and TS be drawn. It is obvious, that the tendency which the moon has towards the ſun is along the line LS, and that which the earth has, is along the line TS: let then the former of theſe be reſolved into two others, the one along LA parallel and equal to TS, the other from L to T, along the line LT. The former of theſe tendencies being parallel and equal to that by which the earth tends along the line TS, alters not the ſituation of the two bodies L and T with reſpect to each other, that is, it diſturbs not the motion of the body L; but the other along LT increaſes its tendency towards T. And this increaſe will be to the tendency the moon has to A, which is the ſame which the earth has to S, as the diſtance LT to LA, or TS: that is, the gravity of the moon towards the earth in the quadratures is augmented by the action of the ſun, and that augmentation is to the tendency which the earth has to the ſun, as the length of the line LT, or the diſtance of the moon from the earth, to TS the diſtance of the earth from the ſun.

Hence the greater the moon's diſtance is from the earth, the diſtance of the ſun remaining the ſame, the greater will this increaſe of the moon's gravity towards the earth be. But if the diſtance of the moon from the earth remains the ſame, and the diſtance of the ſun be augmented, this additional increaſe will be the leſs in the ratio of the cube of that diſtance. For, if TS be increaſed while LT remains the ſame, LT will be ſo much the leſs with reſpect to TS, that is, the increaſe will be diminiſhed in the ratio of the ſun's diſtance: but when TS the diſtance of the ſun is increaſed, the abſolute force of the ſun, and with it the above-mentioned increaſe, will be diminiſhed alſo in proportion to the ſquare of that diſtance; conſequently, taking in both conſiderations, it will upon the whole be diminiſhed in the ratio of the cube of that diſtance.

Let now the moon be in one of its ſyzygies at M, then will the tendency it has to the ſun, more than that which the earth has, which is farther off at T, be to that which the earth has, as the difference of the ſquares of SM and ST is to the ſquare of SM: but the difference between the ſquares of SM and ST has nearly the ratio to the ſquare of SM, which twice MT, that is MO, has to SM; becauſe the difference between the ſquares of two numbers whoſe difference is very ſmall with reſpect to either of them (as the difference between SM and ST is with reſpect to the diſtance of S) has little more than double the ratio to the ſquare of the leſſer number, that the difference between the numbers themſelves has to the leſſer number. The tendency therefore which the moon, when at M, has to the ſun, more than that which the earth has, is to that which the earth has, nearly at MO, or twice TL, to SM, or becauſe of the ſun's great diſtance, as twice LT to TS. Her tendency therefore to the earth is now diminiſhed in that ratio: but as was ſhewn above, it was augmented in the quadratures in the ratio only of LT to TS. The diminution here is therefore nearly double of the augmentation there. And whereas that augmentation, when the diſtance of the ſun remains the ſame, was ſhewn to increaſe with the diſtance of the moon; but when the diſtance of the moon remains the ſame, to decreaſe with the cube of the ſun's diſtance; this diminution being always nearly double of that, will do the ſame.

When the moon is in the other ſyzygy at O, it is attracted towards the ſun leſs than the earth is by the difference of the ſquares of SO and ST; which, in effect, is the ſame thing as if the earth were not attracted at all towards S, and the moon were attracted the contrary way; ſo that its tendency to the earth is here alſo diminiſhed, as well as when it was at M, and almoſt in the ſame degree; for, on account of the ſun's great diſtance, the difference between the ſquares of SO and ST is nearly the ſame as between ST and SM.

Or thus: the annual courſe of the moon round the ſun being performed in the ſame time that the earth's is, it ought to be retained in that courſe by the ſame force that the earth is; whereas when the moon comes to M, the action of the ſun upon it is greater than it is upon the earth, by the difference of the ſquares of SM and ST; and when the moon is at O, it is leſs than it is upon the earth by the difference between the ſquares of ST and SO: ſo that in the former caſe the moon is drawn too much towards the ſun, and in the latter too little; and therefore in both caſes its tendency towards the earth is diminiſhed, and almoſt in the ſame degree; becauſe, as was obſerved above, the difference of the abovementioned ſquares is nearly the ſame in either caſe.

Next, let the moon be in a point of her orbit between the quadrature and the ſyzygy. Then being nearer the ſun than the earth is, ſhe will be attracted with a ſtronger force: let it be expreſſed by *l*S produced to D, till *l*D be of ſuch a length, that TS being put to expreſs the action of the ſun upon the earth, *l*D may expreſs the ſtronger force of the ſun upon the moon: and let *l*D be reſolved into two others, one of which let be *la*, equal and parallel to TS, then will the other be *a*D, or its equal and parallel *l*G. This *l*G is the only diſturbing force upon the moon at L, the other L*a* being parallel to TS, affects the moon juſt as the ſun does the earth; and thus alters not their ſituations with reſpect to each other. Let then this figure with the line LG be transferred to fig. 9. This force LG may be reſolved into LI and LH, the one a tangent to the orbit of the moon, and the other a perpendicular thereto: the former accelerates the motion of the moon when going from the quadrature at O to the ſyzygy at B; and will retard it when going from B to R. The other, when H falls upon TL produced, as in this figure, diminiſhes the tendency of the moon towards the earth; and when it falls between L and T, it augments it.

Plate 11. Fig. 9.

Thus the nearer the moon is to its ſyzygies, the greater will be its velocity; and the nearer it is to the quadratures, the ſlower it will move; becauſe one of the forces into which LG is reſolvable, accelerates its motion from the quadrature to the ſyzygies; and retards it as much from thence to the quadratures.

COR. Hence the moon in her monthly revolution is, by the action of the ſun, alternately accelerated and retarded.

PROP. CLIX. The moon deſcribes equal areas in equal times only at the ſyzygies and quadratures, and deviates from this law the fartheſt in the octants.

The

Plate 11. Fig. 9. The diſturbing force being reſolved into two others, one of them, at the quadratures, or ſyzygies, will be found to point from or towards T the centre of the earth directly, and therefore will not hinder the moon from deſcribing equal areas in equal times; the other likewiſe, in thoſe places will be found to tend towards the centre of the ſun, and therefore neither of them will prevent the moon there from deſcribing equal areas in equal times, that is, will not at the quadratures diſturb the moon's motion at all.

But when the moon is in the octants, as at L, this force being reſolved into two others, one of them, as LH, will point directly to or from the centre of the earth, and therefore will increaſe or diminiſh the moon's tendency towards the earth, but not hinder it from deſcribing equal areas in equal time. But the other, as LI, or HG, points neither towards the centre of the earth, nor ſun, and therefore, in the octants, prevents its deſcribing equal areas in equal times. But this being the mid-way-between the quadrature and the ſyzygy, in both which places this diſturbing force doth not prevent the moon from deſcribing equal areas in equal times, it follows, that at the octants, this diſturbing force will be greateſt of all.

Schol. Hence it has always been found more difficult to obtain the moon's place in the octants, ſo as to agree with obſervation, than at the quadratures or ſyzygies.

PROP. CLX. The orbit of the moon is more curved in the quadratures, and leſs in the ſyzygies, than it would be if it were only attracted by the earth.

For its motion (by Prop. CLVIII.) being accelerated during its progreſs from the quadratures to the ſyzygies, in the ſyzygies its motion will be quicker than it ought otherwiſe to be, and therefore its centripetal force leſs than it would otherwiſe be. It will therefore at the ſyzygies deſcribe the portion of a larger curve, which conſequently will be leſs curved than a ſmaller. On the other hand, while the moon paſſes from the ſyzygies to the quadratures, its motion is continually retarded, and therefore, at the quadratures, its motion will be ſlower than it would otherwiſe be. At the quadratures, therefore, the moon will deſcribe the portion of a leſſer curve, which therefore will be more curved than a larger curve.

PROP. CLXI. When the earth is in its perihelion, the periodical time of the moon will be the greateſt; when the earth is in its aphelion, the periodical time of the moon will be the leaſt.

Since the irregularities explained in the three preceding Propoſitions proceed from the action of the ſun, it follows, that where the action of the ſun is greateſt, the irregularities ariſing from it will be greateſt too. But the nearer the earth is to the ſun, the greater will be the action of the ſun upon the moon: and the more the moon tends towards the ſun, the leſs will it tend towards the earth. When therefore the earth is at the perihelion A, and conſequently at its leaſt diſtance from the ſun S, the action of the ſun upon the moon will be greateſt, and deſtroy more of its tendency towards the earth than at any other diſtance, as SE, SC, SD, &c. Therefore when the earth is at the perihelion A, the moon will deſcribe a larger orbit about the earth, than when the earth, is at any other diſtance from the ſun, and conſequently, her periodical time will then be the longeſt. Plate 11. Fig. 10.

But the earth is at its perihelion in the winter, and conſequently, then the moon will deſcribe the outermoſt circle about the earth, and her periodical time will be the longeſt: which agrees with obſervation. For the ſame reaſon, when the earth is at its aphelion B, the tendency of the moon towards the earth will be the greateſt, and conſequently, her periodical time the leaſt. And in this caſe, which will be in the ſummer, it will deſcribe the innermoſt circle about the earth.

PROP. CLXII. The line of the moon's *apſis* goes forward when the moon is in ſyzygy, and backwards when it is in quadrature; but it goes farther forwards than backwards each time, ſo that at length it performs a revolution according to the order of the ſigns.

Plate 11. Fig. 10.

Since the moon deſcribes an elliptical orbit CEDF about the earth S, placed in one of its *foci*, and ſince its centripetal force towards the earth, by means of the action of the ſun (by Prop. CLVIII.) is continually increaſing, or decreaſing, but not equably, that is, ſometimes leſs, and ſometimes more, than in the inverſe duplicate ratio of the diſtance of the moon from the earth, therefore, the line of the moon's *apſis* AB will be continually going backwards or forwards; that is, the axis AB will not always lie in that ſituation, but go backwards into the ſituation CD, or forwards into the ſituation EF. Since however, taking one whole revolution of the moon about the earth, the action of the ſun more diminiſhes the tendency of the moon towards the earth than it augments it, therefore the motion of the *apſes* forwards, exceeds their motion backwards. Upon the whole, therefore, the *apſes* of the moon's orbit go forwards, or according to the order of the ſigns. Their revolution is completed in about 9 years.

PROP. CLXIII. The excentricity of the moon's orbit is varied in every revolution of the moon, and is greateſt when the moon is in ſyzygy, and leaſt when it is in quadrature: and the orbit is moſt of all excentrical when the line of the *apſis* is in the ſyzygies, and leaſt of all excentrical when this line is in the quadratures.

Plate 11. Fig. 10.

Becauſe the moon deſcribes an excentrical orbit CEDB about the earth S, and the action of the ſun upon it ſometimes increaſes its tendency towards the earth, and ſometimes diminiſhes it, that is, makes its gravity towards the earth increaſe or decreaſe too faſt: if, while the moon aſcends from her lower *apſis* A, its gravity towards the earth decreaſes too faſt, inſtead of deſcribing the path DBF, and coming to the higher *apſis* at B, it will run out into a curve beyond DBF, that is, the orbit will become more excentric, or farther from a circle. On the other hand, if the moon is paſſing from her higher *apſis* B, to her lower A, and its gravity towards the earth, by the action of the ſun, increaſes too faſt, it will approach nearer to the earth than the curve CAE, and deſcribe a curve within CAE, or a portion of an orbit leſs excentric, or nearer to a circle, than CEDF. And if we compare ſeveral revolutions of the moon together, we ſhall find, that when the line of the *apſis* is in the ſyzygies, the excentricity will be the greateſt of all, becauſe in that ſituation, the difference between the tendency which the moon has to the earth in one of the *apſes*, and that which it has in the oppoſite one, is the greateſt of all; whereas, when the line of the *apſis* is in the quadrature, this difference is the leaſt, and therefore the lunar excentricity will be ſo too.

COR. When the gravity of the moon towards the earth decreaſes too faſt, the excentricity of her orbit will increaſe: and when her gravity towards the earth increaſes too faſt, the excentricity of her orbit will decreaſe; and the orbit itſelf will approach nearer to a circle.

PROP. CLXIV. The line of the nodes moves backwards, but not uniformly; when it is in the ſyzygies it ſtands ſtill, and moves faſteſt in the quadratures.

Plate 11. Fig. 11. When the line of the nodes is in the ſyzygies, as CD, the plane of the moon's orbit paſſes through the centre of the ſun S, as well as through that of the earth E; whence, the diſturbing force acting in the direction of the line of the nodes, and conſequently in the plane of the lunar orbit, the moon is not drawn out of the plane of its orbit by the ſun. But when the line of the nodes is in any other ſituation, and the moon not in one of the nodes, it is continually drawn out of the plane of its own orbit, on that ſide on which the ſun lies. For inſtance, if the plane of its orbit CGDF produced paſſes above the ſun, the ſun draws it downwards; if, on the contrary, the plane of its orbit produced paſſes below the ſun, it draws it upwards. Hence it follows, that when the line of the nodes is not in the ſyzygies, and the moon having paſſed either of the nodes, has got out of the plane of the ecliptic ACBD on either ſide of it, the action of the ſun occaſions the moon to return back to the plane of the ecliptic ſooner than it otherwiſe would do. But where the moon enters that plane, there is the next node; ſo that each node does, as it were, come towards the moon: and the nearer the line of the nodes is to the quadratures, the greater is this effect, becauſe, in that caſe, the ſun is the fartheſt of all from the plane of the lunar orbit produced. So that the line of the nodes goes backwards the faſteſt of all, when it is in the quadratures; and not at all in the ſyzygies.

PROP. CLXV. The inclination of the lunar orbit is liable to change, and is greateſt when the nodes are in the ſyzygies, and leaſt when they are in the quadratures.

Plate 11. Fig. 12. When the nodes are in the quadratures A, B, and the moon in its orbit AGBF has paſſed A, and is approaching the ſyzygy which is next to the ſun, the action of the ſun upon the moon prevents its aſcending ſo high, that is, departing ſo far from the plane of the ecliptic ADBC; whence the inclination of its orbit to the ecliptic will become leſs, and it will come to conjunction with the ſun at H, making an angle with the ecliptic HAD, leſs than GAD. As the moon goes on to the next quadrature B, the action of the ſun upon the moon, in its deſcent towards the node, haſtens its deſcent, and thus, bringing it down to the ecliptic at K ſooner than it would otherwiſe arrive there, increaſes the inclination of the plane of its orbit as much as it was diminiſhed in aſcending from A to H. And for the ſame reaſon, while the moon paſſes from B to the oppoſite ſyzygy F, the action of the ſun

decreaſes the inclination of its orbit, and increaſes it again on its paſſage from thence to A the next quadrature.

When the nodes are in the ſyzygies, C, D, the plane of the moon's orbit produced, paſſes through the centre of the ſun; and conſequently, not being affected by the action of the ſun, its inclination is neither increaſed nor diminiſhed.

But while the nodes are paſſing from the ſyzygies C, D, to the quadratures A, B, the inclination of the moon's orbit is diminiſhed in every revolution of the moon; and while they are paſſing from thence to the ſyzygies, it is continually increaſing. Suppoſe the nodes in the octants at O and L, and the plane of AGBF, the orbit of the moon, ſo inclined to ADBC, the ecliptic, that if produced it will paſs above the ſun S. When the moon is nearer the ſun than the earth is, it is attracted towards the ſun more than the earth is; and when farther off, the earth is attracted more than the moon is, that is, the moon is as it were attracted the other way. Hence, whilſt the moon is *aſcending* from the ecliptic in paſſing from O to P, the diſturbing force being towards S, and the orbit above S, the moon will not riſe ſo high as P, and the inclination of its orbit will be diminiſhed while it is paſſing over 90 degrees from the node O. In going from a point below P to the next quadrature B, which is 45 degrees, the diſturbing force being ſtill towards S, becauſe the moon is as yet nearer the ſun than the earth is, and the moon now *deſcending* towards the ecliptic, the attraction of the ſun will haſten its deſcent, and therefore cauſe it to move in a plane which will make with the plane of the ecliptic a larger angle than before; that is, in paſſing from P to B the inclination of the orbit is increaſed. But when the moon has paſſed B, and is moving towards L, the diſturbing force acting, in the plane of the ecliptic, *from* the ſun, and the moon ſtill *deſcending* towards the ecliptic, the diſturbing force, attracting the moon upwards, will retard its deſcent to the ecliptic, and cauſe it to move in a plane which will make a leſs angle with the plane of the ecliptic than before; that is, while it is paſſing from B to the node L, the inclination of its orbit is diminiſhed. Thus, while the moon paſſes from O to L, the inclination of its orbit is diminiſhed during three fourths of the paſſage. In like manner, while the moon is *aſcending* from L to I, becauſe the diſturbing force acts *from* the ſun, the inclination of its orbit is diminiſhed; and while it is *deſcending* from I to A, the diſturbing force ſtill acting from the ſun, the inclination is increaſed. But while it is ſtill deſcending from A to O, becauſe the diſturbing force acts *towards* the ſun, the inclination is diminiſhed. Add to this, that while the moon paſſes from O to P, and from L to I, the diſturbing force is much greater than when it was paſſing from P to L, and from I to O, becauſe the difference between the diſtances of the moon and of the earth from the ſun is greater in the former caſe than in the latter. On the whole, therefore, while the nodes are between A and D, B and F, that is, while they are paſſing from ſyzygy to quadrature, the inclination of the lunar orbit is diminiſhed; for, though the nodes have been ſuppoſed equally diſtant from the quadrature and ſyzygy, it is obvious that the like effects muſt happen, though different in degree, when they are nearer to the one than the other.

Next, let the nodes be in the octants I, P, between A and F, and B and G. While the moon is *aſcending* from the node I towards the quadrature A, the diſturbing force acting *from* the ſun, it will be drawn upwards, and the inclination of its orbit will be hereby increaſed. In *aſcending* from A to O, the diſturbing force acting *towards* the ſun, its aſcent will be diminiſhed, or the inclination of its orbit leſſened: but in *deſcending* from O to the node P, the diſturbing force ſtill acting towards the ſun, it will be drawn downwards, and conſequently, the inclination of its orbit will be increaſed. Thus, during one whole revolution of the moon in this poſition of its nodes, the inclination of its orbit will be increaſed through three fourths of its paſſage. And this will be true, as in the other caſe, when the nodes are not in the octants. Alſo, for the reaſon mentioned in the other caſe, the force which increaſes the inclination of the orbit is, while it acts, ſuperior to that which diminiſhes it. While the nodes, therefore, are paſſing from the quadratures to the ſyzygies, the inclination of the moon's orbit is increaſing. From all which it is manifeſt, that the inclination of the lunar orbit is the leaſt when the line of the nodes is in quadrature, and the moon in ſyzygy, and greateſt when the line of the nodes is in ſyzygy.

PROP. CLXVI. The nodes of the moon are at reſt, when the line of the nodes is in ſyzygy; they move *in antecedentia*, or from eaſt to weſt, when the line of the nodes is in quadrature; and alſo when it is between quadrature and ſyzygy; but their regreſs in one revolution, is, in this caſe, leſs than when the line of the nodes is in quadrature.

Plate 11. Fig. 12. When the line of the nodes is in ſyzygy, becauſe the diſturbing force acts in the plane of the moon's orbit, it cannot change the inclination of that plane to the ecliptic; whence, the common interſection of the two planes, or the line of the nodes, is immoveable. If, whilſt the line of the nodes is in AB, the moon is paſſing from A through G to B, being conſtantly drawn towards the plane of the ecliptic by the diſturbing force, it will come to the plane ſooner than it would have done if no ſuch force had acted upon it, that is, before it has deſcribed 180°. or is arrived at B.

In like manner, while the moon is paſſing from B to A, through F, being drawn towards the plane of the ecliptic by the diſturbing force, it will croſs the ecliptic ſooner than it would otherwiſe have done, that is, before it arrives at A. Conſequently, the nodes will have changed their places, and moved in a contrary direction to the moon. In any other poſition of the line of the nodes, the diſturbing force will, for the ſame reaſon, cauſe the line of the nodes to move *in antecedentia*, though in a leſs degree: becauſe, whilſt the moon is deſcribing the greater part of its orbit, it is drawn by the diſturbing force (as was ſhewn in the laſt Prop.) towards the ecliptic, and conſequently is made to croſs the ecliptic ſooner than it would otherwiſe have done, that is, the nodes are, on the whole, in one revolution of the moon, made to move in a direction contrary to that of the moon: but this regreſs is leſs than when the line of the nodes is in quadrature, becauſe, during part of the revolution in this oblique poſition of the line of the nodes, the nodes move *in conſequentia*, or in the ſame direction with the moon, namely, whilſt the diſturbing force (as was ſhewn in the laſt Prop.) draws the moon from the plane of the ecliptic; whereas, when the line of the nodes is in quadrature, they move *in antecedentia* during the whole revolution.

Schol. 1. The nodes perform one revolution, or paſs through every part of the ecliptic, in about 19 years.

SCHOL. 2. All the irregularities of the moon are greater when the earth is in its perihelion, than when it is in its aphelion, becauſe the effect of the ſun's action, whereby they are produced, is inverſely as the cube of its diſtance from the earth. They are alſo greater when the moon is in conjunction with the ſun, than in oppoſition, for the ſame reaſon; for the earth and moon, taken together, are nearer the ſun in the former ſituation of the moon, than they are in the latter.

C H A P. III.

Of the SPHEROIDICAL FORM *of the* EARTH.

PROP. CLXVII. In the daily revolution of the earth round its axis, the centrifugal force diminiſhes the weight of bodies more at the equator than in any other place on the ſurface of the earth, in the duplicate ratio of the ſemidiameter to the coſine of the latitude of the place.

Plate II. Fig. 13.

Let PEP*e* be the earth, PP the axis, E*e* the equator. As the earth revolves upon its axis, every place on its ſurface, except the two poles, deſcribes a circle, the plane of which is perpendicular to the axis, and the radius of which is the diſtance of that place from the axis. Thus, a body placed at A will in one revolution of the earth deſcribe a circle, the ſemidiameter of which will be AB, which with the plane in which it lies will be perpendicular to the axis PP. In like manner, CE is the ſemidiameter of a circle deſcribed by the revolution of a place in the equator. But CE is the ſemidiameter of the earth, and AB is the coſine of latitude of the place A; for AB is the ſine of AP, the complement of AE, which is the latitude of the place. And a body at E, revolving in a circle whoſe radius is CE, performs its revolution in the ſame time with a body at A, revolving in a circle whoſe radius is AB. But where the periodical times are equal, the centrifugal forces are as the radii, by Book II. Prop. LXXVII. Whence the body at E has its centrifugal force as much greater than the body at A, as the radius CE is greater than the radius AB; and univerſally, the centrifugal force at the equator is to the centrifugal force at any other place on the ſurface of the earth, as the ſemidiameter of the earth to the coſine of the latitude of the place. And ſince it is manifeſt that the gravity muſt be diminiſhed as much as the centrifugal force is increaſed, the gravity of a body at the equator is as much leſs than that of a body at any other place on the earth, as the ſemidiameter of the earth is greater than the coſine of the latitude of the place.

Moreover, if the centrifugal forces at E and A were equal, they would diminiſh the weights of bodies unequally, on account of the different directions in which theſe forces act. The centrifugal force at A, acting obliquely upon the force of gravitation towards C, can only diminiſh this force by ſuch a part of its action as is oppoſite to the direction of gravitation, that is, reſolving A*b* which may expreſs the centrifugal force at A into A*a*, *ab*, the part of the centrifugal force which will act to diminiſh the gravity of the body at A, will be to the whole centrifugal force at A, as A*a* to A*b*. Whereas at E, the whole centrifugal force, acting in direct oppoſition to the force of gravitation, will operate to diminiſh the weight of a body at E. Hence the force which acts to diminiſh the weight of a body, that is, the diminution at

E is to the ſame at A, as the whole centrifugal force A*b* to the part A*a*. But A*b* is to A*a*, as CE to BA : for, the triangles A*ab* and ABC being ſimilar, A*b* is to A*a*, as AC or EC to BA. Therefore, from the different directions in which the centrifugal forces act at E and A, the weight at E is as much more diminiſhed than at A, as EC the ſemidiameter is greater than AB the coſine of the latitude of the place A.

The centrifugal force then diminiſhes the force of gravitation in the ratio of EC to AB, both becauſe the centripetal force at E is greater than at A, and becauſe it acts directly at E, but obliquely at A. Therefore the centrifugal force diminiſhes the weight of a body at E, more than at A, in the duplicate ratio of CE to BA, that is, as much more at E than at A, as the ſquare of CE is greater than the ſquare of BA.

Schol. It is found by calculation from this Prop. that gravity at the equator is diminiſhed by the centrifugal force in the ratio of 288 to 289.

Cor. 1. If the diurnal motion of the earth round its axis was about 17 times faſter than it is, the centrifugal force would, at the equator, be equal to the power of gravity, and all bodies there, would entirely loſe their weight. But if the earth revolved ſtill quicker than this, they would all fly off.

Cor. 2. Since a place in the equator deſcribes a circle of 24930 miles in 24 hours (Prop. III. Cor.) it is evident that the velocity with which that place moves, is at the rate of about 17.3 miles per minute. The velocity in any parallel of latitude decreaſes in the proportion of the coſine of latitude to radius. Thus, for the latitude of London, ſay, as Rad. : Coſ. 51° 30′ : : velocity of the equator : velocity of London ; by logarithms, as 10.00000 : 9.794150 : : 1.232046 : 1.026196 = 10.6 miles ; that is, the city of London moves about the axis of the earth at the rate of more than 10½ miles in a minute of time.

PROP. CLXVIII. The earth is an oblate ſpheroid, elevated at the equator, and depreſſed at the poles.

It has been found by obſervation, that a pendulum, ſhorter by 2.169 lines, is required to vibrate ſeconds at the equator than at the poles; but (from Book II. Prop. XLIII. and XLIV.) the lengths of pendulums vibrating in the ſame time are as the gravities at the places where they vibrate ; therefore the gravity at the poles is greater than at the equator. And it has been found by Sir I. Newton, that this difference of gravity is ſo much greater than would ariſe from the centrifugal force alone, that the ratio of the equatorial diameter of the earth to the polar diameter, muſt be as 230 to 229, which makes the equatorial diameter exceed the polar by about 34 miles.

Cor. 1. Hence bodies near the poles are heavier than the ſame bodies towards the equator ; (1.) Becauſe they are nearer the earth's centre, where the whole force of the earth's attraction is accumulated. (2.) Becauſe their centrifugal force is leſs on account of their diurnal motion being ſlower. For both theſe reaſons, bodies carried from the poles towards the equator, gradually loſe their weight.

Cor. 2. The degrees of latitude upon the earth's ſurface are longer at the poles than at the equator. For an arc of a meridian near the poles is leſs curved than near the equator, that is, it is an arc of a larger circle; whence a degree meaſured upon that arc muſt be greater than upon an arc of the ſame meridian at the equator.

Cor. 3. The tendency of a heavy body, on any part of the ſurface of the earth between the poles and the equator, is not directly towards the centre, but towards ſome point between the centre and the equator.

Schol. The point towards which a body in any given place will tend may be determined. For (by Prop. CLXVII.) as radius EC is to the coſine of latitude of the place AB, ſo is the centrifugal force at E to the centrifugal force at A in the direction A*b*. Produce, therefore, the line BA to *b*, till A*b* has the ſame ratio to AC, as the quantity laſt found has to gravity upon the ſurface of the earth. Complete the parallelogram A*b*C*c*; the point ſought will be *c*, and the tendency of the body will be along A*c*; thus ſuppoſe the latitude of the place 51°. 46′. The centrifugal force at the equator is found to be that of gravity, as 1 to 289: hence, as radius to the coſine of 51°. 46′. ſo is 1 to ,618, which is the centrifugal force at A. Conſequently, the centrifugal force at A is to the force of gravity, as ,618 to 289: therefore, by the conſtruction, A*b* or C*c*, is to AC in that ratio. The ratio of AC to C*c* being thus found, as AC is to C*c*, or as 289 is to ,618, ſo will the ſine of the angle of latitude AC*c*, or 51°. 46′. be to 5′. nearly, which is the angle required, meaſuring the deviation of the line of direction of falling bodies at the given latitude from a line drawn to the centre of the earth.

Plate 11.
Fig. 15

CHAP. IV.

Of the PRECESSION *of the* EQUINOXES.

Def. LXIV. A *Periodical Year*, is the time in which the ſun completes its revolution through the ecliptic.

Def. LXV. A *Tropical Year*, is the time in which the ſun completes its revolution ſetting out from any ſolſtitial or equinoctial point, and returning to the ſame.

PROP. CLXIX. The equinoctial points move *in antecedentia*, or go backwards from eaſt to weſt, contrary to the order of the ſigns.

It is found from obſervation, that the equator and ecliptic do not always interſect each other in the ſame points, but that the points of interſection change their place, moving from eaſt to weſt, whilſt the inclination of the planes remains the ſame. This motion is called the *preceſſion of the equinoxes*, becauſe it carries the equinoctial points *in precedentia ſigna*.

PROP. CLXX. The preceſſion of the equinoxes makes the tropical year ſhorter than the periodical year.

If, while the ſun moves in the order of the ſigns, the equinoctial point moves in the contrary direction, it is manifeſt, that the ſun muſt arrive at the ſolſtitial or equinoctial point from which it ſet out, before it arrives at the ſame place in the zodiac, or muſt complete the tropical year ſooner than the periodical year.

The tropical year is obſerved to be 365 days, 5 hours, 49 minutes; the periodical year, 365 days, 6 hours, 4 minutes, 56 ſeconds.

PROP. CLXXI. The preceſſion of the equinoxes cauſes the poles of the equator to deſcribe a circle from eaſt to weſt about the poles of the ecliptic.

In this preceſſion, the plane of the equator revolves from eaſt to weſt, cutting the ecliptic, which, with its axis, is at reſt, in ſucceſſive points. But while the plane of the equator is revolving, its axis muſt revolve with it the ſame way. And, ſince the plane of the equator is always equally inclined to that of the ecliptic, the axis of the equator muſt always have the ſame inclination to the axis of the ecliptic: conſequently, the poles of the equator will revolve round the poles of the ecliptic, always preſerving the ſame diſtance from each other; that is, the poles of the equator will deſcribe a circle about the poles of the ecliptic.

Exp. The preceſſion of the equinoxes, and the revolution of the pole of the equator about that of the ecliptic, may be thus repreſented on the celeſtial globe. Let the broad wooden horizon repreſent the ecliptic; place the axis of the globe perpendicular to the wooden circle; the ecliptic on the globe will then make an angle of 23°. 30′. with the wooden horizon: conſequently, if the wooden horizon repreſents the ecliptic, the circle which commonly repreſents the ecliptic will now repreſent the equator; and the two points in which this circle cuts the wooden horizon will repreſent the equinoctial points. If the globe, in this poſition, be turned ſlowly round from eaſt to weſt, theſe points of interſection will move round the ſame way, while the inclination of the circle which now repreſents the equator to that which repreſents the ecliptic remains the ſame; whence the preceſſion of the equinoxes is properly repreſented. Again, the axis and poles of the globe now repreſenting thoſe of the ecliptic, the axis and poles of the ecliptic, marked on the globe, will repreſent thoſe of the equator; and in turning the globe round from eaſt to weſt, the points which repreſent the poles of the equator, will revolve the ſame way round the poles of the globe which repreſent thoſe of the ecliptic, and the axis of the ſuppoſed equator will always make the ſame angle with the plane of the ſuppoſed ecliptic.

PROP. CLXXII. The preceſſion of the equinoxes is cauſed by the action of the ſun and moon on that exceſs of matter about the equatorial parts of the earth, by which from a perfect ſphere it becomes an oblate ſpheroid.

Plate 11. Fig. 12. Let ADCB be the planet of the ecliptic, S the ſun, E the earth, and AFBG a ring encompaſſing the earth at any diſtance, as Saturn is encompaſſed by its ring. Let the half of this ring AGB towards the ſun be above the plane of the ecliptic, and the other half below it:

then, a line paſſing through A and B will be the line of the nodes of this ring. If it be ſuppoſed that this ring moves round its centre E, the ſame way in which the moon moves round the earth, it is obvious that every point of this ring will be acted upon by the diſturbing force of the ſun in the ſame manner as the moon was ſhewn to be acted upon in Prop. CLVIII. &c. Particularly, the motion of the nodes of this ring, and conſequently of the whole ring which moves with theſe nodes, and its inclination to the plane in which its centre moves, will be affected in the ſame manner with the orbit of the moon: whence, its nodes when in ſyzygies will ſtand ſtill, and its inclination will be greateſt; but in all other ſituations, the nodes will go backwards, and faſteſt of all when in the quadratures, at which time the inclination of the ring will be the leaſt. This will be the caſe whatever be the thickneſs of the ring, or its diſtance from the centre.

If this ring be ſuppoſed to adhere to the earth, it is obvious that it will ſtill have the motions deſcribed above, and that in this ſituation, the earth itſelf muſt participate of theſe motions. Now the earth being an oblate ſpheroid, having its equitorial diameter longer than that which paſſes through its poles, this redundancy of matter, by which the form of the earth departs from a perfect ſphere, may be conſidered as a portion of the ſuppoſed ring, which receives from the action of the ſun the motions abovementioned, and communicates them to the earth. Hence the equinoctial points, which are the nodes of the ring, when they are in ſyzygy, that is, at the equinox, will ſtand ſtill, and the inclination of the equator to the plane of the ecliptic will be the greateſt: in all other ſituations they will go backwards, and faſteſt when in quadrature at the ſolſtices; and the inclination of the plane of the equator to that of the ecliptic is then the leaſt.

COR. Hence the axis of the earth, being perpendicular to the plane of the equator, changes therewith its inclination to the plane of the ecliptic twice in every revolution of the earth about the ſun. For inſtance, it increaſes whilſt the earth is moving from the ſolſtitial to the equinoctial, and diminiſhes as much in its paſſage from the equinoctial to the ſolſtitial points: which phenomenon is called the Nutation of the Poles.

SCHOL. This preceſſion of the equinoxes is found to be 50 ſeconds of a degree, every year, weſtward or contrary to the ſun's annual motion; ſo that with reſpect to the fixed ſtars, the equinoctial points fall backwards 30 degrees in 2160 years, whence the ſtars will appear to have gone 30 degrees forward, with reſpect to the ſigns of the ecliptic, which are reckoned from the equinoctial point. Thus the ſtars which were formerly in Aries are now in Taurus, &c. This period is completed in 25,920 years.

CHAP. V.

Of the TIDES.

PROP. CLXXIII. The tides are cauſed by the attraction of the moon and of the ſun.

Let *ApLn* be the earth, and C its centre; let the dotted circle PN repreſent a maſs of water covering the ſurface of the earth; let M, *m*, be the moon; S, *s*, the ſun in different ſituations. Plate 11. Fig. 14.

ſituations. Becauſe the power of gravity diminiſhes as the ſquares of the diſtances increaſe, (by Prop. CXLVIII.) the waters on the ſide of the earth A are more attracted by the moon at M than the central parts of the earth C, and the central parts are more attracted than the waters on the oppoſite ſide of the earth at L: conſequently (as was ſhewn concerning the moon) the waters on the ſide L will be as it were attracted *from* the centre of the earth, or will recede from thence. Therefore, while the moon is at M, the waters will riſe towards *a* and *l* on the oppoſite ſides of the earth A, L; while, by the oblique attraction of the moon, the waters at P and N will be depreſſed.

Or thus: becauſe (by Prop. CLV.) the moon and earth are continually revolving about their common centre of gravity, ſuppoſe *a*; the points A, C, L, deſcribing circles about this common centre in the ſame periodical times, the forces required to retain them in theſe circles (as may be inferred from Book II. Prop. LXXV.) will be to each other as their diſtances from the centre *a*A, *a*C, *a*L. Conſequently, the point L requires a greater force than C, and C than A, to retain it in its orbit. Now theſe points are retained in their reſpective circles by the moon at M; and conſequently the point L, which is moſt remote, and therefore requires the greateſt force, is attracted the leaſt, whilſt A, the neareſt point, is attracted the moſt. Thus, the water about A being attracted too much, and that about L too little, both will have their gravity diminiſhed by the action of the moon, and will endeavour to leave the centre C; while the water at P and N, having their gravity increaſed by the ſame cauſe, will ſubſide. Hence the form of the water on the ſurface of the earth will become an oblong ſpheroid.

This oval of waters keeps pace with the moon in its monthly courſe round the earth; while the earth, by its daily rotation about its axis, preſents each part of its ſurface to the direct action of the moon, twice each day, and thus produces two floods and two ebbs. But becauſe the moon is in the mean time paſſing from weſt to eaſt in its orbit, it comes to the meridian of any place later than it did the preceding day; whence the two floods and ebbs require nearly 25 hours to complete them. The tide is at the greateſt height, not when the moon is in the meridian, but ſome time afterwards, becauſe the force by which the moon raiſes the tide continues to act for ſome time after it has paſſed the meridian.

As the moon thus raiſes the water in one place, and depreſſes it in another, the ſun does the ſame; but in a much leſs degree, on account of the ſmall ratio of the ſemidiameter of the earth to the diſtance of the ſun; for, as it was ſhewn of the moon, that the force of the ſun by which it diſturbs its motion is as the diſtance of the moon from the earth to that of the ſun from the ſame, ſo, in this caſe, the force of the ſun to diſturb the waters is as the ſemidiameter of the earth to the diſtance of the ſun, which ratio is very ſmall.

Cor. The moon being neareſt to the earth when in perigee, its attraction muſt then be ſtrongeſt, and the effect or elevation of the waters, greateſt. And the earth being in its perihelion at the winter ſolſtice, the ſun's power to produce tides is greateſt at that time.

PROP. CLXXIV. The tides are greateſt at the new and full moons, and leaſt at the firſt and laſt quadratures; and the higheſt of the former tides are near the time of the equinoxes.

Plate 11. Fig. 14.

When the moon is in conjuction or oppoſition with the ſun, as M, *m*, S, the tides which each endeavours to raiſe are in the ſame place; whence they are then greateſt, and are called ſpring tides. But, when the moon is in the firſt or laſt quarter, the ſun being in the meridian when the moon is in the horizon, as M, *s*, depreſſes the water where the moon raiſes it; whence the tides are then leaſt, and are called neap tides. On the full and new moons, which happen about the equinoxes, when the luminaries are both in the equator or near it, the tides are the greateſt of all: for firſt, the two eminences of water are at the greateſt diſtance from the poles, and hence the difference between ebb and flood is more ſenſible; for if thoſe eminences were at the poles, it is obvious we ſhould not perceive any tide at all: ſecondly, the equatorial diameter of the earth produced paſſes through the moon, which diameter is longer than any other, and conſequently there is greater diſproportion between the diſtances of the zenith, centre and nadir, from the centre of gravity of the earth and moon, in this ſituation than in any other; and thirdly, the water riſing higher in the open ſeas, ruſhes to the ſhores with greater force, where being ſtopped, it riſes higher ſtill; for it not only riſes at the ſhores in proportion to the height it riſes to in the open ſeas, but alſo according to the velocity with which it flows from thence againſt the ſhore. The ſpring tides, which happen a little before the vernal and after the autumnal equinox, are the greateſt of all, becauſe the ſun is nearer the earth in the winter than in the ſummer.

PROP. CLXXV. When the moon is in the northern hemiſphere, it produces a greater tide while it is in the meridian above the horizon, than when it is in the meridian below it; when in the ſouthern hemiſphere, the reverſe.

Plate 11. Fig. 15.

Let AFHD repreſent the earth, whoſe centre is T, and axis PO, the point P the north pole, and O the ſouth pole, EQ the equator, FH a parallel to it on the ſouth ſide, and KD another parallel to it on the north ſide. Let the fluid ſurrounding the earth form itſelf into an oblong ſpheroid, whoſe longer axis HK produced, paſſes through the moon at L. The right lines TK, or TH, drawn from the centre T, will repreſent the greateſt height of the water in thoſe places. Then, ſuppoſing NM perpendicular to KH, TN, or TM, will denote the leaſt height, and will repreſent the height of the water in all parts of the globe through which the circle NM paſſes. The right lines TE, TF, TH, TQ, TD, will ſhew the height of the water in thoſe reſpective places E, F, H, Q, D.

Let us now conſider ſome place in particular, which, by the diurnal motion of the earth, deſcribes the parallel KD. When that place is at K, the height of the water TK is the greateſt, that is, it will be high water, and the moon L will be in the meridian. But afterwards, when that place comes to X, the height of the water will be the leaſt, that is, it will be low water; and when the ſame place comes to D, it will be high water again. But becauſe TK is greater than TD, therefore, in the preſent caſe, when the moon is on the north ſide of the equator, or in the northern ſigns, the height of the ſea, or tide, will be greater when the moon is in the part of the meridian which is above the horizon, than when it is in the meridian, and below it. Hence it is that the moon, when it is in the northern ſigns, makes the greateſt tides on our ſide the equator when it is above the earth.

Again, TH, on the ſouth ſide the equator, is longer than TF; and therefore, to a place that deſcribes the parallel FH, the greateſt height of the water, when the moon is in the northern ſigns, is when it is on that part of the meridian that is below the horizon of that place, and the leaſt tides when it is above the horizon. For the like reaſon, when the moon is in the ſouthern ſigns, the greateſt tides on the other ſide of the equator will be when it is below our horizon, and the leaſt tides when it is above it.

Cor. Hence it is evident, that, when the moon is in the equator, the two tides are equally high. For then the longer axis HK, of the oblong ſpheroid coincides with the equator EQ, and the two points of high water on any parallel, being equally diſtant from the higheſt points of the tides, are conſequently at the ſame height.

Schol. What hath been ſaid of the tides, muſt be underſtood upon ſuppoſition, that the globe of the earth is covered entirely with water to a conſiderable depth: but continents which ſtop the tide, ſtraits between them, iſlands, and the ſhallowneſs of the ſea in ſome places, which are all impediments to the courſe of the water, cauſe many exceptions to what hath been above laid down. Theſe exceptions can only be explained from particular obſervations on the nature of tides at different places.

BOOK VII. PART III.

Of the FIXED STARS.

Def. LXVII. Thoſe bodies which always appear in the heavens at the ſame diſtance from each other, are called *Fixed Stars ;* becauſe they do not appear to have any proper motion of their own.

PROP. CLXXVI. The fixed ſtars are luminous bodies.

Becauſe they appear as points of ſmall magnitude when viewed through a teleſcope, they muſt be at ſuch immenſe diſtances, as to be inviſible to the naked eye if they borrowed their light ; as is the caſe with reſpect to the ſatellites of Jupiter and Saturn, although they appear of very diſtinguiſhable magnitude through a teleſcope. Beſides from the weakneſs of reflected light, there can be no doubt but that the fixed ſtars ſhine with their own light. They are eaſily known from the planets, by their twinkling.

Schol. The number of ſtars, viſible at any one time, to the naked eye, is about 1000 : But Dr. Herſchel, by his magnificent improvements of the reflecting teleſcope, has diſcovered that the whole number is great beyond all conception.

PROP. CLXXVII. The fixed ſtars appear of different magnitudes.

The magnitudes of the fixed ſtars appear to be different from one another, which difference may ariſe either from a diverſity in their real magnitudes, or diſtances ; or from both theſe cauſes acting conjointly. The difference in the apparent magnitude of the ſtars is ſuch as to admit of their being divided into ſix claſſes, the largeſt being called ſtars of the firſt magnitude, and the leaſt, which are viſible to the naked eye, ſtars of the ſixth magnitude. Stars only viſible by the help of glaſſes are called teleſcopic ſtars.

Schol. 1. It muſt not be inferred that all the ſtars of each claſs appear exactly of the ſame magnitude ; there being great latitude given in this reſpect ; even thoſe of the firſt magnitude appear almoſt all different in luſtre and ſize. There are alſo other ſtars of intermediate magnitudes, which, as aſtronomers cannot refer to any one claſs, they, therefore, place them between two. *Procyon*, for inſtance, which Ptolemy makes of the firſt magnitude, and Tycho of the ſecond, Flamstead lays down as between the firſt and ſecond.

So that inſtead of 6 magnitudes, we may ſay that there are almoſt as many orders of ſtars, as there are ſtars; ſuch conſiderable varieties being obſervable in their magnitude, colour, and brightneſs.

Schol. 2. To the bare eye the ſtars appear of ſome ſenſible magnitude, owing to the glare of light ariſing from the numberleſs reflections of the rays in coming to the eye; this leads us to imagine that the ſtars are much larger than they would appear, if we ſaw them only by the few rays which come directly from them, ſo as to enter the eye without being intermixed with others.

Exp. Examine a fixed ſtar of the firſt magnitude through a long and narrow tube; which, though it takes in as much of the ſky as would hold a thouſand ſuch ſtars, ſcarcely renders that one viſible.

Schol. 3. There ſeems but little reaſon to expect that the real magnitudes of the fixed ſtars will ever be diſcovered with certainty; we muſt, therefore, be contented with an approximation, deduced from their parallax, (if this ſhould ever be aſcertained) and the quantity of light they afford us compared with the ſun. To this purpoſe, Dr. Herschel informs us that with a magnifying power of 6450, and by means of his new micrometer, he found the apparent diametar of α Lyræ to be $0''.335$.

Dr. Herschel's method of finding the annual parallax of the fixed ſtars is by obſerving how the angle between two ſtars very near to each other varies in oppoſite parts of the year. The following is the moſt ſimple caſe given by that great aſtronomer. Let G and E be two ſtars ſituated in a line with the earth at A, and ſuppoſed perpendicular to the diameter AB of the earth's orbit, and when the earth is at B obſerve the angle GBE. Let P = the angle AGB, or the annual parallax of G; p the angle GBE found by obſervation; M, m, the angles under which the diameters of G and E appear, and draw GH perpendicular to BG. Then $p : P :: GH : AB :: GE : AE ::$ (becauſe $M : m :: AE : AG$) $M - m : M$, hence $P = \frac{p \times M}{M - m}$ the parallax G. If G be a ſtar of the firſt magnitude, and E one of the third, and $p = 1''$. then $P = 1'' \frac{1}{2}$. See Phil. Tranſ. Vol. LXXII.

Plate II. Fig. 16.

This theory is only true upon the ſuppoſition that the ſtars are all of the ſame magnitude; and that a ſtar of the ſecond magnitude is at double the diſtance of one of the firſt, and ſo on: Theſe ſuppoſitions are certainly *not* founded on any analogy from the known and well eſtabliſhed principles of that ſyſtem of bodies to which we belong.

Schol. 4. The ingenious obſervation of Kepler upon the magnitudes and diſtances of the fixed ſtars, deſerves to be introduced here, and the more ſo, as he was followed in the conjecture by the great Dr. Halley. He obſerves, that there can be only 13 points upon the ſurface of a ſphere as far diſtant from each other as from the centre; and ſuppoſing the neareſt fixed ſtars to be as far from each other as from the ſun, he concludes that there can be only 13 ſtars of the firſt magnitude. Hence at twice that diſtance from the ſun there may be placed four times as many, or 52: At three times that diſtance, nine times as many, or 117; and ſo on. Theſe numbers will give pretty nearly the number of ſtars of the firſt, ſecond, third, &c. magnitudes. Dr. Halley farther remarks, that if the number of ſtars be finite, and occupy only a part of ſpace, the outward ſtars would be continually attracted to thoſe within, and in time would unite into one. But if the number be infinite, and they occupy an infinite ſpace, all the parts would be nearly in equilibrio, and conſequently each fixed ſtar being drawn in oppoſite directions would keep its place, or move on till it had found an equilibrium. Phil. Tranſ. N. 364. See alſo the introductory remarks of Dr. Herſchel to a paper on the changes of the fixed ſtars. Phil. Tranſ. 1796.

PROP. CLXXVIII. The fixed ſtars are divided into conſtellations, or ſyſtems of ſtars.

The ancients, that they might the better diſtinguiſh the ſtars with regard to their ſituation in the heavens, divided them into ſeveral conſtellations, that is, ſyſtems of ſtars, each ſyſtem conſiſting of ſuch as are near each other. And to diſtinguiſh theſe ſyſtems from one another, they gave them the names of ſuch men or things as they fancied the ſpace they took up in the heavens repreſented. To theſe, ſeveral new conſtellations have been added by modern aſtronomers.

Schol. The following table contains the names of the conſtellations, and the number of ſtars obſerved in each by different aſtronomers.

The Ancient Conſtellations.		*Ptolemy.*	*Tycho.*	*Hevelius.*	*Flamſtead.*
Urſa minor	The Little Bear	8	7	12	24
Urſa major	The Great Bear	35	29	73	87
Draco	The Dragon	31	32	40	80
Cepheus	Cepheus	13	4	51	35
Bootes, *Arctophilax*	Bootes	23	18	52	54
Corona Borealis	The Northern Crown	8	8	8	21
Hercules, *Engonaſin*	Hercules kneeling	29	28	45	113
Lyra	The Harp	10	11	17	21
Cygnus, *Gallina*	The Swan	19	18	47	81
Caſſiopea	The Lady in her Chair	13	26	37	55
Perſeus	Perſeus	29	29	46	59
Auriga	The Waggoner	14	9	40	66
Serpentarius, *Ophiuchus*	Serpentarius	29	15	40	74
Serpens	The Serpent	18	13	22	64
Sagitta	The Arrow	5	5	5	18
Aquila, *Vultur*	The Eagle	15	12	23	71
Antinous	Antinous		3	19	

The

The Ancient Conſtellations.		*Ptolemy.*	*Tycho.*	*Hevelius.*	*Flamſtead.*
Delphinus	The Dolphin	10	10	14	18
Equulus, *Equi ſectio*	The Horſe's Head	4	4	6	10
Pegaſus, *Equus*	The Flying Horſe	20	19	38	89
Andromeda	Andromeda	23	23	47	66
Triangulum	The Triangle	4	4	12	16
Aries	The Ram	18	21	27	66
Taurus	The Bull	44	43	51	141
Gemini	The Twins	25	25	38	85
Cancer	The Crab	23	15	29	83
Leo	The Lion	35	30	49	95
Coma Berenices	Berenice's Hair		14	21	43
Virgo	The Virgin	32	33	50	110
Libra, *Chelæ*	The Scales	17	10	20	51
Scorpius	The Scorpion	24	10	20	44
Sagittarius	The Archer	31	14	22	69
Capricornus	The Goat	28	28	29	51
Aquarius	The Water-bearer	45	41	47	108
Piſces	The Fiſhes	38	36	39	113
Cetus	The Whale	22	21	45	97
Orion	Orion	38	42	62	78
Eridanus, *Fluvius*	Eridanus, *the River*	34	10	27	84
Lepus	The Hare	12	13	16	19
Canis major	The Great Dog	29	13	21	31
Canis minor	The Little Dog	2	2	13	14
Argo Navis	The Ship	45	3	4	64
Hydra	The Hydra	27	19	31	60
Crater	The Cup	7	3	10	31
Corvus	The Crow	7	4		9
Centaurus	The Centaur	37			35
Lupus	The Wolf	19			24
Ara	The Altar	7			9
Corona Auſtralius	The Southern Crown	13			12
Piſcis Auſtralis	The Southern Fiſh	18			24

The New Southern Conſtellations.

Columba Noachi	Noah's Dove	10
Robur Carolinum	The Royal Oak	12
Grus	The Crane	13

The New Southern Conſtellations.

Phœnix	The Phenix	13
Indus	The Indian	12
Pavo	The Peacock	14
Apus, *Avis Indica*	The Bird of Paradiſe	11
Apis *Muſca*	The Bee or Fly	4
Chamæleon	The Cameleon	10
Triangulum Auſtrale	The South Triangle	5
Piſcis volans, *Paſſer*	The Flying Fiſh	8
Dorado, *Xiphias*	The Sword Fiſh	6
Toucan	The American Gooſe	9
Hydrus	The Water Snake	10

Hevelius's Conſtellations made out of the unformed Stars.

		Hevelius.	*Flamſtead.*
Lynx	The Lynx	19	44
Leo minor	The Little Lion		53
Aſterion & Chara	The Grey Hounds	23	25
Cerberus	Cerberus	4	
Vulpecula & Anſer	The Fox and Gooſe	27	35
Scutum Sobieſki	Sobieſki's Shield	7	
Lacerta	The Lizard	10	16
Camelopardalus	The Camelopard	32	58
Monoceros	The Unicorn	19	31
Sextans	The Sextant	11	41

Schol. 1. Stars not included in any conſtellation are called *unformed* ſtars. Beſides the names of the conſtellations, the ancient Greeks gave particular names to ſome ſingle ſtars, or ſmall collections of ſtars: thus, the cluſter of ſmall ſtars in the neck of the bull, was called the *Pleiades*; five ſtars in the bull's face, the *Hyades*; a bright ſtar in the breaſt of *Leo*, the *Lyon's Heart*; and a large ſtar between the knees of *Bootes*, *Arcturus*.

Schol. 2. The conſtellations may be repreſented on two plane ſpheres projected on a great circle, or on the convex ſurface of a ſolid ſphere, as on the celeſtial globe, or moſt perfectly on the concave ſurface of a hollow ſphere. If the celeſtial globe be made uſe of, after rectifying it to the time of the night, the ſtars may be found, by conceiving a line drawn from the centre of the globe through any ſtar in the heavens, and its repreſentation upon the globe. Greek letters have been added by Bayer to ſtars in the ſeveral conſtellations of his catalogue, (α being affixed to the largeſt ſtar) by means of which any ſtar may be eaſily found.

Schol. 3. Twelve of theſe conſtellations lie upon the ecliptic, including a ſpace about 15°. broad, called the *Zodiac*, within which all the planets move. The conſtellation Aries about 2000 years ago, lay in the firſt ſign of the ecliptic; but on account of the preceſſion of the equinoxes, it now lies in the ſecond. Prop. CLXXII. Schol. In the foregoing table *Antinous* was made out of the unformed ſtars near *Aquila*: And *Coma Berenices* out of the unformed ſtars near the *Lyon's* tail. They are both mentioned by Ptolemy, but as unformed ſtars. The conſtellations as far as the triangle, with *Coma Berenices*, are *northern*; thoſe after *Piſces* are *ſouthern*.

PROP. CLXXIX. The luminous part of the heavens, called the *Milky Way*, conſiſts of fixed ſtars too ſmall to be ſeen by the naked eye.

This is found from obſervations made with teleſcopes.

" In a paper on the Conſtructions of the Heavens, Dr. Herſchel ſays it is very probable, that the great ſtratum called the milky way is that in which the ſun is placed, though perhaps not in the centre of its thickneſs, but not far from the place where ſome ſmaller ſtratum branches from it. Such a ſuppoſition will ſatisfactorily, and with great ſimplicity, account for all the phenomena of the milky way, which, according to this hypotheſis, is no other than the appearance of the projection of the ſtars contained in this ſtratum, and its ſecondary branch."

In another paper on the ſame ſubject, he ſays, " that the milky way is a moſt extenſive ſtratum of ſtars of various ſizes admits no longer of the leaſt doubt; and that our ſun is actually one of the heavenly bodies belonging to it is as evident."

" We will now retreat to our own retired ſtation in one of the planets attending a ſtar in the great combination with numberleſs others; and in order to inveſtigate what will be the appearances from this contracted ſituation, let us begin with the naked eye. The ſtars of the firſt magnitude, being in all probability the neareſt, will furniſh us with a ſtep to begin our ſcale; ſetting off, therefore, with the diſtance of Sirius or Arcturus, for inſtance, as unity, we will at preſent ſuppoſe, that thoſe of the ſecond magnitude are at double, and thoſe of the third at treble the diſtance, and ſo forth. Taking it, then, for granted, that a ſtar of the ſeventh magnitude is about ſeven times as far from us as one of the firſt, it follows that an obſerver, who is encloſed in a globular cluſter of ſtars, and not far from the centre, will never be able, with the naked eye, to ſee to the end of it: For ſince, according to the above eſtimations, he can only extend his view about ſeven times the diſtance of Sirius, it cannot be expected that his eyes ſhould reach the borders of a cluſter, which has, perhaps, not leſs than fifty ſtars in depth every where around him. The whole univerſe, therefore, to him, will be compriſed in a ſet of conſtellations, richly ornamented with ſcattered ſtars of all ſizes. Or if the united brightneſs of a neighbouring cluſter of ſtars ſhould, in a remarkably clear night, reach his ſight, it will put on the appearance of a ſmall, faint, nebulous cloud, not to be perceived without the greateſt attention. Allowing him the uſe of a common teleſcope, he begins to ſuſpect that all the milkyneſs of the bright path which ſurrounds the ſphere may be owing to ſtars. By increaſing his power of viſion, he becomes certain, that the milky way is indeed no other than a collection of very ſmall ſtars, and the nebulæ nothing but cluſters of ſtars."

Dr. Herſchel then ſolves a general problem for computing the length of the viſual ray: That of the teleſcope, which he uſes, will reach to ſtars 497 times the diſtance of Sirius. Now (by Prop. B. Cor. 1. p. 347.) Sirius cannot be nearer than 100.000 × 194.000.000 miles, therefore Dr. Herſchel's teleſcope will, at leaſt, reach to 100.000 × 194.000.000 × 497 miles.

And Dr. Herſchel ſays, that in the moſt crowded part of the milky way, he has had fields of view that contained no leſs than 588 ſtars, and theſe were continued for many minutes, ſo that, in a quarter of an hour, he has ſeen 116,000 ſtars paſs through the field view of a teleſcope of only 15′. aperture: and at another time, in 41 minutes, he ſaw 258.000 ſtars paſs through the field of his teleſcope. Every improvement in his teleſcopes has diſcovered ſtars not ſeen before, ſo that there appears no bounds to their number, or to the extent of the univerſe. See Phil. Tranſ. Vol. LXXIV. and LXXVI.

Schol. 1. There are ſpots in the heavens, called *Nebulæ*, ſome of which conſiſt of cluſters of teleſcopic ſtars, others appear as luminous ſpots of different forms. The moſt conſiderable is one in the mid-way between the two ſtars on the blade of Orion's ſword, marked θ by Bayer, diſcovered in the year 1656 by Huygens; it contains only ſeven ſtars, and the other part is a bright ſpot upon a dark ground, and appears like an opening into brighter regions beyond. Dr. Halley and others have diſcovered nebulæ in different parts of the heavens. In the *Connoiſſance des Temps* for 1783 and 1784, there is a catalogue of 103 nebulæ obſerved by Messier and Mechain. But to Dr. Herschel we are indebted for catalogues of 2000 nebulæ and cluſters of ſtars which he himſelf has diſcovered. Some of them form a round compact ſyſtem, others are more irregular, of various forms, and ſome are long and narrow. The globular ſyſtems of ſtars appear thicker in the middle, than they would do if the ſtars were all at equal diſtances from each other; they are therefore condenſed towards the centre. That ſtars ſhould be thus accidentally diſpoſed is too improbable a ſuppoſition to be admitted; he ſuppoſes, therefore, that they are brought together by their mutual attractions, and that the gradual condenſation towards the centre, is a proof of a central power of ſuch a kind. He obſerves alſo, that there are ſome additional circumſtances in the appearance of extended cluſters and nebulæ, that very much favour the idea of a power lodged in the brighteſt part. For although the form of them be not globular, it is plain that there is a tendency to ſphericity. As the ſtars in the ſame nebulæ muſt be very nearly all at the ſame relative diſtances from us, and they appear nearly of the ſame ſize, their real magnitudes muſt be nearly equal. Granting, therefore, that theſe nebulæ and cluſters of ſtars are formed by mutual attraction, Dr. Herſchel concludes that we may judge of their relative age by the diſpoſition of their component parts, thoſe being the oldeſt which are moſt compreſſed. He ſuppoſes, and indeed offers powerful arguments to prove, that the milky way is the nebulæ, of which our ſun is one of its component parts. See Phil. Tranſ. Vol. LXXVI. and LXXIX.

Schol. 2. Dr. Herſchel has alſo diſcovered other phenomena in the heavens which he calls *nebulous* ſtars, that is, ſtars ſurrounded with a faint luminous atmoſphere of large extent. Thoſe which have been thus ſtyled by other aſtromomers, he ſays, ought not to have been ſo called, for on examination they have proved to be either mere cluſters of ſtars plainly to be diſtinguiſhed by his large teleſcopes, or ſuch nebulous appearances as might be occaſioned by a multitude of ſtars at a vaſt diſtance. The milky way conſiſts entirely of ſtars; and he ſays, "I have been led on by degrees from the moſt evident congeries of ſtars to other groups in which the lucid points were ſmaller, but ſtill very plainly to be ſeen; and from them to ſuch wherein they could but barely be ſuſpected, until I arrived at laſt to ſpots in which no trace of a ſtar was to be diſcerned. But then the gradation to theſe latter were by ſuch connected ſteps as left no room for doubt but that all theſe phenomena were equally occaſioned by ſtars variouſly diſperſed in the immenſe expanſe of the univerſe."

In the ſame paper is given an account of ſome nebulous ſtars, one of which is thus deſcribed: "Nov. 13, 1790. A moſt ſingular phenomenon! A ſtar of the eighth magnitude, with a faint luminous atmoſphere of a circular form, and of about 3'. in diameter. The ſtar is perfectly in the centre, and the atmoſphere is ſo diluted, faint and equal throughout, that there can be no ſurmiſe of its conſiſting of ſtars, nor can there be a doubt of the evident connexion between the atmoſphere and the ſtar. Another ſtar not much leſs in brightneſs, and in the ſame field of view with the above, was perfectly free from any ſuch appearance." Hence Dr. Herſchel draws the following conſequences: Granting the connexion between the ſtar and the ſurrounding nebuloſity, if it conſiſt of ſtars very remote which gives the nebulous appearance, the central ſtar, which is viſible, muſt be immenſely greater than the reſt; or if the central ſtar be no bigger than common, how extremely ſmall and compreſſed muſt be thoſe other luminous points which occaſion the nebuloſity? As, by the former ſuppoſition, the luminous central point muſt far exceed the ſtandard of what we call a ſtar; ſo in the latter, the ſhining matter about the centre will be too ſmall to come under the ſame denomination; we, therefore, either have a central body which is not a ſtar, or a ſtar which is involved in a ſhining fluid, of a nature totally unknown to us. This laſt opinion Dr. Herſchel adopts. Light reflected from the ſtar could not be ſeen at this diſtance. Beſides, the outward parts are nearly as bright as thoſe near the ſtar. Moreover, a cluſter of ſtars will not ſo completely account for the milkyneſs or ſoft tint of the light of theſe nebulæ, as a ſelf luminous fluid. "What a field of novelty," ſays Dr. Herſchel, "is here opened to our conceptions! A ſhining fluid, of a brightneſs ſufficient to reach us from the regions of a ſtar of the 8th, 9th, 10th, 11th, 12th magnitude, and of an extent ſo conſiderable as to take up 3, 4, 5, or 6 minutes in diameter." He conjectures that this ſhining fluid may be compoſed of the light perpetually emitted from millions of ſtars. See Phil. Tranſ. Vol. LXXXI. p. 1. on Nebulous Stars properly ſo called.

Schol. 3. New ſtars ſometimes appear while others diſappear. Several ſtars, mentioned by ancient aſtronomers are not now to be found: ſeveral are now viſible to the naked eye, which are not mentioned in the ancient catalogues; and ſome ſtars have ſuddenly appeared, and again, after a conſiderable interval, vaniſhed: alſo a change of place has been obſerved in ſome ſtars.

The following are remarkable, and well authenticated examples. The firſt new ſtar we have an accurate account of, is that diſcovered by Cornelius Gemma, on Nov. 8, 1572, in *the Chair of Caſſiopeia*. It exceeded *Sirius* in brightneſs, and *Jupiter* in apparent magnitude. Tycho Brahe obſerved it, and found that it had no ſenſible parallax. It gradually decayed, and after 16 months, diſappeared.

On Auguſt 13, 1596, David Fabricius obſerved a new ſtar in the *Neck of the Whale*, 25°. 45'. of Aries, with 15°. 54'. ſouth latitude. It diſappeared after October in the ſame year; was diſcovered again in 1637.

In the year 1600, William Jansenus diſcovered a changeable ſtar in the neck of the Swan. It was ſeen by Kepler, who wrote a treatiſe upon it, and determined its place to be 16°. 18'. ♒, and 55°. 30'. or 32'. north latitude. Ricciolus ſaw it in 1616, 1621, and 1624, and is certain that it was inviſible from 1640 to 1650. M. Cassini ſaw it again in 1655; it increaſed till 1660; then decreaſed; and at the end of 1661, it diſappeared. In November, 1665, it appeared again, and diſappeared in 1681. In 1715 it appeared, as it does at preſent, of the 6th magnitude.

In 1686 KIRCHER obſerved χ in the Swan, to be a changeable ſtar; and from 20 years obſervations, the period of the return of the ſame phaſes was found to be 405 days; the variations of its magnitude are ſubject to ſome irregularity.

In the year 1604, KEPLER diſcovered a new ſtar near the heel of Serpentarius, ſo very brilliant, that it exceeded every fixed ſtar, and even Jupiter, in apparent magnitude.

MONTANARI diſcovered two ſtars in the *Ship*, marked β and γ, by Bayer, to be wanting. He ſaw them in 1664, but loſt them 1668. He obſerved alſo that β, in *Meduſa's Head*, varied in its magnitude.

Mr. GOODRICKE has determined the periodical variation of Algol, or β, in Meduſa's Head (obſerved by MONTANARI to be variable) to be about 2 days 21 hours. Its greateſt brightneſs is of the ſecond magnitude, and leaſt of the fourth. Phil. Tranſ. Vol. LXXIII.

SCHOL. 4. From an attentive examination of the ſtars with good teleſcopes, many of which appear only ſingle to the naked eye, are found to conſiſt of two, three, or more ſtars. Dr. MASKELYNE had obſerved α *Herculis*, to be a double ſtar. Dr. HORNSBY found π *Bootis* to be double. Other aſtronomers had made ſimilar diſcoveries. But Dr. HERSCHEL, by his highly-improved teleſcopes has found about 700, of which, not more than 42 had been obſerved before.

The following are a few of the moſt remarkable:

α *Herculis*, FLAM. 64, a beautiful double ſtar; the two ſtars very unequal; the largeſt red, and the ſmalleſt blue, inclining to green.

α *Geminorum*, FLAM. 66, double, a little unequal, both white; with a power of 146 their diſtance appears equal to the diameter of the ſmalleſt.

ϵ *Lyræ*, FLAM. 4, and 5, a double-double ſtar; at firſt ſight it appears double at a conſiderable diſtance, and by a little attention each will appear double; one ſet are equal, and both white; the other unequal, the largeſt white, and the ſmalleſt inclined to red.

β *Lyræ*, FLAM. 10, quadruple, unequal, white; but three of them a little inclined to red.

λ *Orionis*, FLAM. 39, quadruple, or rather a double ſtar, and has two more at a ſmall diſtance; the double ſtar conſiderably unequal, the largeſt white; ſmalleſt, pale roſe-colour.

μ *Herculis*, FLAM. 86, double, very unequal; the ſmall ſtar is not viſible with a power of 278, but is ſeen very well with one of 460; the largeſt is inclined to a pale red, ſmalleſt, duſkiſh.

α *Lyræ*, FLAM. 3, double, very unequal, the largeſt a fine brilliant white, the ſmalleſt duſky; it appears with a power of 227. Dr. HERSCHEL meaſured the diameter of this fine ſtar, and found it to be $0''.3553$.

The examination of double ſtars with a teleſcope, is a very excellent and ready method of proving its powers. Dr. HERSCHEL recommends the following method. The teleſcope, and the obſerver, having been ſome time in the open air, adjuſt the focus of the teleſcope to ſome ſingle ſtar of nearly the ſame magnitude, altitude, and colour of the ſtar, to be examined; attend to all the phenomena of the adjuſting ſtar as it paſſes through the field of view; whether it be perfectly round, and well defined, or affected with little appendages playing about the edge, or any other circumſtances of the like kind. Such deceptions may be detected by turning the object-glaſs a little in its cell, when theſe appendages will turn the ſame way. Thus you may detect the imperfections of the inſtrument, and therefore will not be deceived when you come to examine the double ſtar. Phil. Tranſ. Vol. LXXII. and LXXV.

Schol. 5. The number of ſtars is unknown. The catalogue publiſhed by Bayer contains 1160; that by Flamstead, which includes many teleſcopic ſtars, contains 3000. But the moſt complete catalogue, is that publiſhed by the Rev. Mr. Wollaston, in 1789.

PROP. CLXXX. The longitude of the fixed ſtars increaſes, while their latitude remains the ſame.

Becauſe the vernal equinoctial point (by Prop. CLXXI.) moves weſtward, the diſtance between any given ſtar and that point, that is, its longitude, will increaſe. But ſince this change is produced by the preceſſion of the equinoxes, which is performed round the axis of the ecliptic, this motion will make no change in the diſtance of the fixed ſtar from the ecliptic, that is, in its latitude.

Cor. Hence the conſtellations of the zodiac are to the eaſt of thoſe ſigns or arcs of the zodiac which are called by the ſame names. The firſt part of the conſtellation Aries, by the preceſſion of the equinoxes, has gone ſo far to the eaſt, ſince the names were firſt given to the ſigns, that it is now 30°. from the firſt degree of Aries in the line of the ecliptic.

Def. LXVIII. The *Annual Parallax* of a heavenly body, is the change of its apparent place, as it is viewed from the earth in its annual motion.

Plate 11. Fig. 16.

If ADBC be the orbit of the earth, S the ſun, and A, B, the earth in oppoſite parts of its orbit: the change in the apparent place of any body, as viewed from A and from B, is its annual parallax.

PROP. CLXXXI. The annual parallax of any heavenly body is proportional to the angle which a diameter of the earth's orbit would ſubtend, if it was viewed from that body.

If when the earth is at A, the fixed ſtar E appears at or near the pole, and when the earth has paſſed to the oppoſite point B, a different ſtar F appears at or near the ſame pole, the ſtar E will have changed its place in reſpect of the pole; for when the pole is at F, the ſtar E which was at or near it before, is at the diſtance EF from it: the apparent length of this diſtance EF (by Def. LXVIII.) is the ſtar's annual parallax. Now if AB, the diameter of the earth's orbit, was to be viewed from the ſtar E, it would ſubtend the angle AEB; but, becauſe the axis of the earth is always parallel to itſelf, AE and BF, which coincide with the axis, are likewiſe parallel; whence (El. I. 29.) the angle EBF, ſubtended by EF, is equal to AEB, ſubtended by AB; and AEB is the parallactic angle.

Cor. The annual parallax of any heavenly body is inverſely as its diſtance from the earth: for the angle AEB (by Book VI. Prop. LXIX.) is inverſely as the diſtance of AB, the axis of the earth's orbit.

PROP. A. If the diſtance of an object be greater than 100.000 times the baſe, the angles at the ſtations will not ſenſibly differ from right angles; conſequently the lines drawn from the object to the ſtations, are, phyſically ſpeaking, parallel,

Suppoſe one of the angles to be 9°. Then, ſince the moſt accurate inſtruments for the menſuration of angles cannot be depended upon to leſs than 2″. the tangent of which is to radius as 1 to 103.132. See Hutton's Logarithms.

The angle, the tangent of which is to radius as 1 to 100,000, is 2″.06, or very little more than two ſeconds.—It has been ſeen that Dr. Herſchel depends upon his inſtruments for the accuracy of meaſuring quantities much leſs than 2″.

PROP. B. The parallax of an object, the diſtance of which is above 100,000 times greater than that between the two ſtations of obſervation, is inſenſible,

If the object be at a greater diſtance from either ſtation than 100,000 times the baſe, the angle at one of the ſtations being 90°, the angle at the other will be more than 89° 59′ 57″.9, the difference of which angle and 90°. being ſcarcely more than 2″. is too ſmall to become ſenſible by obſervation.

Cor. 1. If the parallax of an object (obſerved with an inſtrument ſufficiently exact to meaſure an angle of 2″.) be inſenſible, the diſtance of it from either ſtation cannot be leſs than 100,000 times the baſe, from the extremities of which it is obſerved.

Schol. It is to be remarked, that, though the diſtance of the object cannot be leſs than 100,000 times the baſe, yet it may be greater in any aſſignable ratio.

Cor. 2. Lines drawn from any given points in a baſe, to an object, may be eſteemed, in practice, parallel, without any ſenſible error, if the diſtance of the object be more than 100,000 times the baſe.

Cor. 3. Rays, therefore, diverging from any point in the ſun's diſc upon the ſurface of the earth, may be eſteemed parallel, if their diſtance from each other do not exceed about 970 miles at the earth's ſurface; becauſe 970 is to the diſtance of the earth from the ſun in a proportion of 1 to 100,000.

PROP. CLXXXII. The fixed ſtars have no ſenſible annual parallax.

When the place of the ſtar E is obſerved by the beſt inſtruments from oppoſite points of the earth's orbit, its apparent place in the heavens remains the ſame, which could not be the caſe if the angle of its parallax were ſo much as two ſeconds. Plate 11. Fig. 16.

Cor. 1. Hence it appears, that the fixed ſtars are ſo remote, that a diameter of the earth's orbit bears no proportion to their diſtance, or (by Prop. CLXXXI.) that a diameter of the earth's orbit, if viewed from one of the fixed ſtars, would appear as a point.

Cor. 2. The diſtance of the ſtars muſt be greater than 100,000 times the baſe, from the extremities of which it is obſerved; that is, greater than 100,000 times the diameter of the orbit of the earth, or greater than 100,000 × 194,000,000.

Cor. 3. Two planes drawn parallel to each other, and paſſing through the extremities of a diameter of the orbit of the earth, if produced, will appear to coincide with the ſame great circle of the heavens; becauſe the diameter of the earth's orbit, when ſeen from the fixed ſtars, ſubtends an angle leſs than 2″. In the ſame manner, if a plane, paſſing through the earth's centre, be parallel to a plane drawn to the ſurface, theſe planes, when produced, apparently coincide with the ſame great circle in the heavens.

Cor. 4. The parallax of a fixed ſtar, being not more than 2″. the ſun when viewed from that ſtar, would appear under an angle leſs than $\frac{32' \, 6''}{200,000}$, or leſs than $\frac{1''}{100}$, and therefore could not be diſtinguiſhed from a point.

Schol. Since bodies equal in magnitude and ſplendour to the ſun, being placed at the diſtance of the fixed ſtars, would appear to us as the fixed ſtars now do, it may be ſuppoſed probable, that the fixed ſtars are bodies ſimilar to the ſun, which is the centre of the ſolar ſyſtem. This being the caſe, the reaſon will appear, why a fixed ſtar, when viewed through a teleſcope magnifying 200 times, appears no other than a point. For the apparent diameter of the ſtar being leſs than $\frac{1}{100}$ part of a ſecond when magnified 200 times, will ſubtend an angle leſs than 2″. at the eye of the ſpectator, obſerving it in the teleſcope.

The parallax of the fixed ſtar, when viewed from the oppoſite parts of the earth's orbit, is here aſſumed 2″. but it is probable that the parallax of the neareſt ſtar is much leſs, and conſequently the diſtance greater, in the ſame proportion, as the parallax is leſs.

PROP. CLXXXIII. The motion of the earth, and the progreſſive motion of light, will make a fixed ſtar, which has no ſenſible parallax, deviate from its true place in the direction in which the earth moves.

Plate 11. Fig. 17. If a ſtar S paſſes through the zenith of any place when the earth is at A, it will (by laſt Prop.) paſs through the zenith of the ſame place when the earth is at B, the oppoſite extremity of the earth's orbit. Conſequently, ſuch a ſtar might be ſeen through a vertical teleſcope in the ſame perpendicular at any point of the earth's orbit, if the motion of light from the ſtar were inſtantaneous. But the progreſſive motion of light will cauſe the ſtar to deviate from the perpendicular; for, let the earth be moving from B to A, and let the velocity of light be to the velocity of the earth, as CA to BA, and let CB be the diagonal of the parallelogram formed from CA, BA. Then the direction of a teleſcope, in order to ſee the ſtar S when the earth is arrived at A, muſt be AH, parallel to BC. For, ſuppoſe BC to be a very long, ſlender teleſcope, through which only one ray of light could paſs at a time, or to be the axis of a larger teleſcope. The ſtar S cannot be ſeen through this teleſcope, but

through a teleſcope perpendicular to B, if the earth be ſtationary at B, and the progreſs of light inſtantaneous. But if the teleſcope in the poſition BC were to continue in this poſition, and to move along with the earth to A, ſo as to come into the ſituation AH, when the earth arrives at A, the ſtar S might then be ſeen through it. For, ſince the ſtraight courſe of the ray is the line CA, in which it muſt always be if it comes to the eye without interruption; and ſince the ray cannot come directly along CB the axis of the teleſcope, and arrive at the eye in this axis, unleſs it is always in the axis; that is, ſince the ray, in order to come to the eye muſt be always in the line CA, and alſo in the line CB, it muſt be always in the common interſection of theſe two lines. Now C is the common interſection when the earth is at B; *e* is the common interſection when the earth is at E; *f*, when it is at F; *g*, when it is at G; and A, when at A; the teleſcope, at each ſtation, being ſucceſſively in the ſituations CB, EE, FF, GG, HA. Thus the common interſection deſcends down the line CA, while the earth moves from B to A; and, ſince the velocity of light is to that of the earth, as CA to BA, a ray of light will likewiſe have deſcended down CA, while the earth was moving from B to A. Therefore, in the whole motion of the teleſcope, the ray will have been in the common interſection of the line CA, and the axis of the teleſcope, and conſequently will have paſſed along the axis of the teleſcope, and will come without interruption to the eye at A.

Thus it appears, that by the progreſſive motion of light, a ray which, coming from S, enters, at C, a teleſcope in the ſituation CB, will arrive at the eye, when the teleſcope, carried along BA with a velocity which is to that of the ray of light, as BA to CA, is come into the ſituation HA; and conſequently (Book VI. Prop. II.) the eye will ſee the ſtar through the teleſcope in the direction AH the axis of the teleſcope; that is, ſome point in the line AH produced will be the apparent place of the ſtar. Thus the ſtar's apparent place has deviated from its true place S in the direction BA, in which the earth was moving, ſo that if the motion of the earth is from north to ſouth, the ſtar which appeared in the zenith of the place when the earth was at B, will appear to the ſouthward of the zenith when the earth is arrived at A, and the reverſe when the earth is moving from ſouth to north.

According to Bradley's obſervations made on the ſtar γ in the conſtellation Dragon, this ſtar deviated ſouthward from the zenith from Dec. till March, when it had departed from the zenith $20''$. From that time till June its ſouthern deviation decreaſed, after which it deviated northward, and in September appeared about $20''$. towards the north of its ſtation in June, from which time till December, it continued returning to its firſt ſituation. Thus the deviation of the ſtar was always in the direction of the earth's motion, and contrary to that of any deviation which might be ſuppoſed to ariſe from the annual parallax of the ſtar. But ſuch a deviation could not happen unleſs the earth moved, and the motion of light was progreſſive; for if the earth did not move, ſince the ſtar is fixed, no alteration could be made in the apparent place of the ſtar by the progreſſive motion of its rays in a vertical direction: and if the earth moves, and the propagation of light were inſtantaneous, the earth's velocity would be nothing in reſpect of the velocity of light, or BA with reſpect to CA would be nothing; whence the angle ACB, and its alternate angle CAH, would vaniſh, and AH would become coincident with AC, and conſequently the ſtar would have no deviation from its true place. Hence we may conclude from the deviation of the ſtar above deſcribed, both that the earth moves, and that the motion of light is progreſſive.

Cor. From theſe obſervations it is found, that the velocity of ſtar-light is ſuch as carries it through a ſpace equal to the ſun's diſtance from the earth in $8'.\ 13''$.

Schol. Sir Isaac Newton has ſhewn that the ſun, by its attractive power, retains the planets belonging to our ſyſtem in their orbits; he has likewiſe pointed out the method whereby the quantity of matter contained in the ſun may be accurately determined. Dr. Bradley has aſſigned the velocity of the ſolar light with a degree of preciſion exceeding our utmoſt expectation. Galileo and others have aſcertained the rotation of the ſun upon its axis, and determined the poſition of its equator. By means of the tranſit of Venus over the diſc of the ſun, our mathematicians have calculated its diſtance from the earth; its real diameter and magnitude; the denſity of the matter of which it is compoſed; and the laws of the fall of heavy bodies on its ſurface.

In the year 1779, there was a ſpot on the ſun which was large enough to be ſeen by the naked eye; it was divided into two parts, and muſt have been 50,000 miles in diameter: this phenomenon may be accounted for, from ſome natural change of an atmoſphere. For if ſuch of the fluids which enter into its compoſition be of a ſhining brilliancy, whilſt others are merely tranſparent, then any temporary cauſe which ſhould remove the lucid fluid will permit us to ſee the body of the ſun through the tranſparent ones. If an obſerver were placed on the moon, he could ſee the ſolid body of the earth only in thoſe places where the tranſparent fluids of our atmoſphere would permit him. In others, the opaque vapours would reflect the light of the ſun without permitting his view to penetrate the ſurface of our globe. He would probably find that our planet had occaſionally ſome ſhining fluids in its atmoſphere, ſuch as the northern lights. And there is good reaſon to believe, that all the planets emit light in ſome degree; for the illumination which remains on the moon in a total eclipſe cannot be entirely aſcribed to the light which may reach it by the refraction of the earth's atmoſphere. For, in ſome caſes, as in the eclipſe of 1790, the focus of the ſun's rays refracted by the earth's atmoſphere, muſt be many thouſand miles beyond the moon.

There are appearances alſo which denote a phoſphoric quality in the atmoſphere of Venus.

Dr. Herſchel ſuppoſes, that the ſpots in the ſun, are mountains on its ſurface, which, conſidering the great attraction exerted by the ſun upon bodies placed at its ſurface, and the ſlow revolution it has about its axis, he thinks may be more than 300 miles high, and yet ſtand very firmly. He ſays, that in Auguſt, 1792, he examined the ſun with ſeveral powers from 90 to 500. And it evidently appeared, that the black ſpots are the opaque ground or body of the ſun; and that the luminous part is an atmoſphere, which being intercepted or broken, gives us a glimpſe of the ſun itſelf.

Hence he concludes, that the ſun has a very extenſive atmoſphere, which conſiſts of elaſtic fluids that are more or leſs lucid and tranſparent; and of which, the lucid ones furniſh us with light. This atmoſphere, he thinks, is not leſs than 1843, nor more than 2765 miles in height; and he ſuppoſes that the denſity of the luminous ſolar clouds need not be exceedingly more than that of our aurora borealis, in order to produce the effects with which we are acquainted.

The ſun, then, appears to be a very eminent, large, and lucid planet, evidently the firſt and only primary one belonging to our ſyſtem. Its ſimilarity to the other globes of the ſolar ſyſtem, with regard to its ſolidity;—its atmoſphere;—its ſurface diverſified with mountains and vallies;—its rotation on its axis;—and the fall of heavy bodies on its ſurface;

face ;—leads us to ſuppoſe that it is moſt probably inhabited, like the reſt of the planets, by beings whoſe organs are adapted to the peculiar circumſtances of that vaſt globe.

If it be objected, that from the effects produced at the diſtance of 97,000,000 miles, we may infer, that every thing muſt be ſcorched up at its ſurface :—We reply, that there are many facts in natural philoſophy which ſhew that heat is produced by the ſun's rays only when they act on a calorific medium : they are the cauſe of the production of heat by uniting with the matter of fire which is contained in the ſubſtances that are heated ; as the colliſion of the flint and ſteel will inflame a magazine of gunpowder, by putting all the latent fire which it contains, into action.

On the tops of mountains of ſufficient height, at the altitude where clouds can ſeldom reach to ſhelter them from the direct rays of the ſun, we always find regions of ice and ſnow. Now if the ſolar rays themſelves conveyed all the heat we find on this globe, it ought to be hotteſt where their courſe is the leaſt interrupted. Again ; our aëronauts all confirm the coldneſs of the upper regions of the atmoſphere ; and ſince, therefore, even on our earth the heat of the ſituation depends upon the readineſs of the medium to yield to the impreſſion of the ſolar rays, we have only to admit, that on the ſun itſelf, the elaſtic fluids compoſing its atmoſphere, and the matter on its ſurface are of ſuch a nature as not to be capable of any extenſive affection of its own rays ; and this ſeems to be proved by the copious emiſſion of them ; for if the elaſtic fluids of the atmoſphere, or of the matter contained on the ſurface of the ſun, were of ſuch a nature as to admit of an eaſy chemical combination with its rays, their emiſſion would be very much impeded. Another well known fact is, that the ſolar focus of the largeſt lens thrown into the air, will occaſion no ſenſible heat in the place where it has been kept for a conſiderable time, although its power of exciting combuſtion, when proper bodies are expoſed, ſhould be ſufficient to fuſe the moſt refractory ſubſtances.

It is by analogical reaſoning that we conſider the moon as inhabited. For it is a ſecondary planet of conſiderable ſize, its ſurface is diverſified, like that of the earth, with hills and vallies. Its ſituation with reſpect to the ſun, is much like that of the earth ; and by a rotation on its axis, it enjoys an agreeable variety of ſeaſons, and of day and night. To the moon, our globe would appear a capital ſatellite, undergoing the ſame changes of illumination as the moon does to the earth. The ſun, planets, and the ſtarry conſtellations of the heavens, will riſe and ſet there as they do here : and heavy bodies will fall on the moon as they do on the earth. There ſeems, then, only to be wanting, in order to complete the analogy, that it ſhould be inhabited like the earth.

It may be objected, that, in the moon, there are no large ſeas ; and its atmoſphere (the exiſtence of which is doubted by many) is extremely rare, and unfit for the purpoſes of animal life ;—that its climates, its ſeaſons, and the length of its days and nights, totally differ from ours ;—that without denſe clouds, which the moon has not, there can be no rain, perhaps no rivers and lakes.

In anſwer to this, it may be obſerved, that the very difference between the two planets ſtrengthens the argument. We find even on our own globe, that there is a moſt ſtriking diſſimilarity in the ſituation of the creatures that live upon it. While man walks on the ground, the birds fly in the air, and the fiſhes ſwim in the water. We cannot ſurely object to the conveniences afforded by the moon, if thoſe that are to inhabit its regions are fitted

to

to their conditions as well as we on this globe of ours. The analogy already mentioned eſtabliſhes a high probability that the moon is inhabited.

Suppoſe, then, an inhabitant of the moon, who has not properly conſidered ſuch analogical reaſonings as might induce him to ſurmiſe that our earth is inhabited, were to give it as his opinion, that the uſe of that great body, which he ſees in his neighbourhood, is to carry about his little globe, in order that it may be properly expoſed to the light of the ſun, ſo as to enjoy an agreeable and uſeful variety of illumination, as well as to give it light by reflection, when direct light cannot be had :—Should we not condemn his ignorance and want of reflection ? The earth, it is true, performs thoſe offices which have been named, for the inhabitants of the moon, but we know that it alſo affords magnificent dwelling-places to numberleſs intelligent beings.

From experience, therefore, we affirm, that the performance of the moſt ſalutary offices to inferior planets, is not inconſiſtent with the dignity of ſuperior purpoſes ; and in conſequence of ſuch analogical reaſonings, aſſiſted by teleſcopic views, which plainly favour the ſame opinion, we do not heſitate to admit that the ſun is richly ſtored with inhabitants.

This way of conſidering the ſun, is of the utmoſt importance in its conſequences. That ſtars are ſuns can hardly admit of a doubt. Their immenſe diſtance would effectually exclude them from our view, if their light were not of the ſolar kind. Beſides, the analogy may be traced much farther : the ſun turns on its axis ; ſo does the ſtar Algol ; ſo do the ſtars called β Lyræ, δ Cephei, η Antinoi, o Ceti, and many more, moſt probably all. Now from what other cauſe can we, with ſo much probability, account for their periodical changes ? Again ; our ſun's ſpots are changeable—ſo are the ſpots on the ſtar o Ceti. But if ſtars are ſuns, and ſuns are inhabitable, we ſee at once what an extenſive field for animation opens to our view.

It is true, that analogy may induce us to conclude, that ſince ſtars appear to be ſuns, and ſuns, according to the common opinion, are bodies that ſerve to enlighten, warm, and ſuſtain a ſyſtem of planets, we may have an idea of numberleſs globes that ſerve for the habitation of living creatures. But if theſe ſuns themſelves are primary planets, we may ſee ſome thouſands of them with the naked eyes, and millions with the help of teleſcopes ; and, at the ſame time, the ſame analogical reaſoning ſtill remains in full force with regard to the planets which theſe ſuns may ſupport.

Moreover, from the obſervations of Dr. Herſchel, on the compreſſed cluſters of ſtars, it appears, that in many inſtances there cannot be ſpace for the revolutions of a ſyſtem of planets and comets, and therefore it is highly probable that theſe ſuns are capital primary planets which exiſt for themſelves, and are connected together in one great ſyſtem for mutual ſupport. See a very curious and valuable paper on the nature and conſtruction of the ſun and fixed ſtars, by Dr. Herſchel ; read to the Royal Society, Dec. 18, 1794. From this paper, the foregoing ſcholium has been taken. See alſo Dr. Herſchel's paper on the periodical ſtar, α Herculis ; with remarks, tending to eſtabliſh the rotatory motion of the ſtars on their axes. Phil. Tranſ. 1796.

APPENDIX TO THE ASTRONOMY,

Containing SOLAR *and* LUNAR TABLES, *with their Explanation and Uſe, and the* PROJECTION OF ECLIPSES, *ſelected from* "EWING'S PRACTICAL ASTRONOMY."

EXPLANATION OF THE TABLES.

TABLES *of the Mean Motions of Celeſtial Objects.*

THE idea that the ſun, moon, and ſtars, performed all their motions in circles, was, perhaps, one of the firſt which men received concerning theſe very diſtant bodies. The regular returns of day and night, of the ſeaſons of the year, and of almoſt all things in the viſible world, would ſerve to confirm it : and although it is well known that the orbits of the planets are not perfect circles, it is equally known that they differ very little from circles; and therefore modern aſtronomers retain the idea, and form tables of the motions of the planets, as if their orbits were circles, and their motions always uniform, paſſing over equal ſpaces in equal times : Such are called *Tables of Mean Motion;* and the longitude of a planet computed from ſuch Tables for any given time is called its *mean longitude.*

TABLE I. II. *Contain the Sun's Mean Motion, and the Preceſſion of the Equinoctial Points in Julian Years.*

The aſtronomical year is that ſpace of time wherein the earth moves round the ſun, or wherein the ſun ſeems to move round the whole ecliptic from any point, ſuch as the vernal equinox to the ſame again; which, according to the moſt accurate obſervations, conſiſts of 365 natural days, 5 hours, 48 minutes, 54¾ ſeconds; but in civil reckoning there are two kinds of years, common and biſſextile. A common year conſiſts of 365 days, and the biſſextile of 366. Theſe are called *Julian years,* from Julius Ceſar, who introduced this method of computation.

Mr. Mayer makes the ſun's mean motion in 365 days to be 11s. 29° 45′ 40″.7. Admitting his numbers, we have,

Year.	S.	°	′	″
1	11	29	45	40.7
2	11	29	31	21.4
3	11	29	17	2.1
4	0	0	1	51.1

For the 4th year,

	S.	°	′	″
To the 3d year	11	29	17	2.1
Add mo. for 1 year	11	29	45	40.7
And for 1 day			59	8.3
The 4th year =	0	0	1	51.1

That is, in four years the earth goes four times round the ſun, and 1′ 51″ more. In the ſame manner, the ſun's mean motion may be found for any number of Julian years, as in the Table.

The yearly motion of the earth's or ſun's apogee is found to be 1′ 6″; which being ſubtracted from the ſun's yearly motion in longitude, the remainder is the ſun's mean anomaly for 1 year; and for any number of years the mean anomaly is found in the ſame manner as the mean longitude.

It has been found by obſervation, that the equinoctial points move backward, or contrary to the order of the ſigns, at the rate of 50″.3 yearly, called the *preceſſion of the equinoxes;* and multiplying 50″.3 by 2, 3, 4, 5, &c. the numbers in Table II. are found.

TABLE III. *Contains the Sun's Mean Longitude and Anomaly, with the Obliquity of the Ecliptic, for years current of the Chriſtian Æra.*

The Table is compoſed in this manner: The mean longitude and anomaly of the ſun with the obliquity of the ecliptic, are found for the noon of the laſt day of Dec. 1760, which is accounted his mean longitude, &c. for the ſucceeding year 1761. To theſe numbers add the ſun's mean motions for 20 years, taken from Table I.; the ſums are the ſun's mean longitude and anomaly for 1781. To theſe add the mean motions for 10 years, and the mean longitude and anomaly for 1791 are known. And for the following years add the ſun's mean motions in one year to his mean longitude and anomaly for the preceding year continually; remembering to add the motion of 1 day more for every biſſextile year, until the number of years which the Table is to contain be completed.

The annual differences of the obliquity of the ecliptic are very ſmall, and not always regular. It appears by the Table that the difference is only about 25″ in 60 years; therefore the obliquity of the ecliptic may be ſtated at 23° 28′ during the next 50 years without ſenſible error.

When the ſun's mean longitude and anomaly are wanted for any year which is not in the Table, take the mean motions for the intermediate years from Table I. and if the year is before 1761, ſubtract them; if after it, add them to the numbers for 1761, and the remainder or ſum will be the mean longitude and anomaly required.

In theſe additions 12 ſigns are to be rejected as often as they occur; and in the ſubtractions 12 ſigns muſt be borrowed when neceſſary.

TABLE IV. Contains the ſun's mean motions for the days of the year, diſtributed into 12 kalendar months. At the noon of January 1ſt there is one aſtronomical day paſt, becauſe it began at the noon of December 31ſt; and therefore the ſun's mean motions in one day are put down. Theſe being multiplied by 2, 3, 4, 5, &c. (allowance being made for the fractions in one day's motion) the products are the numbers of the Table; remembering to place them properly, *viz.* the motions for 31 days at the laſt day of January; for 32 days at the 1ſt day of February; and ſo on until the 31ſt of December.

TABLE V. Contains the ſun's mean motions for hours, minutes, and ſeconds, which may be underſtood from what has been ſaid of his motions for days.

TABLE VI. *Equations of the Sun's Centre.*

The ſun's mean and true longitudes differ more or leſs in every point of the earth's orbit except two, *viz.* the aphelion and perihelion; and theſe differences are called *equations of the ſun's centre.* One cauſe of the difference is, that the earth's orbit is an ellipſe, in the periphery whereof the true longitude is reckoned from the vernal equinox, and the mean longitude is reckoned from the ſame point in the circumference of a circle, whoſe diameter is the greater axis of the ellipſe. The circle and ellipſe coincide only in the aphelion and perihelion; and therefore when the ſun or earth is in either of theſe points, the mean and true longitudes are the ſame, and there is no equation.

Another cauſe of the difference between the mean and true longitudes ariſes from the unequal motion of the earth in its orbit: for while the earth proceeds from its aphelion to its perihelion, its motion is continually accelerated; and from its perihelion to its aphelion, its motion is continually retarded; and this is true of every planet.

When the ſun appears in the earth's aphelion, his longitude is about 3s. 9°, and his anomaly is 0, becauſe it is reckoned from that point: and when he appears in the earth's perihelion, his longitude is nearly 9s. 9°, and his anomaly is 6 ſigns. In the firſt ſix ſigns of anomaly, the equation found in the Table is to be ſubtracted from the mean longitude; in the other ſix ſigns it is to be added, and the remainder or ſum is the true longitude.

TABLE VII. *Contains the Logarithms of the Earth's Diſtances from the Sun.*

Becauſe the earth's orbit is an ellipſe, and the ſun in one of its foci, the earth's diſtance from the ſun varies every moment: for it is greateſt in aphelion, and decreaſes as the earth proceeds from thence towards the perihelion, where it is leaſt. The ſeveral diſtances decreaſe in the firſt ſix ſigns of anomaly, and increaſe in the other ſix. When the anomaly is 3 or 9 ſigns, the earth is at its mean diſtance from the ſun, expreſſed by 1000000. In all other points the diſtance is either greater or leſs than the mean.

The earth's diſtance from the ſun being calculated for every degree of anomaly, the logarithms of theſe diſtances are contained in the Table, and are of uſe in computing the longitudes of the other planets.

TABLE VIII. *The Sun's Declination to every Degree of his Longitude.*

At the time of either equinox the ſun is in the equinoctial circle, but at all other times he appears at ſome diſtance from it, either north or ſouth; and this diſtance is called his *declination.*

At the vernal equinox the ſun has no declination; but from that point his declination increaſes northward, until he comes to the ſummer ſolſtice, where it is greateſt; and from thence it decreaſes until the ſun is at the autumnal equinox, when again it is nothing; and then changes its name from north to ſouth, and increaſes ſouthward to the winter ſolſtice, when again it is greateſt; and from thence it decreaſes until the ſun appears in the vernal equinox.

Thoſe points of the ecliptic, which are equally diſtant from the equinoxes or ſolſtices, being alſo equally diſtant from the equinoctial circle, have the ſame declination; and therefore the declination being calculated for every degree of the firſt quadrant of the ecliptic, anſwers for the whole; for the beginning of the ſign Taurus, of Virgo, of Scorpio, and of Piſces, are all at the ſame diſtance from the equinoctial circle, and conſequently theſe points have the ſame declination; only in the two firſt the declination is north, and in the two laſt it is ſouth: and the ſame is true of all other points of the ecliptic which are equally diſtant from the equinoctial or ſolſtitial points.

TABLE IX. *The Sun's Apparent Semidiameter and Hourly Motion.*

When the earth is in its aphelion, the ſun's diameter appears leaſt, and his apparent motion ſloweſt; his diameter at that time is only 31′ 34″, and his hourly motion 2′ 23″. While the ſun moves apparently from the aphelion to the perihelion, his diameter and hourly motion increaſe with his anomaly, and are greateſt in the perihelion; his diameter being then 32′ 38″, and his hourly motion 2′ 33″. The difference between the aphelion and perihelion diameter is 64″, and between the hourly motions 10″.

The Table contains the ſun's ſemidiameter and hourly motion to every 10° of mean anomaly; which, by taking proportional parts for the intermediate degrees, will ſerve to find them for any given anomaly. They are frequently of uſe in Aſtronomy, eſpecially in calculating eclipſes.

TABLE X. Contains the equation of time for every degree of the ſun's longitude; and by uſing proportion for the minutes and ſeconds, the equation may be found for any given longitude. For example, Let it be required to find the equation of time when the ſun's longitude is 6s. 10° 50′ 30″. *Anſ.* — 11m. 12s.

s.	°		m.	s.		°	s.	′	″	s.
6	10	gives	10	57	As	1 :	18 ::	50	30 :	15
6	11	——	11	15	Add				10	57
	Diff.			18	True equation				11	12

TABLE XI. *The Sun's Longitude every Day at Noon.*

The conveniency of having the ſun's longitude nearly true for the noon of every day in the year, inclined me to inſert this Table. The reaſon why it could not be made accurate is, a common year of 365 days differs from the time wherein the ſun ſeems to move round the whole ecliptic by almoſt 6 hours, or a day in four years; which cauſes a difference in the ſun's longitude at noon on the ſame days of different years; and therefore, to have made the Table more perfect, it muſt have been calculated for four years, which was not thought expedient.

TABLES *of the Moon's Mean Motions.*

Theſe Tables are made in the ſame manner as thoſe of the ſun or earth, the moon's period being known: But her motions are more in number than thoſe of the earth; for beſides her mean longitude and anomaly the longitude of her node muſt be known, in order to calculate her place for any given time.

The earth is in one of the foci of the moon's orbit, and is the centre of her motion; for the moon revolves round the earth in the ſame manner as the earth moves round the ſun.

The moon's period, or the time wherein ſhe moves once round the earth, is 27 days, 7 hours, 43 minutes, 5 ſeconds; and to find the moon's mean motion in a common year of 365 days, the proportion is,

As the moon's period 27d. 7h. 43m. 5s.
Is to her whole orbit or 360°;
So is a common year of 365 days
To 13 revolutions, and 4s. 9° 23′ 5″.

The 13 revolutions are rejected, and 4s. 9° 23′ 5″ are taken for the moon's motion in 365 days.

For the Moon's Mean Motion in one Day.

As the moon's period 27d. 7h. 43m. 5s.
Is to the whole circle or 360°;
So is one day
To the moon's mean motion in 1 day 13° 10′ 35″.

Having the moon's mean motion in 1 year and 1 day, the Table of her mean longitudes for Julian years is made by multiplying the motion for 1 year by 2, 3, 4, &c. and to the 4th adding the motion for 1 day, the products are the mean longitudes for theſe years: and by multiplying the motion for 1 year by the ſeveral numbers of years, and adding the motion in 1 day to every 4th or biſſextile year, the Table may be made to any extent.

To calculate the Moon's Mean Anomaly.

The moon's apogee moves once round her whole orbit in 8 years 309 days 8 hours 20 minutes; or (adding 2 days for leap years) in 3231 days 8 hours 20 minutes. Then,

As 3231d. 8h. 20m.
Is to the whole circle or 360°;
So is a common year of 365 days
To the mo. of the ☽'s apogee in 1 year 40° 39′ 50.″

		s	°	′	″
From the ☽'s m. mot. in lon. during 1 year		4	9	23	5
Subtract the motion of her apogee in ditto	—	1	10	39	50
There remains the ☽'s mean anomaly in 1 year		2	28	43	15

To find the Moon's Mean Anomaly for one Day.

Divide the motion of her apogee in a year, viz. 40° 39′ 50″ by 365, and the quotient 6′ 41″ is the motion of the moon's apogee for 1 day.

		°	′	″
From the moon's motion in longitude for 1 day		13	10	35
Subtract the motion of her apogee for ditto	—		6	41
There remains the moon's mean anom. for 1 day		13	3	54

Having found the moon's mean anomaly for 1 year and 1 day, the ſeveral mean anomalies in the Table are found in the ſame manner as the longitudes, viz. by multiplying the mean anomaly for 1 year by 2, 3, 4, &c. and adding the motion for 1 day to every 4th or biſſextile year.

To find the Mean Motion of the Moon's Node.

The moon's node moves backward round her whole orbit in 18 years 224 days 5 hours; therefore, for its motion in one year,

As 18 years 224 days 5 hours
Is to the whole circle or 360°,
So is a year of 365 days
To the motion of the ☽'s node in 1 year 19° 19′ 43″.

For the motion of the moon's node in 1 day, divide 19° 19′ 43″ by 365, and the quotient, which is nearly 3′ 11″, is its motion for 1 day.

The motion of the moon's node for 1 year and 1 day being known, its motion for any number of years is found in the ſame manner as thoſe of mean longitude and anomaly, viz. by multiplying one year's motion by 2, 3, 4, &c. and adding the motion of a day to every fourth or biſſextile year.

TABLE II. *Containing the Moon's Mean Longitude and Anomaly, with the Longitude of her Node, for current Years.*

To make the Table, find, either by calculation or a good obſervation, the moon's mean longitude and anomaly, with the longitude of her node, on the noon of the laſt day of December preceding the year, where the Table is to begin, which here is the year 1760, and theſe are the mean places for 1761, the firſt in our Table. For 1781, the next year in the Table, take the mean motions for 20 years from Table I. and add the longitude and anomaly to thoſe for 1761, but ſubtract the motion of the node; the reſults are the mean places for 1781: then take the mean motions for 10 years, and apply them in the ſame manner to the numbers for 1781, and the mean places for 1791 will be known.

For the following years the Table is carried on by the continual addition of the mean longitude and anomaly for 1 year, as alſo the motion for 1 day more every 4th or biſſextile year, and ſubtracting the motion of the node.

TABLE III. *Contains the Moon's Mean Motions for Days.*

It begins with the mean motions for one day already found. Theſe being multiplied by every number of days from 1 to 365, place the products at the proper days of the ſeveral months, and the Table is made.

TABLE IV. *Contains the Moon's Mean Motions for Hours, Minutes, and Seconds.*

The numbers are found by dividing the mean motions for 1 day by 24, the quotients are the mean motions for an hour; and theſe again divided by 60, give the mean motions for 1 minute, &c. The motions for the different numbers of hours and minutes are found by multiplication.

The moon's motions are affected with many inequalities, and therefore many equations are neceſſary to reduce her mean longitude to the true. The method of finding the arguments, of taking out the equations, and applying them, is deſcribed in Prob. VII.

The Tables for finding the moon's latitude, parallax, diameter, hourly motion in longitude and latitude, at any given time, are ſufficiently plain from their titles; and the method of applying them is deſcribed in Prob. VIII—XIII.

TABLES *of the Mean Motion of the Moon from the Sun.*

The moon's mean motion in a common year of 365 days is 4s. 9° 23′ 5″, over and above 13 revolutions; and the ſun's apparent mean motion in the ſame time is 11s. 29° 45′ 40″.

	S.	°	′	″	
From -	4	9	23	5	
Subtract -	11	29	45	40	
The remainder	4	9	37	25	is the moon's mean motion from the ſun in a common year.

In the ſame manner, the moon's mean motion from the ſun may be found for a day, an hour, or for any number or parts of theſe.

Having the moon's mean motion from the ſun in a year and a day, it may be found for any number of years by multiplication; remembering to add the motion for one day to every four years motion for leap year.

To make a Table of the moon's mean motion from the ſun for current years of the Chriſtian era, ſubtract the ſun's mean longitude from the moon's for every year which the Table is to contain; and the remainders will be the numbers expreſſing the moon's mean motion from the ſun; as in the Table.

The Table for months is made by ſubtracting the ſun's mean longitude from the moon's on the laſt day of the preceding month; and the remainder is the moon's mean motion from the ſun on the firſt day of the following month. The Tables for days, hours, minutes, and ſeconds, are made in the ſame manner; and the method of uſing theſe Tables is deſcribed in Prob. XIV.

TABLE *of Mean New Moons in March, with the Mean Anomalies of the Sun and Moon; and the Sun's Mean Diſtance from the Moon's Node.*

To make Table I. Calculate the time of mean new moon in March, for the year with which the Table is to begin, by the rule, Prob. XIV. or by any other method: Calculate alſo the ſun and moon's mean anomalies for that time, and the ſun's mean diſtance from the moon's node, from the Tables of mean motion, and write them down in order.

For the following years. If the new moon falls after the 11th day of March, add 12 lunations to the time for the former year; and if the next is a common year, ſubtract 365 days from the ſum; the remainder is the time of mean new moon in March the next year; but if the next be a leap year, ſubtract 366 days. If the mean new moon falls before the 11th of March, add 13 mean lunations, and ſubtract 365 or 366 days from the ſum, according as the next is a common or biſſextile year.

Calculate alſo the ſun and moon's mean anomalies, with the ſun's mean diſtance from the moon's node, for 12 or 13 lunations, and add them to thoſe of the former year; and the numbers for the ſecond year in the table will be known. Proceed in the ſame manner for every ſucceeding year until the Table is completed.

TABLE II. *Contains 13½ Mean Lunations, with the Anomalies, and the Sun's Mean Diſtance from the Moon's Node.*

The numbers are computed from the Tables of mean motion thus: Take the mean anomalies of the ſun and moon out of the Tables for 29 days 12 hours 44m. 3s.; take alſo the mean motion of the ſun, and of the moon's node, out of their proper Tables for the ſame time; their ſum is the ſun's mean diſtance from the moon's node. Having found the numbers for one lunation, multiply them by 2, 3, 4, &c. until there are 13 products, and divide the numbers for one lunation by 2 for the half lunation.

By theſe Tables the times of mean ſyzygies for any month of a given year, within the limits of Table I. may be found.

The Tables of equations for reducing the mean to the true times of new and full moons, in the following pages, depend on the mean anomalies of the ſun and moon, with the ſun's mean diſtance from the moon's node, and are expreſſed in time for the conveniency of calculation. The method of applying them is given in Problem XVI.

Conſtruction and Uſe of Logiſtical Logarithms.

Logiſtical Logarithms are artificial numbers deduced from common logarithms, and made for minutes and ſeconds either of degrees or time, uſed in working proportions, wherein ſome or all of the given terms are ſexageſimals.

The numbers which compoſe the Table of logiſtical logarithms are regulated by its extent. For if the Table contains no more than the minutes and ſeconds in 1°, or 3600″, the logiſtical logarithm of 1″ is the logarithm of 3600″, *viz.* 3.5563; and all the following numbers are the differences between 3.5563 and the logarithms of the ſeveral numbers of ſeconds from 1 to 3600″. Therefore to calculate the logiſtical logarithm of any quantity leſs than 1°, ſubtract the logarithm of the given number of ſeconds from 3.5563; the remainder is the logiſtical logarithm required.

Exam. Let it be required to find the logiſtical logarithm of 10′, or 600″.

From the common log. of 3600 = 3.5563
Subtract the common log. of 600 = 2.7781

There remains the logiſtical log. of 10 = 0.7782

But if the Table is to contain more than 1°, ſuch as in Dr. Maſkelyne's Proportional Logarithms, which extends to 3°, or 10800″; then, in ſuch a Table, the proportional or logiſtical logarithm of 1″ is the logarithm of 10800, *viz.* 4.0334; and all the reſt are the differences between 4.0334 and the logarithms of the ſeveral numbers of ſeconds from 1 to 10800″.

Exam. Let it be required to find the logiſtical or proportional log. of 10′ or 600″.

From the log. of - 10800 = 4.0334
Subtract the log. of - 600 = 2.7781

Remains the prop. log. of - 10′ 1.2553

Hence there may be as many different ſyſtems of logiſtical or proportional logarithms as any one chooſes to aſſume different extents of the Table. Our Table extends only from 1″ to 1°, which is ſufficient for common uſe. Logiſtical logarithms conſiſt of four figures beſides the index.

The numbers on the head of the Table are either degrees or minutes; and thoſe in the left hand column, on the ſide, are minutes or ſeconds; but if the numbers on the head be hours or minutes, thoſe in the left hand column, on the ſide, will be minutes or ſeconds of time. Therefore theſe numbers change their denomination as occaſion requires. The ſecond line on the head of the Table are the numbers of ſeconds in the minutes which ſtand over them.

To take the logiſtical logarithm of any number of minutes and ſeconds out of the Table, the given number muſt be within the limits of the Table.

Find the minutes on the head of the Table, and the ſeconds in the left hand column on the ſide, and under the minutes and oppoſite to the ſeconds ſtands the logiſtical logarithm required.

Examples.

	′	″		
Given numbers	0	40	Logiſt. log.	1.9542
	1	10		1.7112
	7	40		8935
	9	15		8120
	50	0		792
	50	37		739

A logiſtical logarithm being given, to find the number of minutes and ſeconds anſwering.

If the given logarithm be found in the Table, the minutes are on the head of the column and the ſeconds on the ſide; but if be not found exactly in the Table, take the next greater than it, and the minutes and ſeconds anſwering are found on the head and ſide of the Table, as before.

Examples.

L. Log.	Values. m.	s.
1.7112	1	10
9823	6	15
6746	12	41
5175	18	13

When the logiſtical logarithm of any number leſs than 3600 is required, find it, or the neareſt leſs, in the ſecond line on the head of the Table; and below the number on the head, and oppoſite to its exceſs on the ſide, ſtands the logiſtical logarithm. Thus the logiſtical logarithm of 1276 is 4505.

Logiſtical logarithms are uſed in finding a fourth proportional, when ſome, or all of the given terms, are minutes and ſeconds either of motion or time: and the method of operation is the ſame as in working with common logarithms, that is, by adding the logarithms of the ſecond and third terms, and ſubtracting the logarithm of the firſt term from the ſum, the remainder is the logarithm of the fourth term, or anſwer.

Exam. 1. Suppoſe the ſun's hourly motion was 2′ 27″, and it were required to find his motion in 47m. 10s. The anſwer is 1′ 55″.

As 60m.	0
To 2′ 27″	13890
So 47m. 10s.	1045
To 1′ 55″	14935

2. Suppoſe the hourly motion of the moon from the ſun is 32′ 8″, and the moon is 58′ 12″ behind the ſun: In what time will the moon overtake the ſun? *Anſ.* 1h. 48m. 40s.

N. B. In ſuch queſtions, when the fourth proportional would exceed 60m. which is beyond the limit of the Table, take $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$, of the third term, and multiply the fourth term, when found, by 2, 3, or 4.

As 32′ 8″	2712
To 60m.	0
So 29′ 6″	3143
To 54m. 20s.	431
2	

Doubled is 1h. 48m. 40s. the anſwer.

In like manner, when two of the given terms exceed the limits of the Table, divide all the terms by ſome number, ſuch as 2, 3, or 4, and multiply the anſwer by the ſame number which divided the given terms.

		Half of each term.	
Suppoſe we have,	As 76′ 34″	38′ 17″	1951
	Is to 24h.	12h.	6990
	So is 64′ 20″	32′ 10″	2707
	To 20h. 10m. nearly	10h. 5m.	9697
	Doubled	20h. 10m.	7746

PROBLEMS,

PROBLEMS, SHEWING THE USE AND APPLICATION OF THE TABLES, AND PROJECTION OF ECLIPSES.

PROB. I. *The Longitudes of two Places, and the Time at one of them being given, to find the correſponding Time at the other.*

REDUCE the difference of the longitudes to time, by allowing at the rate of 15° to an hour, and add it to the given time for a place towards the eaſt; but ſubtract it for a place toward the weſt.

EXAM. 1. Suppoſe the time at Greenwich to be 6h. 7′ 8″ P. M. required the correſponding time at Cambridge, in longitude 71° 6′ 45″ weſt.

	h.	′	″	
Time at Greenwich	6	7	8	P. M.
Diff. long. 71° 6′ 45″ = —	4	44	27	
Time at Cambridge	1	22	41	P. M.

EXAM. 2. The time at Cambridge being 7h. 43′ 57″ A. M. required the correſponding time at Greenwich.

	h.	′	″	
Time at Cambridge	7	43	57	A. M.
Diff. long. 71° 6′ 45″ = +	4	44	27	
Time at Greenwich		28	24	P. M.

EXAM. 3. The time at Hamburgh, in longitude 10° 38′ E. being 6h. 30′ 40″ A. M. required the correſponding time at Cambridge, in longitude 71° 6′ 45″ W.

	h.	′	″	
Time at Hamburgh	6	30	40	A. M.
Diff. long. 71° 17′ 23″ = —	5	26	59	
Time at Cambridge	1	3	41	A. M.

PROB. II. *To calculate the true Longitude of the Sun for any given Time and Place by the Tables.*

When the given time is apparent, reduce it to mean time, and if the given place be not in the meridian of the Tables, that is, Greenwich, reduce it to that meridian by Prob. I. and then write down the years, months, days, hours, &c. under one another in a column on the left hand.

Take out of the Tables the ſun's mean longitude and anomaly, anſwering to each part of the time, and write them down on the right hand of the former numbers; then add them as they ſtand in their ſeveral columns, rejecting 12 ſigns, or any multiple thereof, and the ſum will be the mean longitude of the ſun for the given time.

To reduce the mean longitude to the true: Enter the Table with the ſun's mean anomaly, and take out the equation of the centre, making proportion for the odd minutes, &c. and if the ſign of the ſun's mean anomaly be at the head of the Table, ſubtract the equation from the mean longitude; but if it be at the foot of the Table, add it to the mean longitude, and the true longitude of the ſun will be had.

Exam. 1. Required the ſun's longitude 1796, Feb. 19. 11h. 21m. 30s. in the forenoon, apparent time, in the meridian of Edinburgh.

	D.	h.	m.	s.
Given time February - -	18	23	21	30
Edinburgh meridian W. - -	+		12	25
Equation of time - -	+		14	14
Mean time at Greenwich -	18	23	48	9

	M. Long.				M. Anom.			
	S.	°	′	″	S.	°	′	″
1796 - -	9	10	51	52	6	1	23	42
Feb. 18. biſſex. -	1	17	18	40	1	17	18	31
23h. - -			56	40			56	40
48m. - -			1	58			1	58
9s. - -	10	29	9	10	7	19	40	51
Equation of the centre	+	1	28	30				
Sun's true longitude	11	0	37	40				

2. Required the ſun's longitude 1795, June 22, at noon, apparent time, at Greenwich.

	h.	m.	s.
Mean time - - -	0	0	0
Equation of time - -	+	1	28
	0	1	28

	M. Long.				M. Anom.			
	S.	°	′	″	S.	°	′	″
1795 - -	9	10	7	2	6	0	39	58
June 22. - -	5	20	31	1	5	20	30	30
0h. 1m. - -				2				2
28″ - -				1				1
	3	0	38	6	11	21	10	31
Equation of the centre		+	17	23				
Sun's true longitude	3	0	55	29				

PROB. III. *The Mean Anomaly of the Sun being given, to find the Logarithm of his Diſtance from the Earth.*

With the mean anomaly of the ſun take out the logarithm of his diſtance from the earth from the Table.

Exam. Let the ſun's mean anomaly be 7s. 19° 40′ 51″, what is the logarithm of his diſtance from the earth?

For 7s. 19° the log. is	-	4.995224
For 40′ 51″ the part is	-	+ 69
Anſwer	-	4.995293

PROB. IV. *To find the Apparent Semidiameter, and alſo the Hourly Motion of the Sun for any given Time.*

Calculate the ſun's mean anomaly for the given time, and with the mean anomaly of the ſun take out his ſemidiameter and hourly motion from the Table.

Exam. Required the ſun's apparent ſemidiameter and hourly motion 1796, February 19th, 11h. 21m. in the forenoon.

The ſun's mean anomaly at that time is 7s. 19° 42′, which gives his ſemidiameter 16′ 13″ and his hourly motion 2′ 31″.

PROB. V. *The Longitude of the Sun, or any Point of the Ecliptic, being given, to find the correſponding Declination.*

With the given longitude take the declination out of the Table.

Exam. Let the ſun's longitude be 11s. 0° 37′ 40″, what is his declination? *Anſ.* 11° 15′ 47″ ſouth.

To longitude 11s. 0° the declination is 11° 29′ 5″ S. decreaſing; the difference between this and the next is 21′ 12″: Then,

		° ′ ″	L. L.		° ′ ″
As	-	1 0 0	0		
To	-	0 21 12	4518	From	11 29 5
So is	-	0 37 40	2022	Subtract	— 13 18
To	-	0 13 18	6540	Declinat.	11 15 47 S.

PROB. VI. *The Time of the Year and the Sun's Declination being given, to find his Longitude; or the Declination of a Star, or of any Point of the Ecliptic, being given, to find its Longitude.*

With the given declination take the longitude out of the ſame Table as in the preceding problem: Thus, take the declination in the Table neareſt leſs than that given, and write down the correſponding longitude; marking whether the declination be increaſing or decreaſing: then ſubtract the declination found in the Table from the next greater, the remainder is the difference of declination for 1° of longitude. Subtract alſo the declination found in the Table from that given; and then, as the difference of declination for 1° of longitude is to 1°, ſo is the difference between the given declination and that found in the Table to a part, to be applied to the longitude already found; the reſult is the true longitude.

Exam. February 19, 1796, the ſun's declination is 11° 15′ 47″ S. what is his longitude? *Anſ.* 11s. 0° 37′ 39″.

1ſt, To declin. 11° 7′ 53″ S. the long. is 11s. 1°, the declination decreaſing.

	°	′	″
2d, From the given declin.	11	15	47
Subtract the next leſs in the Table	11	7	53
Difference	0	7	54
3d, Alſo from the next greater	11	29	5
Subtract the next leſs in the Table	11	7	53
Diff. declin. for 1° long.	0	21	12

	°	′	″	L. L.
4th, As	0	21	12	4518
To	1	0	0	0
So is	0	7	54	8805
To	0	22	21	4287

To be ſubtracted.

	s.	°	′	″
From the longitude found	11	1	0	0
Subtract the prop. part		—	22	21
There remains the true longitude	11	0	37	39

PROB. VII. *To calculate the true Longitude of the Moon for any given Time by the Tables.*

Calculate the true longitude of the ſun and his mean anomaly for the given time by Prob. II.

From the Tables of the moon's mean motion, take out the mean longitude, anomaly, and aſcending node, for the given year; under which write down the mean motions for the month, day, hours, &c.

Add the numbers in the columns of the mean longitude and anomaly, rejecting 12 ſigns or any multiples thereof when they occur; but from the longitude of the node for the given year ſubtract the ſum of all the numbers below it, borrowing 12 ſigns when neceſſary; and thus the moon's mean longitude, mean anomaly, and the mean longitude of her node, will be obtained.

For the Arguments of the Equations.

The ſun's mean anomaly is the firſt argument. Subtract the true longitude of the ſun from the mean longitude of the moon, the remainder is the mean diſtance of the moon from the ſun, of which take the double.

To and from the double diſtance of the moon from the ſun add and ſubtract the firſt argument, the ſum and remainder are the ſecond and third arguments.

Add and ſubtract the moon's mean anomaly to and from the double diſtance of the moon from the ſun for the fourth and fifth arguments.

To and from the fifth argument add and ſubtract the firſt argument, and the ſixth and ſeventh arguments are found.

Subtract the firſt argument from the moon's mean anomaly, the remainder is the eighth argument.

Subtract the moon's mean anomaly from the moon's mean diſtance from the ſun, the remainder is the ninth argument.

Subtract the true longitude of the ſun from the mean longitude of the moon's node, and the remainder is the tenth argument.

With theſe arguments take the ſeveral equations out of the Tables, and find their ſum.

With the mean anomaly of the ſun take out of the Tables the equations of the node and of the moon's mean anomaly. Apply the equation to the longitude of the node, and its equated longitude will be known. Correct the mean anomaly of the moon by its own equation, and alſo by the ſum of the ten former equations, and the correct anomaly of the moon will be had; which is the eleventh argument, with which take out of the Table the equation of the centre. Correct the mean longitude of the moon both by the ſum of the ten former equations and by the equation of the centre, and the equated longitude of the moon will be known.

Subtract the ſun's true longitude from the equated longitude of the moon, the remainder is the twelfth argument, with which take the variation out of the Table, and apply it to the equated longitude of the moon.

Subtract the correct longitude of the node from the longitude of the moon twelve times corrected, and from double of the remainder ſubtract the moon's correct anomaly, the remainder is the 13th argument; with which take the thirteenth equation from the Table and apply it to the moon's longitude laſt found, which will give the moon's longitude in her orbit.

From the moon's longitude in her orbit ſubtract the correct longitude of the node, the remainder is the fourteenth argument; with which take out of the Table the reduction of the moon's place in her orbit to the ecliptic. And, laſtly, with the mean longitude of the moon's node take the equation of the equinoxes from the Table; and apply both the reduction and the equation of the equinoxes to the moon's longitude in her orbit, according to their ſigns, and the true ecliptic longitude of the moon will be found.

Exam. It is required to find the moon's longitude 1795, June 22, at noon, apparent time, in the meridian of Greenwich.

The equation of time is + 1 m. 28 s. therefore the mean time is 0h. 1 m. 28 s.
Sun's longitude 3 s. 0° 55′ 29″. Mean anomaly, 11 s. 21° 10′ 31″.

	☽'s m. Long.	m. Anom.	☊		Equations.	
	S. ° ′ ″	S. ° ′ ″	S. ° ′ ″		+ ′ ″	− ′ ″
1795 -	1 5 37 57	6 13 36 29	4 9 57 43			
June 22. -	3 29 31 0	3 10 14 35		1		1 42
0h. 1m. -	33	33	− 9 9 40	2		0 48
28s. -	15	15		3		0 52
			4 0 48 3	4	0 50	
	5 5 9 45	9 23 51 52	−1 24	5	20 36	
Sum of 10 eqs.	+ 19 0	−3 18		6		0 14
			4 0 46 39	7		0 19
	5 5 28 45	9 23 48 34		8		0 36
Equat. centre	+ 5 35 20	+ 19 0		9	1 14	
				10	0 51	
☽'s equat. long.	5 11 4 5	9 24 7 34 = Arg. 11th.				
Variation	+ 20 48				+ 23 31	−4 31
					− 4 31	
	5 11 24 53					
13th -	+ 46				+ 19 0	
☽'s orbit long.	5 11 25 39					
Reduction	−6 43					
	5 11 18 56					
Equinoxes	− 15					
☽'s eclip. long.	5 11 18 41					

Arguments found.

	s.	°	′	″
☽ - -	5	5	9	45
☉ - -	3	0	55	29
☽ from ☉ -	2	4	14	16
Doubled -	4	8	28	32
☉ Anom. ±	11	21	10	31
2d = - -	3	29	39	3
3d = - -	4	17	18	1
	4	8	28	32
☽ Anom. ± -	9	23	51	52
4th = - -	2	2	20	24
5th = - -	6	14	36	40
±	11	21	10	31

Arguments found.

	s.	°	′	″
6th = - -	6	5	47	11
7th = - -	6	23	26	9
	9	23	51	52
	—11	21	10	31
8th = - -	10	2	41	21
	2	4	14	16
	—9	23	51	52
9th = - -	4	10	22	24
☋ - -	4	0	48	3
☉ - -	—3	0	55	29
10th = -	0	29	52	34
☽'s eq. long.	5	11	4	5

Arguments found.

	s.	°	′	″
	—3	0	55	29
12th = -	2	10	8	36
☽'s long. equat.	5	11	4	5
☋	—4	0	46	39
	1	10	17	26
Doubled -	2	20	34	52
☽'s Anom. —	9	24	7	34
13th = -	4	26	27	18
☽ in orbit -	5	11	25	39
☋ - -	—4	0	46	39
14th = -	1	10	39	0

PROB. VIII. *The Moon's Longitude, Anomaly, and Node, being given, to find her Latitude by the Tables.*

Subtract the corrected longitude of the moon's node from the moon's longitude in her orbit, the remainder is the firſt argument of latitude.

From the moon's longitude in her orbit ſubtract the ſun's longitude; and from double of the remainder ſubtract the firſt argument of latitude, the remainder is the ſecond argument.

Subtract the moon's mean anomaly from the firſt argument of latitude, and the remainder is the third argument.

From the third argument ſubtract the moon's mean anomaly for the fourth argument; alſo ſubtract the moon's mean anomaly from the fourth argument, and the remainder is the fifth argument. With theſe arguments take out the numbers with their proper ſigns from the Tables, and take the ſum of thoſe which are affirmative, and alſo the ſum of the negative; the difference of theſe ſums, with the ſign of the greater, will be the latitude of the moon; and will be north if marked with the ſign +, but ſouth if marked with the ſign —.

Exam. It is required to find the moon's latitude when her longitude in her orbit is 5s. 11° 25′ 39″, mean anomaly 9s. 23° 51′ 52″, and node 4s. 0° 46′ 39″. The ſun's longitude is 3 s. 0° 55′ 29″.

Arguments found.

	S.	°	′	″
☽'s orbit lon.	5	11	25	39
☊ -	—4	0	36	39
Arg. 1ſt =	1	10	49	0
☽ -	5	11	25	39
☉ -	—3	0	55	29
	2	10	30	10
Doubled -	4	21	0	20
1ſt -	—1	10	49	0

Arguments found.

	S.	°	′	″
2d = -	3	10	11	20
1ſt - -	1	10	49	0
☽'s Anom.	—9	23	51	52
3d = -	3	16	57	8
	—9	23	51	52
4th = -	5	23	5	16
	—9	23	51	52
5th -	7	29	13	24

Equations.

	+			—
	°	′	″	″
1	3	21	43	
2	0	8	40	
3				16
4				2
5				16
	3	30	23	34
	—		34	
	+3	29	49	

Anſ. The moon's lat. is 3° 29′ [49″ N.

PROB. IX. *The Moon's Longitude, with the Arguments for finding it, being known, to find the Moon's Equatorial Parallax or the Horizontal Parallax of the Moon for any Place on the Equator.*

With the 5th, 11th, and 12th arguments of longitude, take as many numbers from the Tables, with their proper ſigns; and from the ſum of the affirmative numbers ſubtract the ſum of the negative, the remainder is the moon's equatorial parallax, or the horizontal parallax of the moon to a place on the equator.

EXAM. Let the moon's longitude be 5s. 11° 18′ 41″, and the arguments as below; required her equatorial parallax. *Anſ.* 56′ 5″.

	S.	°	′	″	+ ′	+ ″	− ″
5th Arg. -	6	14	36	40	0	36	
11th - - -	9	24	7	34	55	49	
12th -	2	10	8	36			20
					+ 56	25	
					—	20	
Moon's equatorial parallax					56	5	

PROB. X. *The Equatorial Parallax of the Moon being given and the Latitude of a Place, to find the Horizontal Parallax of the Moon at that Place; and to reduce the Apparent Latitude of the Place to the Centre of the Earth, ſuppoſing the Earth is an Oblate Spheroid.*

Enter the Table, with the given apparent latitude of the place on the ſide, and the equatorial parallax at the top, and making proportion, if neceſſary, find the reduction of parallax; which ſubtracted from the equatorial parallax, leaves the horizontal parallax required.

With the apparent latitude of the place take the reduction of latitude out of the ſame Table; which being ſubtracted from the apparent latitude, leaves the latitude reduced.

N. B. The horizontal parallax and latitude thus found are what ſhould be employed in computing the moon's parallaxes in longitude, latitude, and declination; which will now be performed by the common rules, founded on the ſuppoſition of the earth's being a ſphere.

EXAM. Let the equatorial parallax be 56′ 5″, and the apparent latitude of the place 51° 28′ 40″; the moon's horizontal parallax and the reduced latitude of the place are required.

	′	″		°	′	″
Equatorial parallax	56	5	Latitude -	51	28	40
Reduction - -	—	9	Reduction -	—	14	42
Horizontal parallax	55	56	Reduced lat.	51	13	58

PROB. XI. *The Equatorial Parallax being given, to find the Moon's Horizontal Diameter.*

Enter the Table, with the moon's equatorial parallax, and take out her horizontal diameter.

EXAM. The moon's equatorial parallax being 56′ 5″, the diameter is 30′ 34″.

PROB. XII. *To find the Moon's Hourly Motion in Longitude; her Longitude in her Orbit, with the Arguments for finding it, being known.*

With the 5th, 11th, and 12th arguments of longitude, take out the numbers from the Tables, the ſum of theſe, regard being had to their ſigns, is the moon's hourly motion in longitude.

Exam. Let the moon's longitude be 5s. 11° 25′ 39″, and the arguments as below; required her hourly motion in longitude.

		s.	°	′	″	+ ′	+ ″	− ″
5th Arg.	-	6	14	36	40	0	41	
11th	-	9	24	7	34	31	20	
12th	-	2	10	8	36			31
						+32	1	
						−	31	
Anſwer	- -					31	30	

PROB. XIII. *To find the Moon's Hourly Motion in Latitude, the Arguments for finding her Latitude being known.*

With the firſt and ſecond arguments of latitude, take two numbers out of the Tables, their ſum if they have the ſame ſign, or their difference if they have contrary ſigns, is the moon's hourly motion in latitude; which tends to the north if it has the ſign +, but to the ſouth if it has the ſign —.

Exam. It is required to find the moon's hourly motion in latitude, the arguments of latitude being,

		s.	°	′	″	′	″
1ſt Arg.	-	1	10	39	0	+2	15
2d	- -	3	10	21	21	—	1
Anſwer	-					+2	14 tending north.

PROB. XIV. *To find the Time of the Mean Syzygies in any given Year and Month.*

From the Tables of the mean motion of the moon from the ſun take out the motions for the given year and month; add them, and ſubtract the ſum from 12 ſigns: then, if the time of mean conjunction or new moon be ſought, the remainder, or the neareſt leſs than it, being found in the Table of days, will give the day of mean new moon; and after ſubtracting the number found in the Table, the remainder or the next leſs is to be found in the Table of hours, which will give the hour of the day when it is mean new moon; and after another ſubtraction, the remainder is to be found in the Table of minutes for the minute of mean new moon, and the next remainder will give the ſeconds. But if the time of mean oppoſition or full moon be ſought, add 6 ſigns to the firſt remainder, and the ſum being found in the Table of days, will give the day of mean full moon: the hours and minutes are to be found as before.

EXAM. 1. Required the time of mean new moon in January 1797. *Anſ.* Jan. 27d. 7h. 40m. 21s.

	s.	°	′	″	
1797	—0	26	57	10	
	12	0	0	0	
Remains	11	3	2	50	
27 days	—10	29	9	0	
		3	53	50	
7 hours		—3	33	20	
			20	30	
40 minutes			—20	19	5‴
21 ſeconds				10	55

In this operation the Tables give the moon's mean diſtance from the ſun on the 1ſt day of January 0s. 26° 57′ 10″, which is ſubtracted from 12 ſigns to find how much the moon wants of a conjunction with the ſun: the remainder ſhows that ſhe wants 11s. 3° 2′ 50″. In the Table of days it is found that ſhe moves over 10s. 29° 9′ in 27 days; therefore the mean new moon happens on the 27th day of January.

Subtract 10s. 29° 9′ from 11s. 3° 2′ 50″, there remains 3° 53′ 50″, which in the Table of hours gives 7 hours; and ſubtracting 3° 33′ 20″, there remains 20′ 30″, which gives 40 minutes; and by ſubtracting again there remains 10″ 55‴, and this gives 21 ſeconds.

2. It is required to find the time of mean full moon in January 1797. *Anſ.* Jan. 12d. 13h. 18m. 18s.

	s.	°	′	″	
1797	—0	26	57	10	
	12	0	0	0	
	11	3	2	50	
Add	6	0	0	0	
	5	3	2	50	
12 days	—4	26	17	20	
		6	45	30	
13 hours		—6	36	12	
			9	18	
18 minutes			—9	8	35‴
18 ſeconds				9	25

By this problem the times of mean new and full moon may be calculated for every month in the year, or any longer time, with very little trouble; for having found the time of mean conjunction and oppoſition in the month of January, add to theſe times a mean lunation, viz. 29d. 12h. 44m. 3s. continually, rejecting the days in the month wherein the mean new or full moon is required, and the times will be known.

Exam. Let it be required to find the mean new and full moons in every month of the year 1797.

Mean full moon.						Mean new moon.					
		D.	h.	m.	s.			D.	h.	m.	s.
January	-	12	13	18	18	January	-	27	7	40	21
One lunation	+	29	12	44	3	One lunation	+	29	12	44	3
February	-	11	2	2	21	February	-	25	20	24	24
March	-	12	14	46	24	March	-	27	9	8	27
April	-	11	3	30	27	April	-	25	21	52	30
May	-	10	16	14	30	May	-	25	10	36	33
June	-	9	4	58	33	June	-	23	23	20	36
July	-	8	17	42	36	July	-	23	12	4	39
Auguſt	-	7	6	26	39	Auguſt	-	22	0	48	42
September	-	5	19	10	42	September	-	20	13	32	45
October	-	5	7	54	45	October	-	20	2	16	48
November	-	3	20	38	48	November	-	18	15	0	51
December	-	3	9	22	51	December	-	18	3	44	54

Note. The mean and true ſyzygies never happen at the ſame time, except when the moon is in or very near her apogee or perigee; for in theſe points many of her inequalities either vaniſh or are very ſmall: when the moon is in any other point of her orbit, there is ſome interval of time between the mean and true conjunctions or oppoſitions. The greateſt is about 14 hours.

PROB. XV. *The Time of Mean Conjunction or New Moon being given, to find the true Time.*

Calculate the longitudes of both ſun and moon for the time of mean conjunction; (Prob. II.—VII.) and if they are equal to one another, the mean and true conjunctions happen at the ſame time; but if they differ, ſubtract the leaſt from the greateſt. Find the hourly motions of both ſun and moon at the time, (Prob. IV—XII.) and ſubtract the ſun's hourly motion from the moon's, the remainder is the hourly motion of the moon from the ſun; and then,

As the moon's hourly motion from the ſun
Is to one hour or 60 minutes;
So is the difference between the ſun and moon's long.
To the time between the mean and true conjunction.

If the moon's longitude be leſs than the ſun's, the interval is to be added to the time of mean conjunction; but if it be greater, ſubtract the interval: the ſum or remainder is the time of true conjunction.

When it is required to find the time of true conjunction very accurately, the ſun and moon's longitude muſt be calculated again for the time found; and their difference, if there be any, turned into time, and applied to the time laſt found, will give the true time of conjunction.

If the difference be great, it will be neceſſary to renew the operation again.

Exam. It is required to find the time of true conjunction in January 1797; the time of mean conjunction or new moon being January 27th, 7h. 40m. 21s. in the meridian of Greenwich. *Anſ.* Jan. 27d. 13h. 40m.

The ſun's longitude then is 10s. 8° 28′ 38″, and his mean anomaly 6s. 28° 3′ 51″.

For the Moon's Longitude.

	m. Long.				Apogee.				☊			
	S.	°	′	″	S.	°	′	″	S.	°	′	″
1797 -	10	7	34	43	0	24	6	53	3	1	15	6
Jan. 27th -	11	25	45	46	11	22	45	17				
7h. -		3	50	35		3	48	38	—	1	25	47
40m. -			21	57			21	46				56
21s. -				11				11				5
	10	7	33	12	0	21	2	45	2	29	48	18
10 equations			+ 25	18			—10	25			— 4	25
	10	7	58	30	0	20	52	20	2	29	43	53
Equat. of centre	—	2	9	7			+25	18				
	10	5	49	23	0	21	17	38	= 11th arg.			
Variation -			— 3	15								
	10	5	46	8								
13th -			— 1	5								
☽'s orb. long.	10	5	45	3								
☉'s long. -	10	8	28	38								
Short of conj.		2	43	35								

	Arguments.			
	S.	°	′	″
1	6	28	3	51
2	5	26	12	59
3	5	0	5	17
4	0	19	11	53
5	11	7	7	6
6	6	5	10	14
7	4	8	2	32
8	5	22	58	54
9	11	8	1	49
10	4	21	19	40
12	11	27	20	45
13	1	20	46	52

	Equations.			
	+		—	
	′	″	′	″
1			5	15
2			0	4
3			0	38
4	0	19		
5	30	53		
6			0	11
7	0	37		
8	0	5		
9	0	31		
10			0	59
	+32	25	7	7
	— 7	7		
Sum	+25	18		

	′	″
Moon's hourly motion - -	29	47
Sun's ditto. - -	2	33
Hourly motion ☽ from ☉ - -	27	14

As 27′ 14″
To 1h.
So 2° 43′ 35″
To 6h. 0m. 24s.

	D.	h.	m.	s.
Mean conjunction January -	27	7	40	21
Interval add - -	+	6	0	24
Anſ. New Moon January -	27	13	40	45

The time of true oppoſition or full moon is found in the ſame manner as the time of conjunction. The operation is to be continued until the difference of the ſun and moon's longitude be 6 ſigns.

The rules here given for calculating the times of new and full moon are always good; but the tedioufneſs of the operation has given occaſion to the invention of other Tables, whereby the problem may be ſolved with leſs trouble. The beſt of theſe were publiſhed among Dr. Halley's Aſtronomical Tables in the year 1749, but not as his invention. The ingenious Mr. James Ferguſon having adapted them to the Gregorian ſtyle, and made ſome alterations in the arrangement, printed them in his Aſtronomy about the year 1760. Such of them as were judged proper for this work are to be found in the Tables of mean new moons, &c. extended to the year 1821.

Theſe Tables contain the times of mean conjunction or new moon in March for every year, which is made the firſt month with the mean anomalies of the ſun and moon, and alſo the ſun's mean diſtance from the moon's node at the time, with equations for reducing the mean to the true time of conjunction or oppoſition, whereby theſe times may be found for any month of a given year within the limits of the Table.

PROB. XVI. *To calculate the Time of New or Full Moon for any Month of a given Year by the Tables.*

1ſt, For the Month of March.

Write down the time of mean new moon in March for the given year, with the mean anomalies of the ſun and moon, and alſo the ſun's mean diſtance from the moon's aſcending node, out of the Table.

When the time of Full Moon in March is required.

If the new moon happens before the 15th of the month, add half a lunation, with the anomalies, &c. to the former numbers for new moon, the ſum is the time of full moon; but if it happens after the 15th, ſubtract half a lunation with the anomalies from the numbers for new moon, and the time of mean full moon in March will be known.

2d, For any Month after March.

When the time of the mean new or full moon is required in any month after March, take out the numbers for March as before, and under them write down as many lunations with their anomalies as the given month is after March; and by the ſum of theſe, the time of mean new or full moon may be known, together with the mean anomalies and the ſun's diſtance from the moon's node; which are the arguments for finding the ſeveral equations, to reduce the time of the mean ſyzygies to the true.

With the ſum of the days enter the Table of days, under the given month; and oppoſite to that number in the left hand column is the day of the mean ſyzygies; but if the ſum be leſs than any of thoſe under the given month, add a lunation with the anomalies to the former numbers, and then enter the Table with the ſum under the given month, and in the left hand column is the day of the month required.

The time of mean ſyzygy being known, to find the true by the Tables.

1. With the ſun's mean anomaly enter the Table, and take out the firſt equation, (making proportion for the odd minutes, &c.) and apply it to the time of the mean ſyzygy, according to its ſign.

2. With the ſun's mean anomaly take the equation of the moon's mean anomaly out of the Table, and apply it according to its ſign, and the moon's equated anomaly will be known.

3. With the moon's equated anomaly take the ſecond equation out of the Table, which being applied to the former time, according to its ſign, the reſult will be the time of the ſyzygy very nearly.

4. Subtract the moon's equated anomaly from the ſun's mean anomaly, and with the remainder take the third equation out of the Table, and apply it to the former equated time according to its ſign.

Laſtly, with the ſun's mean diſtance from the moon's node take the fourth equation out of the Table, and apply it to the laſt found equated time, according to its ſign, and the reſult is the time of the true ſyzygy.

Exam. 1. Required the time of full moon in January, 1797.

	D. h. m. s.	☉'s anom. S. ° ′ ″	☽'s anom. S. ° ′ ″	☉'s diſt. from ☊. S. ° ′ ″
1796, March -	8 11 39 44	8 8 3 47	3 7 2 14	8 0 26 47
½ lunation -	+ 14 18 22 2	0 14 33 10	6 12 54 30	0 15 20 7
Full in March	23 6 1 46	8 22 36 57	9 19 56 44	8 15 46 54
Add 10 lunations	295 7 20 30	9 21 3 14	8 18 10 4	10 6 42 20
1797, January -	12 13 22 16	6 13 40 11	6 8 6 48	6 22 29 14
1ſt equation -	+ 1 0 30	—6 8 29 38	+ 22 50	arg. 4th equat.
	12 14 22 46	0 5 10 33	6 8 29 38	
2d equation -	— 1 20 45	arg. 3d equat.	arg. 2d. equat.	
	12 13 2 1			
3d equation -	— 26			
4th equation -	+ 1 6			
Anſ. January -	12 13 2 41			

2. It is required to find the time of conjunction, or new moon, in the month of Auguſt, 1820.

	D. h. m. s.	☉'s Anomaly. S. ° ′ ″	☽'s Anomaly. S. ° ′ ″	☉'s diſt. from ☽'s ☊. S. ° ′ ″
1820, March -	14 1 42 45	8 12 25 31	6 24 37 23	11 19 35 34
5th luna. -	147 15 40 15	4 25 31 37	4 9 5 2	5 3 21 10
Auguſt -	8 17 23 0	1 7 57 8	11 3 42 25	4 22 56 44
1ſt equat. -	— 2 31 47	—11 2 44 51	— 57 34	arg. 4th equat.
	8 14 51 13	2 5 12 17	11 2 44 51	
2d equat. -	— 4 47 7	arg. 3d equat.	arg. 2d equat.	
	8 10 4 6			
3d equat. -	— 4 27			
4th - -	— 1 29			
Anſ. Aug. -	8 9 58 10			

Note 1. Theſe examples are wrought for the meridian of Greenwich; but the anſwers may be reduced to the time under any other meridian by adding or ſubtracting the difference of longitude in time to or from them, according as the place is ſituated to the eaſt or weſt of Greenwich.

2. Theſe tables give the times of new and full moon with little trouble, and ſufficiently true for common uſe; being rarely above 1 or 2 minutes wide of the truth: but when it is required to find the moment of conjunction or oppoſition accurately, calculate the longitudes of the ſun and moon for the time found by the tables; and if they are the ſame, or differ 6 ſigns, according as the new or full moon is required, the time is truly found; but if not, take their difference. Find the hourly motion of the moon from the ſun; and then,

As the hourly motion of the moon from the ſun
Is to 60 minutes;
So is the difference between the ſun and moon's long.
To a number of minutes, &c.

which, applied to the time formerly found, will give the true time.

3. The preciſe moment of conjunction or oppoſition is ſeldom neceſſary except in calculating eclipſes.

4. In the preceding examples the time of orbit conjunction or oppoſition is taken for the anſwer; that is, the time when the moon's longitude in her orbit is the ſame with the ſun's longitude in the ecliptic, is taken for the time of conjunction; and the time when the moon's orbit longitude differs 6 ſigns from the ſun's longitude, is taken for the time of oppoſition.

The anſwers are in mean time; and if the equation of time be applied to them with a contrary ſign to that in the Table, the apparent time will be known.

To calculate more accurately the time of new moon in Auguſt 1820: The anſwer by our Tables is Auguſt 8d. 9h. 58m. 10s. The ſun's longitude at this time is 4s. 16° 9′ 44″, and his mean anomaly 1s. 7° 23′ 13″.

The calculation of the moon's longitude for the ſame time follows.

	☽'s m. Long.	☽'s anom.	☽'s ☊.
	S. ° ′ ″	S. ° ′ ″	S. ° ′ ″
1820 -	3 19 18 47	11 0 1 6	0 6 25 42
Auguſt 8th	0 18 48 26	11 24 17 51	
9h. - -	4 56 28	4 53 58	— 11 39 0
58m. - -	31 50	31 34	— 1 12
10s. - -	5	5	— 7
	4 13 35 36	10 29 44 34	11 24 45 23
10 equations	— 25 26	+ 12 54	+ 5 28
	4 13 10 10	10 29 57 28	11 24 50 51
Equat. centre	+ 3 1 4	— 25 26	
	4 16 11 14	10 29 32 2	
Variation -	+ 1	arg. 11th.	
	4 16 11 15		
13th equation	— 1 1		
☽'s orb. long.	4 16 10 14		
Sun's long.	4 16 9 44		
Difference -	30		

	Arguments.	Equations. +	Equations. —
	S. ° ′ ″	′ ″	′ ″
1	1 7 23 13	6 37	
2	1 2 14 57		0 29
3	10 17 28 31	0 51	
4	10 24 36 18		0 36
5	0 25 7 10		33 43
6	2 2 30 23	1 49	
7	11 17 43 57		0 10
8	9 22 21 21		0 39
9	0 2 40 29		0 3
10	7 8 35 39	0 57	
			— 35 40
12	0 0 1 30	+ 10 14	+ 10 14
13	10 13 8 46		
			— 25 20

	′ ″
Moon's hourly motion -	28 54
Sun's ditto. -	2 24
Hourly motion ☽ from ⊙	26 30

	L. L.
As 26′ 30″	3549
To 60m.	0
So is 30″	20792
To 1m. 7s.	17243

	D. h. m. s.
Former time Auguſt	8 9 58 10
Interval ſubtract -	— 1 7
True time of new moon	8 9 57 3 which is nearly the ſame with the former.

PROB. XVII. *To find out the Number of Eclipſes there are in any given Year, and in what Months and Days they happen.*

From the Table of the moon's mean motion take out the mean longitude of the moon's nodes for the given year; and then by the Table of the ſun's longitude for every day of the year, find when the ſun's longitude will be nearly the ſame with that of the nodes, for at theſe times the eclipſes muſt happen. The months when the ſun is in or near the places of the nodes may be called the *node-months.* In this inquiry it may be proper to remember that the moon's nodes move backward about 1° 38′ every month.

To find the Days when Eclipſes happen.

Calculate the times of the mean ſyzygies in the node-months out of the Tables, and alſo the ſun's diſtance from the moon's node; and if the ſun's diſtance from the node at the time of new moon be leſs than 18 degrees, an eclipſe of the ſun may be expected; and if the ſun's diſtance from the node at the time of full moon be leſs than 12 degrees, there may be an eclipſe of the moon. We ſay there may be, becauſe we ſpeak only of mean motions here.

Exam. It is required to find the number of eclipſes in the year 1796; and on what days they happen.

The mean longitude of the moon's north node, on the 1ſt of January 1796, is 3s. 20° 35′, and of the ſouth node 9s. 20° 35′. The ſun enters the ſign Cancer about the 20th of June, and the ſign Capricorn about the 22d of December; therefore the node-months are January, July, and December.

1. Calculate the time of mean new moon in January.

New moon.					☉'s diſt. from ☽'s ☊.				
	D.	h.	m.	s.	s.	°	′	″	
1795, March	20	2	51	8	7	22	24	0	
10 lunats.	+ 295	7	20	30	10	6	42	20	
New ☽ 1796 Jan.	9	10	11	38	5	29	6	20	ſun eclipſed.

The ſun's mean diſtance from the moon's north node being 5s. 29° 6′ 20″, his diſtance from the ſouth node is only 53′ 40″; and therefore the ſun will be eclipſed to ſome place of the earth.

2. For the time of mean new moon in July.

New moon.					☉'s diſt. from ☽'s ☊.				
	D.	h.	′	″	s.	°	′	″	
1796, March	8	11	39	44	8	0	26	47	
4 lunations	+ 118	2	56	12	4	2	40	56	
New ☽ July	4	14	35	56	0	3	7	43	ſun eclipſed.

3. For the time of mean full moon in December.

	New moon.				Sun's diſt. from ☋.				
	D.	h.	m.	s.	S.	°	′	″	
1796, March	8	11	39	44	8	0	26	47	
½ lunation	+ 14	18	22	2		+ 15	20	7	
Full ☽ in March	23	6	1	46	8	15	46	54	
9 lunations	+ 265	18	36	27	+ 9	6	2	6	
Full ☽ Dec.	14	0	38	13	5	21	49	0	moon eclipſed.

The ſun's diſtance from the moon's north node being 5s. 21° 49′, the moon's diſtance from her ſouth node is only 8° 11′, and therefore ſhe is eclipſed.

4. For the time of mean new moon in December.

Add ½ lunation to the numbers for full moon.

	New moon.				Sun's diſt. from ☊.				
	D.	h.	m.	s.	s.	°	′	″	
Full ☽ Dec.	14	0	38	13	5	21	49	0	
½ lunation	+ 14	18	22	2	+ 15	20	7		
New ☽ Dec.	28	19	0	15	6	7	9	7	ſun eclipſed.

We have found that there are four eclipſes in the year 1796, viz. three of the ſun and one of the moon. The times of the mean conjunctions and oppoſitions are:

		D.	h.	m.	s.
Sun is eclipſed - - -	Jan.	9	10	11	38
	July	4	14	35	56
	Dec.	28	19	0	15
And the moon is eclipſed -	—	14	0	38	13

PROB. XVIII. *To calculate an Eclipſe of the Moon.*

1. Calculate the true time of oppoſition by Prob. XVI. and compute the longitude of the ſun, and the orbit longitude of the moon, for that time: if theſe differ 6 ſigns, the time of oppoſition is truly found; but if their difference be leſs or greater than 6 ſigns, mark the defect or exceſs, and find the moon's hourly motion from the ſun; then, as the hourly motion of the moon from the ſun is to one hour, ſo is the defect or exceſs to the interval; which being applied to the time formerly found, gives the true time of oppoſition. This operation muſt be repeated until the longitude of the ſun and the orbit longitude of the moon differ preciſely 6 ſigns.

2. Calculate the moon's latitude, horizontal parallax, and hourly motion, by the Tables; take alſo the ſun and moon's ſemidiameters out of the Tables.

3. Add the ſun's horizontal parallax (which is 9 ſeconds) to the moon's for the time, and from the ſum ſubtract the ſun's ſemidiameter; the remainder is the ſemidiameter of the earth's ſhadow; to which add the moon's ſemidiameter, and the ſum is the ſemidiameter of the moon and earth's ſhadow; from which ſubtract the moon's latitude, and the remainder is called *the parts deficient.* If theſe be leſs than the moon's diameter, the eclipſe will be partial; if they be equal to the moon's diameter, the eclipſe will be total without continuance; and if they be greater than the moon's diameter, the eclipſe will be total with continuance.

Exam. 1. It is required to calculate the eclipſe of the moon, which will happen in December, 1797.

1. Calculate the time of oppoſition.

	D. h. m. s.	Sun's Anom. s. ° ′ ″	Moon's Anom. s. ° ′ ″	☉'s diſt. from ☊. s. ° ′ ″
1797, new ☽ March	27 9 12 24	8 26 25 59	2 12 39 19	9 9 9 48
½ lunation - -	—14 18 22 2	—14 33 10	—6 12 54 30	—15 20 7
Full moon March -	12 14 50 22	8 11 52 49	7 29 44 49	8 23 49 41
9 lunations -	+265 18 36 27	8 21 56 54	7 22 21 4	9 6 2 6
Full moon December	3 9 26 49	5 3 49 43	3 22 5 53	5 29 51 47
1 equation - -	—1 52 47	—3 21 23 13	—42 40	arg. 4th equat.
	3 7 34 2	1 12 26 30	3 21 23 13	Moon eclipſed.
2 equation - -	+8 53 8	arg. 3d equat.	arg. 2d equat.	
	3 16 27 10			
3 equation - - -	—3 11			
4 ditto - - -	—1			
Full moon December	3 16 23 58			

2, Calculate the ſun and moon's longitude for the time of full moon.

For the Sun's Longitude.

	Sun's Long. s. ° ′ ″	Sun's Anom. s. ° ′ ″
1797 - -	9 10 37 33	6 1 8 17
December 3d	11 2 9 47	11 2 8 46
16h. - -	39 25	39 25
23m. - -	57	57
58s. - -	2	2
	8 13 27 44	5 3 57 27
Equation -	—51 45	
Sun's longitude	8 12 35 59	

For the Moon's Longitude.

	Moon's m. Long. s. ° ′ ″	Moon's anom. s. ° ′ ″	Moon's ☊. s. ° ′ ″
1797 -	10 7 34 43	0 24 6 53	3 1 15 6
Dec. 3d -	4 0 26 45	2 22 54 4	
16h. - -	8 47 3	8 42 36	—17 50 45
23m. - -	12 37	12 31	—2 7
58s. - -	32	32	—3
	2 17 1 40	3 25 56 36	2 13 22 11
10 equations	+1 25 1	+9 44	+4 3
	2 18 26 41	3 26 6 20	2 13 26 14
Equat. centre	—5 45 57	+1 25 1	
	2 12 40 44	3 27 31 21	
Variation -	+6	arg. 11th	
	2 12 40 50		
13th equation	—1 13		
Moon's orb. lon.	2 12 39 37		
Sun's lon. -	8 12 35 59		
Paſt oppoſition	3 38		

	Arguments. s. ° ′ ″	Equations. + ° ′ ″	— ′ ″
1	5 3 57 27	0 5 0	
2	5 12 48 49		0 17
3	7 4 53 55	0 0 43	
4	4 4 47 58	0 0 47	
5	8 12 54 46	1 17 15	
6	1 16 52 13	0 1 30	
7	3 8 57 19	0 0 46	
8	10 21 59 9		0 26
9	2 8 29 5		0 19
10	6 0 46 12	0 0 2	
12	6 0 4 45	+1 26 3	—1 2
13	8 0 57 51	—0 1 2	
14	11 29 9 29		
		+1 25 1	

Moon's

	′	″
Moon's hourly motion - -	35	18
Sun's ditto. - - -	2	32
Hourly motion ☽ from ☉ -	32	46

	L. L.
As hourly motion ☽ from ☉ 32′ 46″	2627
To one hour or 60 m. - -	0
So is 3′ 38″ - - -	12178
To 6m. 39s. - - -	9551

	D.	h.	m.	s.
Former time December -	3	16	23	58
Interval ſubtract - -		—	6	39
Time of full moon - -	3	16	17	19

	S.	°	′	″
Sun's longitude	8	12	35	43
Moon's ditto -	2	12	35	43
Reduction -			+	12

For the Moon's Latitude.

Arguments	S.	°	″	″		′	″
1	11	29	13	23	—	4	11
2	0	0	53	23	+		8
3	6	25	15	56	+		6
4	2	29	19	20	—		25
5	11	3	22	44	—		7
Moon's lat. S.					—	4	29

For the Moon's Horizontal Parallax.

Arguments.	S.	°	′	″		′	″
1	8	12	54	46	+	0	11
2	3	27	31	21	+	58	32
3	6	0	4	45	+	0	27
Equatorial parallax -						59	10
Reduction - -					—		9
Horizontal parallax -						59	1

For the time of Reduction.

	L. L.
As Moon's hourly motion from ☉ 32′ 46″	2627
To 1h. or 60m.	0
So is reduction 12″	24771
To time of reduction + 22″.	22144

	′	″
Sun's ſemidiameter -	16	17
Moon's ſemidiameter -	16	8

Angle of the moon's path with the ecliptic 5° 40′

Note. In eclipſes, the diſc of the ſun or moon is ſuppoſed to be divided into twelve equal parts called *digits*, and each of theſe again divided into 60 equal parts or minutes. The magnitude of an eclipſe is eſtimated by the number of digits.

Calculation

Calculation of the Eclipſe.

	′	″
Moon's horizontal parallax -	59	1
Sun's ditto - - -	+	9
Sum - - -	59	10
Sun's ſemidiameter ſubtract -	—16	17
Semidiameter of the earth's ſhadow	42	53
Moon's ſemidiameter add -	+16	8
Semid. of the moon and earth's ſhadow	59	1
Moon's latitude ſubtract -	— 4	29
Remains the parts deficient -	54	32

The eclipſe will be total.

For the Digits eclipſed.

	L. L.
As moon's ſemidiameter 16′ 8″ -	5704
To 6 digits - - -	10000
So parts deficient 54′ 32″ - -	415
To digits eclipſed 20° 16′ - -	4711

	D.	h.	m.	s.
Mean time of orbit oppoſition Dec.	3	16	17	19
Time of reduction add and ſubtract	-		∓	22
Middle of the eclipſe -	3	16	17	41
Ecliptic oppoſition - -	3	16	16	57
To each of theſe add the equation of time			+9	22
Appar. time of the middle of the eclipſe	3	16	27	3
——— of ecliptic oppoſition	3	16	26	19

To find the Scruples of Incidence, or that Part of the Moon's Path which ſhe paſſes over between the beginning and middle of the Eclipſe.

Reduce the ſum of the ſemidiameters of the moon and earth's ſhadow to ſeconds, and alſo the moon's latitude ; find their ſum and difference ; and to the logarithm of their ſum add the logarithm of their difference, and divide the ſum by 2, the quotient is the logarithm of the ſcruples of incidence in ſeconds.

In this example the ſemidiameter of the moon and earth's ſhadow is

	′	″		″	Logarithms.
	59	1	=	3541	
Moon's latitude	4	29	=	269	
Sum -				3810	3.5809450
Difference				3272	3.5148133
					2)7.0957583
Scruples of incidence				3530	3.5478791

or 58′ 50″.

For the time of half duration.

	′ ″		L. L.
As hourly mo. ☽ from ☉ =	32 46		2627
To 1 hour or 60 minutes			0
So ſcruples of incidence	58 50	(29′ 25″)	3096
		h. m. s.	
To time of half duration 1h.	47m. 42s.	(0 53 51)	469
		2	
		1 47 42	

Becauſe the terms of this proportion are ſuch, that the fourth term comes out too great for the Table of logiſtical logarithms, take half of the third term; and then double the fourth term for the anſwer.

	D.	h.	m.	s.
Apparent time of the middle of the eclipſe	3	16	27	3
Time of half duration, ſubtract and add	∓	1	47	42
Beginning - - - -	3	14	39	21
End - - - -	3	18	14	45

In total eclipſes of the moon, ſhe continues ſome time in total darkneſs; to calculate this time, it is neceſſary to find the half length of that part of the moon's path which lies wholly within the earth's ſhadow, which ſome call the *ſcruples of total darkneſs*. The operation is performed thus:

Subtract the moon's ſemidiameter from that of the earth's ſhadow; reduce the remainder, and alſo the moon's latitude, to ſeconds; then find their ſum and difference, and take the half ſum of the logarithms of the ſum and difference, and the number anſwering is the ſcruples of total darkneſs.

In this example,

	′ ″
The ſemidiameter of the earth's ſhadow is	42 53
The moon's diameter - -	— 16 8
	26 45

	′ ″	″	Logarithms.
The difference is	26 45 =	1605	
Moon's latitude	4 29 =	269	
Sum - - -		1874	3.2727696
Difference - - -		1336	3.1258065
			2)6.3985761
Scruples - - - or 26′ 24″		1584	3.1992880

For the time of half duration of total darkneſs.

	′ ″	L. L.
As hourly motion of moon from ſun =	32 46	2627
To one hour or 60 minutes - -		0
So arc ſcruples - - -	26 24	3565
To time of half duration 48m. 21s.		938

	D.	h.	m.	s.
Apparent time of the middle of the eclipſe	3	16	27	3
Half duration of total darkneſs ſubt. and add		∓	48	21
Beginning of total darkneſs - -	3	15	38	42
End of ditto. - - -	3	17	15	23

From the calculation it appears, that the moon will be eclipſed at London, 1797, Dec. 3d.

	Appar. time. h.	m.
Beginning of the eclipſe - - -	14	39
Beginning of total darkneſs - - -	15	38
Ecliptic oppoſition - - - -	16	26
Middle of the eclipſe - - - - -	16	27
End of total darkneſs - - - -	17	15
End of the eclipſe - - - -	18	14

Digits eclipſed 20° 16′ from S. ſide of earth's ſhadow.
Duration of the eclipſe 3 h. 35 m.
Duration of total darkneſs 1 h. 36 m.

Exam. 2. Calculation of an eclipſe of the moon which will happen 1802, Sept. 11th.

Mean time of orbit oppoſition, Sept. 11 d. 10 h. 40 m. At that time ſun's longitude 5 s. 18° 25′ 16″; moon's longitude 11 s. 18° 25′ 26″, latitude north 37′ 6″ aſcending. Reduction — 1′ 40″, time of reduction 2 m. 48 s.

	′	″
Moon's hourly motion - - -	38	4
Sun's ditto.	— 2	26
Moon's hourly motion from ſun -	35	38

	°	′	″
Moon's ſemidiameter - - -	0	16	43
Sun's ditto. - - - -	0	15	57
Moon's horizontal parallax - - -	0	61	17
Angle of the moon's path with the ecliptic is -	5	38	0

Calculation of the Eclipſe.

	′	″
Moon's horizontal parallax - -	61	17
Sun's ditto. - - -		+ 9
Sum - - - -	61	26
Sun's ſemidiameter ſubtract - -	— 15	57
Semidiameter of earth's ſhadow	45	29
Moon's ſemidiameter, add - -	+ 16	43
Semidiameter of moon and earth's ſhadow	62	12
Moon's latitude, ſubtract - -	— 37	6
Parts deficient - - -	25	6

For the digits eclipſed.

	°	′	″	L. L.
As moon's ſemidiameter	0	16	43	5550
To 6 digits				10000
So parts deficient	0	25	6	3785
To digits	9	0	0	8235

	D.	h.	m.	s.
Mean time of orbit oppoſition	11	10	40	0
Time of reduction, ſubtract and add			∓2	48
Middle of the eclipſe	11	10	37	12
Ecliptic oppoſition	11	10	42	48
Equation of time add to each			+3	31
Apparent middle of the eclipſe	11	10	40	43
——— ecliptic oppoſition	11	10	46	19

For the ſcruples of incidence.

	′	″	″	Logarithms.
Semid. of the ☽ and ⊕'s ſhadow	62	12	= 3732	
Moon's latitude	37	6	= 2226	
Sum			5958	3.7751005
Difference			1506	3.1778250
				2)6.9529255
Scruples			2995	3.4764627
Or	49′	45″		

For the time of half duration.

	′	″	L. L.
As moon's hourly motion from ſun	35	38	2263
To 1 hour or 60m.			0
So $\frac{1}{5}$ ſcruples of incid.	9	59	7789
To $\frac{1}{5}$ of half duration	16	48	5526
		5	

Half duration - - 1h. 24m. 0s.

	D.	h.	m.	s.
Apparent middle of the eclipſe	11	10	40	43
Time of half duration, ſubt. and add		∓1	24	0
Beginning	11	9	16	43
End	11	12	4	43

1802, September 11th, the moon is eclipſed.

	Appar. time.	
	h.	m.
Beginning - - - -	9	16
Middle - - - - -	10	40
Ecliptic oppoſition - - -	10	46
End - - - - -	12	4

Duration 2 hours 48 minutes. Digits eclipſed 9°.

PROB. XIX. *To deſcribe a Figure repreſenting a lunar Eclipſe.* (Plate 14. Fig. 4.)

Exam. 1. It is required to project the eclipſe, calculated in the 1ſt example of the laſt problem.

1. Draw the ſtraight line A B for part of the ecliptic, in which take any point C, and draw CD at right angles to AB, for the axis of the ecliptic; which is drawn downwards, becauſe the moon's latitude at the beginning and middle of the eclipſe is ſouth.

2. Take the ſemidiameter of the moon and earth's ſhadow, 59′ 1″, from any ſcale, and with it, as a radius, deſcribe the circle ADB; the moon's centre is in one point of the circumference of this circle at the beginning, and, in the oppoſite point of it, at the end of a total eclipſe.

3. Take the ſemidiameter of the earth's ſhadow, 42′ 53″, from the ſame ſcale, and with it, from the centre C, deſcribe the circle KLM, to repreſent the earth's ſhadow.

4. Make CD the radius of a line of chords on the ſector, and from it take the angle of the moon's path with the ecliptic 5° 40′, and ſet it from D to E towards the left hand, (becauſe the moon's latitude is ſouth aſcending northward) and draw the line CE for the axis of the moon's orbit.

5. Take the moon's latitude 4′ 29″ from the ſcale, which ſet from C to F on the line CE; and through F draw the ſtraight line PN at right angles to CE; then the points N, F, and P are the places of the moon's centre at the beginning, middle, and end of the eclipſe. Take the moon's ſemidiameter from the ſcale, and with it deſcribe circles from the centres N, F, and P. Theſe ſhall repreſent the moon at theſe points.

To mark the Hours on the Moon's Path.

The middle of the eclipſe is December 3d 16h. 27m. 3s. which, in civil reckoning, is 4h. 27m. in the morning of December 4th, marked on the moon's path by the point F. Now, for the place of four hours, ſay,

As one hour
Is to the moon's hourly motion from the ſun 32′ 46″;
So is 27m. 3s.
To 14′ 46″ the diſtance of 4h. from F.

Take 14′ 46″ from the ſcale, and ſet it from F towards the right hand for the place of 4 hours; then take the moon's hourly motion from the ſun 32′ 46″, and ſet it on the moon's path from 4 to 3, and to 5, and alſo from 5 to 6, and the hours are marked. Divide each of the hour diſtances into ſix equal parts, and the place of the moon's centre will be known at every 10 minutes.

Exam. 2. Required the projection of the eclipſe, calculated in the 2d example of the laſt problem. (Plate 14. Fig. 5.)

This is done in the ſame manner as the former; only, becauſe the moon's latitude is north, the ſemicircles ADB and KLM muſt be above the ſtraight line AB; and becauſe the moon's latitude is aſcending northward, the angle of her path with the ecliptic 5° 38′ muſt be ſet from D to E towards the right hand. Draw CE for the axis of the moon's orbit, and thereon ſet the moon's latitude 37′ 6″ from C to F; and through F draw the ſtraight line NP at right angles to CE; and then the points N, F, and P, are the places of the moon's centre at the beginning, middle, and end of the eclipſe, as in the former example.

Take the moon's ſemidiameter 16′ 43″ from the ſcale, and with it, for a radius, deſcribe circles on the centres N, F, and P, which will repreſent the moon's diſc when ſhe is at theſe points; and then place the hours on the moon's path as directed in the former example.

PREVIOUS to the Calculation and Projection of Solar Eclipſes, it will be proper to premiſe ſome things for explaining the terms uſed in the graphical delineation of them.

Although the earth is nearly a globe, yet when it is viewed from a very diſtant point, ſuch as the centre of the ſun, it will appear as a flat round ſurface, like the moon, to us; and this appearance is called *the earth's diſc*.

By the continual rotation of the earth round its axis, every point on its ſurface deſcribes a circle parallel to the equator; and the circle deſcribed by any point or place is called *the path of the vertex of that place*.

The repreſentations of the paths of places on the earth's diſc are not always of the ſame form, but differ with the ſun's longitude; for when the ſun is in either of the equinoctial points, the paths of all places are repreſented by ſtraight lines; and when the ſun is in any other point of the ecliptic, they are repreſented by ellipſes, more or leſs eccentric according as the ſun is nearer to, or farther from, the equinoctial circle.

In deſcribing theſe paths on the earth's diſc, reſpect muſt be had to the poſition of the axes of the earth and ecliptic at the time.

When the ſun is in any of the aſcending ſigns ♑, ♒, ♓, ♈, ♉, ♊, the northern half of the earth's axis lies to the right hand of the axis of the ecliptic, as ſeen from the ſun; but when he is in any of the deſcending ſigns ♋, ♌, ♍, ♎, ♏, ♐, it lies to the left hand. When the ſun is in either of the ſolſtitial points, the two axes coincide; and when he is in either of the equinoctial points, they form the greateſt angle.

When the ſun is in any of the northern ſigns, Aries, Taurus, &c. the north pole of the earth is in the upper or enlightened part of the earth's diſc; and when he is in any of the ſouthern ſigns, Libra, Scorpio, &c. the north pole of the earth is in the under or obſcure part of the diſc.

The tranſverſe diameter of any path is always at right angles to the earth's axis, and the conjugate diameter coincides with or is a part of it.

PROB. XX. *The Latitude of a Place, the Time of the Year, and the Sun's Longitude, being given, to deſcribe the Earth's Diſc, and the Path of that Place thereon.*

EXAM. It is required to deſcribe the earth's diſc, and the path of a place in latitude 51° 32′ N. 1803, Auguſt 17th, at 8h. 19m. in the morning, apparent time. (Plate 15. Fig. 2.)

Sun's longitude 4s. 23° 24′ 45″, diſtance from the ſolſtice 53° 24′ 45″; declination N. 13° 43′43″.

Draw the ſtraight line AB to repreſent a part of the ecliptic, and let C be the point therein, which is oppoſite to the ſun at the time: from C draw CH at right angles to AB, and CH is the axis of the ecliptic, and H its pole.

From the centre C deſcribe the ſemicircle AHB   repreſent the northern half of the earth's diſc. Make CA or CB the radius of the line of chords on the ſector, and take the chord of $23\frac{1}{2}$ degrees, which ſet from H both ways to *f* and *g*, and draw the ſtraight line *f*V*g*; the north pole of the earth is always in this line.

Make *f*V the radius of the line of ſines on the ſector, and take the ſine of 53° 24′ 45″, the ſun's diſtance from the ſolſtice, which ſet off from V to P towards the left hand, becauſe the ſun is in the ſign Virgo, and draw the ſtraight line CP*h* for the earth's axis; then P is the north pole of the earth. Or the angle contained between the earth's axis and that of the ecliptic may be found more accurately by calculation, thus:

		°	′	
As radius - - -	S.	90	0	10.
Is to the ſine of the ſun's diſt. from the ſolſtice		53	25	9.9047106
So is the tan. of the diſtance of the poles		23	28	9.6376106
To the tan. of the angle contained by the axes		19	13	9.5423212

Now ſet off the chord of 19° 13′ from H to *h*, and join CH, which will cut *fg* in P, the place of the north pole.

Make CA the radius of the line of chords on the ſector; take the chord of 38° 28′, the complement of the latitude, and ſet it off from *h*, both ways, to *e* and *n*, where make marks. Again, take the chord of the ſun's declination 13° 43′ 43″ from the ſector, and ſet it off from the points *e* and *n*, both ways, to D and F, and to M and G; then draw the lines DM, FG, cutting CP in 12 and 12; and the line 12 K 12 is the conjugate axis of the ellipſe, which biſect in K, and through K draw the line 6 K 6 at right angles to CP.

Make CA the radius of the line of ſines on the ſector, and take the ſine of the co-latitude 38° 28′, and ſet it off from K, both ways, to 6 and 6. Theſe hours fall on the circumference of the diſc, when the ſun is in either of the equinoctial points; but at all other times they fall within it. The line 6 K 6 is the tranſverſe axis of the ellipſe.

Make K 6 the radius of the line of ſines on the ſector, and from it take the ſines of 15°, 30°, 45°, 60°, and 75°, and ſet them off from K, both ways, towards 6 and 6; and through theſe points draw lines parallel to CP. Again, make K 12 the radius of the line of ſines on the ſector, and take from it the ſines of 75°, 60°, 45°, 30°, and 15°, and ſet them off on the parallels, beginning with that next to 12 K 12; and through theſe points draw a curve line, which will be the path of the given place. Mark the hours on the path as per Fig. and divide them into halves, quarters, and ſmaller parts, at pleaſure.

The 12 next to P marks midnight, when the ſun's declination is north; and that part of the path is below the diſc; but the other 12 between K and C is noon or mid-day. The path touches the circumference of the diſc a little before 5, which is the time of ſun-riſing; and again a little after 7, the time of ſun-ſetting.

In north latitude, when the ſun's declination is ſouth, the 12 neareſt to P is mid-day, and the other is midnight.

In the ſame manner, the parallel of latitude, or path of any place, may be deſcribed on the earth's diſc for any given time, and is uſed in repreſenting the appearance of an eclipſe of the ſun.

PROB.

PROB. XXI. *To calculate an Eclipſe of the Sun.*

1. Calculate the time of mean conjunction, with the ſun's diſtance from the moon's node, and thereby find if an eclipſe will happen; and if it do, calculate the true time of conjunction, and the ſun and moon's longitudes at that time.

2. Calculate the moon's latitude, horizontal parallax, horary motion, ſemidiameter, and angle of her path with the ecliptic.

3. Calculate the ſun's declination, hourly motion, and his diſtance from the ſolſtice. Having found theſe neceſſary requiſites, write them down in order to be ready for uſe.

Exam. The ſun will be eclipſed in Auguſt, 1803.

The time of mean conjunction is Aug. 17d. 7h. 12m. 24s.
Sun's mean diſt. from the moon's node 6s. 2° 8′ 8″.
And the true conjunction is Aug. 16d. 20h. 30m. 34s. mean time.

	S.	°	′	″
The ſun's longitude then	4	23	25	3
The moon's orbit longitude	4	23	28	49
Paſt conjunction	0	0	3	46

Sun's mean anomaly 1 s. 15° 8′ 15″.

	′	″
Moon's hourly motion	30	57
Sun's ditto ſubtract	2	24
Moon's hourly motion from ſun	28	33

	L. L.
As moon's hourly motion from ⊙ 28′ 33″	3225
To 1 hour or 60 minutes	0
So is difference of longitudes 3′ 46″.	12022
To interval —7 m. 54 s.	8796

	D.	h.	m.	s.
From the time formerly found	16	20	30	34
Subtract the interval		—	7	54
Correct time of conjunction	16	20	22	40
Equation of time		—	3	48
Apparent time of conjunction	16	20	18	54

or 8h. 18m. 54s. in the morning of Aug. 17th.

For the Sun and Moon's places at the corrected time of Conjunction.

m. ′ ″ m. s. ″
As 60 : 2 24 : : 7 54 : 18 to be ſubtracted from ⊙'s long.
60 : 30 57 : : 7 54 : 4′ 4″ to be ſubt. from the ☽'s long.

	S.	°	′	″
And then the ſun's longitude is	4	23	24	45
The moon's ditto.	4	23	24	45

	°	′	″
2. The moon's horizontal parallax, or ſemidiameter of the earth's diſc - - -	0	55	9
3. Sun's diſtance from the neareſt ſolſtice -	53	24	45
4. Sun's declination north - - - -	13	43	43
5. Angle of the moon's path with the ecliptic	5	43	0
6. The moon's latitude ſouth deſcending -	0	0	46
7. The moon's hourly motion from the ſun -	0	28	33
8. The ſun's ſemidiameter 15′ 52″			
9. The moon's ſemidiameter 15 4			
10. The ſemidiameter of the penumbra, or ſum of the ſemidiameters of ſun and moon	0	30	56

In this example the ſun's diſtance from the neareſt ſolſtice is found by ſubtracting 3 ſigns from his longitude. The angle of the moon's path with the ecliptic is taken out of the Table.

The method of finding the other numbers has been taught formerly.

PROB. XXII. *To deſcribe a Figure repreſenting a ſolar Eclipſe, and to find its Magnitude, Beginning, Middle, and End, at London.* (Plate 15. Fig. 2.)

Exam. Project the ſolar eclipſe, calculated in the laſt problem.

Draw the ſtraight line AB to repreſent a part of the ecliptic : take the ſemidiameter of the earth's diſc from a ſcale, and with it as radius deſcribe the ſemicircle AHB for the northern half of the earth's diſc, and thereon deſcribe the path of London, as has been taught in the preceding problem.

Make the ſemidiameter of the diſc CA the radius of the line of chords on the ſector, and take the chord of 5° 43′, the angle of the moon's path, which ſet off on the circumference of the diſc, from H towards the right hand, to M, becauſe the moon's latitude is ſouth deſcending, and draw CM for the axis of the moon's path.

Set off the moon's latitude 46″ ſouth on MC, produced below AB to X; and through X, at right angles to MC, draw the line of the moon's path, which cuts AB on the left hand of X. The point X is the time of conjunction by the Tables.

To find the place of 8 hours on the moon's path.

	L. L.
As 1 hour or 60 minutes	0
To the ☽'s hourly motion from the ☉ 28′ 33″	3225
So is the time of conjunction after 8h. 18m. 52s.	5025
To the diſtance of 8 hours from X 8′ 58″	8250

Take 8′ 58″ from the ſcale, and ſet it off from X towards the left hand for the place of 8 hours on the moon's path.

Take alſo from the ſcale 28′ 33″, the moon's hourly motion from the ſun, and ſet it off both ways from 8 to 9 and 7, as alſo from 7 to 6, &c. and the places of the hours on the moon's path will be known. Divide each hour diſtance into 12 equal parts by dots, and the place of the moon's centre, or rather the centre of the penumbra, will be known at every 5 minutes during the eclipſe.

To find the middle of the Eclipſe.

Apply one ſide of a ſquare to the path of the moon or penumbra's centre, and move it backward and forward until the other ſide of the ſquare cuts the ſame hour and minute on the path of London and of the penumbra's centre, which in this example is at *y* and *u* in 6 hours 32 minutes, the middle of the eclipſe.

Take the ſun's ſemidiameter 15′ 52″ from the ſcale, and with it deſcribe a circle about the point *y* for the ſun's diſc: then take the moon's ſemidiameter 15′ 4″ from the ſcale, and with it deſcribe a circle about the point *u* for the moon's diſc at the middle of the eclipſe. The part of the ſun's diſc cut off by the moon's is the magnitude of the eclipſe as it will appear at London. In this example it is about $3\frac{1}{2}$ digits.

Laſtly, take the ſemidiameter of the penumbra 30′ 56″ from the ſcale; and ſetting one foot of the compaſſes on the moon's path, and the other on the path of London, towards the left hand, carry that extent backwards and forwards until both points fall on the ſame hour and minute in each path, and that is the beginning of the eclipſe at London. With the ſame extent of the compaſſes, and one foot on each path, carry them backwards and forwards towards the right hand; and where both points fall on the ſame time, that will be the end of the eclipſe at London. Theſe trials give,

	h.	m.
The beginning of the eclipſe at - -	5	50
The middle - - - - -	6	32
The end - - - - -	7	30

Duration 1h. 40m. Digits 3° 30′.

Note 1. The projection of a ſolar eclipſe will exhibit the appearance of it more naturally, if ſome alterations be made in the preceding proceſs, adapted to the ſuppoſition, that the obſerver is on the earth. Theſe alterations conſiſt in drawing the axes of the earth and moon's path on the ſide of the axis of the ecliptic contrary to that which is required by the rule, and in numbering the hours in the oppoſite direction. For the relative poſition of the axes, as ſeen from the ſun, is inverted with reſpect to an obſerver of the ſun on the earth. If a projection, made according to the preceding rule, be turned over from right to left, it will appear, if viſible through the paper, to correſpond with one of this conſtruction.

Plate XV. Fig. III. is an example, being the ſame eclipſe projected in this manner, and requires no farther explanation.

Note 2. The ſituation of the point on the ſun's limb, with reſpect to a vertical and a horizontal diameter, where an eclipſe begins, may be eaſily determined by Projection. Thus, with the points in the reſpective paths, where the centres of the ſun and moon are at the beginning of the eclipſe, as centres, and their ſemidiameters as radii, deſcribe circles, touching each other. Draw a line from the centre (C Fig. II. or III. Plate XV.) through the centre of the ſun, and it will give the vertical diameter; and a diameter perpendicular to this will be horizontal. Then the line of chords on a ſector being adapted to the ſemidiameter of the ſun, the arc of the circumference, contained between the point of contact and an extremity of one of theſe diameters, meaſured on the chord line, will give the required ſituation of the point, where the eclipſe begins.

SOLAR AND LUNAR TABLES.

TABLES OF THE SUN's MEAN MOTIONS.

I. The Sun's Mean Longitude and Anomaly in Julian Years.										*II. The Preceſſion of the Equinoctial Points.*		
	Years.	M. Longitude.				M. Anomaly.						
		S.	°	′	″	S.	°	′	″	°	′	″
	1	11	29	45	40	11	29	44	35	0	0	50.3
	2	11	29	31	21	11	29	29	9	0	1	40.7
	3	11	29	17	2	11	29	13	44	0	2	31.
B	4	0	0	1	51	11	29	57	27	0	3	21.4
	5	11	29	47	32	11	29	42	2	0	4	11.7
	6	11	29	33	13	11	29	26	37	0	5	2.1
	7	11	29	18	54	11	29	11	12	0	5	52.4
B	8	0	0	3	43	11	29	54	55	0	6	42.8
	9	11	29	49	23	11	29	39	29	0	7	33.1
	10	11	29	35	4	11	29	24	4	0	8	23.5
	11	11	29	20	45	11	29	8	39	0	9	13.8
B	12	0	0	5	34	11	29	52	22	0	10	4.2
	13	11	29	51	15	11	29	36	57	0	10	54.5
	14	11	29	36	55	11	29	21	31	0	11	44.9
	15	11	29	22	36	11	29	6	6	0	12	35.2
B	16	0	0	7	35	11	29	49	49	0	13	25.6
	17	11	29	53	6	11	29	34	24	0	15	15.9
	18	11	29	38	47	11	29	18	59	0	15	6.3
	19	11	29	24	27	11	29	3	33	0	15	56.6
B	20	0	0	9	17	11	29	47	17	0	16	47.
B	40	0	0	18	33	11	29	34	33	0	33	34
B	60	0	0	27	50	11	29	21	50	0	50	21
B	80	0	0	37	6	11	29	9	6	1	7	8
B	100	0	0	46	23	11	28	56	23	1	23	55
B	200	0	1	32	46	11	27	52	46	2	47	50
B	300	0	2	19	9	11	26	49	9	4	11	45
B	400	0	3	5	32	11	25	45	32	5	35	40
B	500	0	3	51	55	11	24	41	55	6	59	35

III. Sun's Mean Longitude and Anomaly in Years current.										*Obliquity of the Ecliptic 1ſt January.*		
	Years.	M. Longitude.				M. Anomaly.						
	A. D.	S.	°	′	″	S.	°	′	″	°	′	″
	1761	9	10	20	51	6	1	31	11	23	28	16
	1781	9	10	30	7	6	1	18	27	23	28	9
	1791	9	10	5	12	6	0	42	32	23	27	50
B	1792	9	10	50	1	6	1	26	15	23	27	48
	1793	9	10	35	41	6	1	10	49	23	27	47
	1794	9	10	21	22	6	0	55	24	23	27	48
	1795	9	10	7	3	6	0	39	59	23	27	50
B	1796	9	10	51	52	6	1	23	42	23	27	52
	1797	9	10	37	33	6	1	8	17	23	27	54
	1798	9	10	23	13	6	0	52	51	23	27	55
	1799	9	10	8	54	6	0	37	26	23	27	58
	1800	9	9	54	35	6	0	22	1	23	28	0
	1801	9	9	40	16	6	0	6	36	23	28	1
	1802	9	9	25	56	5	29	51	10	23	28	0
	1803	9	9	11	37	5	29	35	45	23	28	0
B	1804	9	9	56	26	6	0	19	28	23	27	58
	1805	9	9	42	6	6	0	4	2	23	27	55
	1806	9	9	27	48	5	29	48	38	23	27	51
	1807	9	9	13	29	5	29	33	13	23	27	48
B	1808	9	9	58	17	6	0	16	48	23	27	44
	1809	9	9	43	57	6	0	1	31	23	27	42
	1810	9	9	29	37	5	29	45	57	23	27	40
	1811	9	9	15	17	5	29	30	32	23	27	39
B	1812	9	10	0	5	6	0	14	15	23	27	39
	1813	9	9	45	45	5	29	58	49	23	27	40
	1814	9	9	31	25	5	29	43	26	23	27	41
	1815	9	9	17	5	5	29	27	58	23	27	43
B	1816	9	10	1	53	6	0	11	41	23	27	46
	1817	9	9	47	33	5	29	56	15	23	27	48
	1818	9	9	33	13	5	29	40	50	23	27	50
	1819	9	9	18	53	5	29	25	24	23	27	52
B	1820	9	10	3	41	6	0	9	7	23	27	52
	1821	9	9	49	22	5	29	55	42	23	27	51
	1841	9	9	58	39	5	29	42	59	23	27	59

TABLE IV. *The Sun's Mean Longitude and Anomaly for Months and Days.*

Days.	January.							
	Longitude.				Anomaly.			
	S.	°	′	″	S.	°	′	″
1	0	0	59	8	0	0	59	8
2	0	1	58	17	0	1	58	17
3	0	2	57	25	0	2	57	24
4	0	3	56	33	0	3	56	32
5	0	4	55	42	0	4	55	41
6	0	5	54	50	0	5	54	49
7	0	6	53	58	0	6	53	57
8	0	7	53	7	0	7	53	6
9	0	8	52	15	0	8	52	14
10	0	9	51	23	0	9	51	21
11	0	10	50	32	0	10	50	30
12	0	11	49	40	0	11	49	38
13	0	12	48	48	0	12	48	46
14	0	13	47	57	0	13	47	54
15	0	14	47	5	0	14	47	2
16	0	15	46	13	0	15	46	10
17	0	16	45	22	0	16	45	19
18	0	17	44	30	0	17	44	27
19	0	18	43	38	0	18	43	35
20	0	19	42	47	0	19	42	43
21	0	20	41	55	0	20	41	51
22	0	21	41	3	0	21	40	59
23	0	22	40	12	0	22	40	8
24	0	23	39	20	0	23	39	16
25	0	24	38	28	0	24	38	23
26	0	25	37	37	0	25	37	32
27	0	26	36	45	0	26	36	40
28	0	27	35	53	0	27	35	48
29	0	28	35	2	0	28	34	57
30	0	29	34	10	0	29	34	5
31	1	0	33	18	1	0	33	12

Days.	February.							
	Longitude.				Anomaly.			
	S.	°	′	″	S.	°	′	″
1	1	1	32	27	1	1	32	21
2	1	2	31	35	1	2	31	29
3	1	3	30	43	1	3	30	37
4	1	4	29	52	1	4	29	46
5	1	5	29	0	1	5	28	53
6	1	6	28	8	1	6	28	1
7	1	7	27	17	1	7	27	10
8	1	8	26	25	1	8	26	17
9	1	9	25	33	1	9	25	26
10	1	10	24	42	1	10	24	35
11	1	11	23	50	1	11	23	42
12	1	12	22	58	1	12	22	50
13	1	13	22	7	1	13	21	59
14	1	14	21	15	1	14	21	7
15	1	15	20	23	1	15	20	15
16	1	16	19	31	1	16	19	23
17	1	17	18	40	1	17	18	31
18	1	18	17	48	1	18	17	39
19	1	19	16	56	1	19	16	47
20	1	20	16	5	1	20	15	56
21	1	21	15	13	1	21	15	4
22	1	22	14	21	1	22	14	11
23	1	23	13	30	1	23	13	20
24	1	24	12	38	1	24	12	28
25	1	25	11	46	1	25	11	36
26	1	26	10	55	1	26	10	45
27	1	27	10	3	1	27	9	52
28	1	28	9	11	1	28	9	0

In the months January and February of biſſextile years take away one day from the time.

Days.	March.							
	Longitude.				Anomaly.			
	S.	°	′	″	S.	°	′	″
1	1	29	8	20	1	29	8	9
2	2	0	7	28	2	0	7	17
3	2	1	6	36	2	1	6	25
4	2	2	5	45	2	2	5	34
5	2	3	4	53	2	3	4	42
6	2	4	4	1	2	4	3	49
7	2	5	3	10	2	5	2	58
8	2	6	2	18	2	6	2	6
9	2	7	1	26	2	7	1	14
10	2	8	0	35	2	8	0	22
11	2	8	59	43	2	8	59	30
12	2	9	58	51	2	9	58	38
13	2	10	58	0	2	10	57	47
14	2	11	57	8	2	11	56	55
15	2	12	56	16	2	12	56	3
16	2	13	55	25	2	13	55	11
17	2	14	54	33	2	14	54	19
18	2	15	53	41	2	15	53	27
19	2	16	52	50	2	16	52	36
20	2	17	51	58	2	17	51	44
21	2	18	51	6	2	18	50	51
22	2	19	50	15	2	19	50	0
23	2	20	49	23	2	20	49	8
24	2	21	48	31	2	21	48	16
25	2	22	47	40	2	22	47	25
26	2	23	46	48	2	23	46	33
27	2	24	45	56	2	24	45	40
28	2	25	45	5	2	25	44	53
29	2	26	44	13	2	26	43	57
30	2	27	43	21	2	27	43	5
31	2	28	42	30	2	28	42	14

Days.	April.							
	Longitude.				Anomaly.			
	S.	°	′	″	S.	°	′	″
1	2	29	41	38	2	29	41	22
2	3	0	40	46	3	0	40	30
3	3	1	39	55	3	1	39	38
4	3	2	39	3	3	2	38	46
5	3	3	38	11	3	3	37	54
6	3	4	37	20	3	4	37	3
7	3	5	36	28	3	5	36	11
8	3	6	35	36	3	6	35	18
9	3	7	34	45	3	7	34	27
10	3	8	33	53	3	8	33	35
11	3	9	33	1	3	9	32	43
12	3	10	32	10	3	10	31	52
13	3	11	31	18	3	11	30	59
14	3	12	30	26	3	12	30	7
15	3	13	29	35	3	13	29	16
16	3	14	28	43	3	14	28	24
17	3	15	27	51	3	15	27	32
18	3	16	27	0	3	16	26	40
19	3	17	26	8	3	17	25	48
20	3	18	25	16	3	18	24	56
21	3	19	24	25	3	19	24	5
22	3	20	23	33	3	20	23	13
23	3	21	22	41	3	21	22	21
24	3	22	21	50	3	22	21	29
25	3	23	20	58	3	22	20	37
26	3	24	20	6	3	24	19	45
27	3	25	19	15	3	25	18	54
28	3	26	18	23	3	26	18	2
29	3	27	17	31	3	27	17	9
30	3	28	16	40	3	28	16	18

The Sun's Mean Longitude and Anomaly for Months and Days.

Days.	May.								Days.	June.								Days.	July.								Days.	Auguſt.							
	Longitude.				Anomaly.					Longitude.				Anomaly.					Longitude.				Anomaly.					Longitude.				Anomaly.			
	S.	°	′	″	S.	°	′	″		S.	°	′	″	S.	°	′	″		S.	°	′	″	S.	°	′	″		S.	°	′	″	S.	°	′	″
1	3	29	15	48	3	29	15	26	1	4	29	49	6	4	29	48	39	1	5	29	23	16	5	29	22	43	1	6	29	56	34	6	29	55	55
2	4	0	14	56	4	0	14	34	2	5	0	48	15	5	0	47	47	2	6	0	22	24	6	0	21	51	2	7	0	55	43	7	0	55	4
3	4	1	14	5	4	1	13	43	3	5	1	47	23	5	1	46	55	3	6	1	21	33	6	1	21	0	3	7	1	54	51	7	1	54	12
4	4	2	13	13	4	2	12	51	4	5	2	46	31	5	2	46	3	4	6	2	20	41	6	2	20	8	4	7	2	53	59	7	2	55	20
5	4	3	12	21	4	3	11	58	5	5	3	45	40	5	3	45	12	5	6	3	19	49	6	3	19	15	5	7	3	53	8	7	3	52	29
6	4	4	11	30	4	4	11	7	6	5	4	44	48	5	4	44	20	6	6	4	18	58	6	4	18	24	6	7	4	52	16	7	4	51	37
7	4	5	10	38	4	5	10	15	7	5	5	43	56	5	5	43	28	7	6	5	18	6	6	5	17	32	7	7	5	51	24	7	5	50	44
8	4	6	9	46	4	6	9	23	8	5	6	43	5	5	6	42	37	8	6	6	17	14	6	6	16	40	8	7	6	50	33	7	6	49	54
9	4	7	8	55	4	7	8	32	9	5	7	42	13	5	7	41	44	9	6	7	16	23	6	7	15	49	9	7	7	49	41	7	7	49	1
10	4	8	8	3	4	8	7	40	10	5	8	41	21	5	8	40	52	10	6	8	15	31	6	8	14	56	10	7	8	48	49	7	8	48	9
11	4	9	7	11	4	9	6	47	11	5	9	40	30	5	9	40	1	11	6	9	14	39	6	9	14	4	11	7	9	47	58	7	9	47	18
12	4	10	6	20	4	10	5	56	12	5	10	39	38	5	10	39	8	12	6	10	13	48	6	10	13	13	12	7	10	47	6	7	10	46	25
13	4	11	5	28	4	11	5	4	13	5	11	38	46	5	11	38	16	13	6	11	12	56	6	11	12	21	13	7	11	46	14	7	11	45	33
14	4	12	4	36	4	12	4	12	14	5	12	37	55	5	12	37	25	14	6	12	12	4	6	12	11	29	14	7	12	45	23	7	12	44	42
15	4	13	3	45	4	13	3	21	15	5	13	37	3	5	13	36	33	15	6	13	11	13	6	13	10	38	15	7	13	44	31	7	13	43	50
16	4	14	2	53	4	14	2	28	16	5	14	36	11	5	14	35	41	16	6	14	10	21	6	14	9	45	16	7	14	43	39	7	14	42	58
17	4	15	2	1	4	15	1	36	17	5	15	35	20	5	15	34	50	17	6	15	9	29	6	15	8	53	17	7	15	42	48	7	15	42	7
18	4	16	1	10	4	16	0	45	18	5	16	34	28	5	16	33	57	18	6	16	8	38	6	16	8	2	18	7	16	41	56	7	16	41	16
19	4	17	0	18	4	16	59	53	19	5	17	33	36	5	17	33	5	19	6	17	7	46	6	17	7	10	19	7	17	41	4	7	17	40	22
20	4	17	59	26	4	17	59	1	20	5	18	32	45	5	18	32	14	20	6	18	6	54	6	18	6	18	20	7	18	40	13	7	18	39	31
21	4	18	58	35	4	18	58	10	21	5	19	31	53	5	19	31	32	21	6	19	6	3	6	19	5	26	21	7	19	39	21	7	19	38	39
22	4	19	57	43	4	19	57	17	22	5	20	31	1	5	20	30	30	22	6	20	5	11	6	20	4	34	22	7	20	38	29	7	20	37	47
23	4	20	56	51	4	20	56	25	23	5	21	30	10	5	21	29	39	23	6	21	4	19	6	21	3	42	23	7	21	37	38	7	21	36	56
24	4	21	56	0	4	21	55	34	24	5	22	29	18	5	22	28	46	24	6	22	3	28	6	22	2	51	24	7	22	36	46	7	22	36	4
25	4	22	55	8	4	22	54	42	25	5	23	28	26	5	23	27	54	25	6	23	2	36	6	23	1	59	25	7	23	35	54	7	23	35	11
26	4	23	54	16	4	23	53	50	26	5	24	27	35	5	24	27	3	26	6	24	1	44	6	24	1	7	26	7	24	35	3	7	24	34	20
27	4	24	53	25	4	24	52	58	27	5	25	26	43	5	25	26	11	27	6	25	0	53	6	25	0	15	27	7	25	34	11	7	25	33	28
28	4	25	52	33	4	25	52	6	28	5	26	25	51	5	26	25	19	28	6	26	0	1	6	25	59	23	28	7	26	33	19	7	26	32	36
29	4	26	51	41	4	26	51	14	29	5	27	24	59	5	27	24	26	29	6	26	59	9	6	26	58	31	29	7	27	32	28	7	27	31	44
30	4	27	50	50	4	27	50	23	30	5	28	24	8	5	28	23	35	30	6	27	58	18	6	27	57	40	30	7	28	31	36	7	28	30	52
31	5	28	49	58	4	28	49	31										31	6	28	57	26	6	28	56	48	31	7	29	30	44	7	29	30	0

The Sun's Mean Longitude and Anomaly for Months and Days.

Days.	September.								Days.	October.								Days.	November.								Days.	December.							
	Longitude.				Anomaly.					Longitude.				Anomaly.					Longitude.				Anomaly.					Longitude.				Anomaly.			
	S.	°	′	″	S.	°	′	″		S.	°	′	″	S.	°	′	″		S.	°	′	″	S.	°	′	″		S.	°	′	″	S.	°	′	″
1	8	0	29	53	8	0	29	9	1	9	0	4	3	9	0	3	13	1	10	0	37	21	10	0	36	26	1	11	0	11	31	11	0	10	30
2	8	1	29	1	8	1	28	17	2	9	1	3	11	9	1	2	21	2	10	1	36	29	10	1	35	34	2	11	1	10	39	11	1	9	38
3	8	2	28	9	8	2	27	25	3	9	2	2	19	9	2	1	29	3	10	2	35	38	10	2	34	43	3	11	2	9	47	11	2	8	46
4	8	3	27	18	8	3	26	33	4	9	3	1	28	9	3	0	38	4	10	3	34	46	10	3	33	50	4	11	3	8	56	11	3	7	55
5	8	4	26	26	8	4	25	41	5	9	4	0	36	9	3	59	46	5	10	4	33	54	10	4	32	58	5	11	4	8	4	11	4	7	3
6	8	5	25	34	8	5	24	49	6	9	4	59	44	9	4	58	54	6	10	5	33	3	10	5	32	7	6	11	5	7	12	11	5	6	11
7	8	6	24	43	8	6	23	58	7	9	5	58	53	9	5	58	2	7	10	6	32	11	10	6	31	15	7	11	6	6	21	11	6	5	20
8	8	7	23	51	8	7	23	6	8	9	6	58	1	9	6	57	10	8	10	7	31	19	10	7	30	23	8	11	7	5	29	11	7	4	27
9	8	8	22	59	8	8	22	14	9	9	7	57	9	9	7	56	18	9	10	8	30	27	10	8	29	30	9	11	8	4	37	11	8	3	35
10	8	9	22	8	8	9	21	22	10	9	8	56	18	9	8	55	27	10	10	9	29	36	10	9	28	39	10	11	9	3	46	11	9	2	44
11	8	10	21	16	8	10	20	30	11	9	9	55	26	9	9	54	35	11	10	10	28	44	10	10	27	47	11	11	10	2	54	11	10	1	52
12	8	11	20	24	8	11	19	38	12	9	10	54	34	9	10	53	42	12	10	11	27	52	10	11	26	55	12	11	11	2	2	11	11	1	0
13	8	12	19	33	8	12	18	47	13	9	11	53	43	9	11	52	51	13	10	12	27	1	10	12	26	4	13	11	12	1	11	11	12	0	8
14	8	13	18	41	8	13	17	55	14	9	12	52	51	9	12	51	59	14	10	13	26	9	10	13	25	12	14	11	13	0	19	11	12	59	16
15	8	14	17	49	8	14	17	2	15	9	13	51	59	9	13	51	7	15	10	14	25	17	10	14	24	19	15	11	13	59	27	11	13	58	24
16	8	15	16	58	8	15	16	11	16	9	14	51	8	9	14	50	16	16	10	15	24	26	10	15	23	28	16	11	14	58	36	11	14	57	33
17	8	16	16	6	8	16	15	19	17	9	15	50	16	9	15	49	24	17	10	16	23	34	10	16	22	36	17	11	15	57	44	11	15	56	41
18	8	17	15	14	8	17	14	27	18	9	16	49	24	9	16	48	31	18	10	17	22	42	10	17	21	44	18	11	16	56	52	11	16	55	48
19	8	18	14	23	8	18	13	35	19	9	17	48	33	9	17	47	40	19	10	18	21	50	10	18	20	52	19	11	17	56	1	11	17	54	57
20	8	19	13	31	8	19	12	43	20	9	18	47	41	9	18	46	48	20	10	19	20	59	10	19	20	0	20	11	18	55	9	11	18	54	5
21	8	20	12	39	8	20	11	51	21	9	19	46	49	9	19	45	56	21	10	20	20	7	10	20	19	8	21	11	19	54	17	11	19	53	13
22	8	21	11	48	8	21	11	0	22	9	20	45	58	9	20	45	5	22	10	21	19	16	10	21	18	17	22	11	20	53	26	11	20	52	22
23	8	22	10	56	8	22	10	8	23	9	21	45	6	9	21	44	13	23	10	22	18	24	10	22	17	25	23	11	21	52	34	11	21	51	29
24	8	23	10	4	8	23	9	16	24	9	22	44	14	9	22	43	20	24	10	23	17	32	10	23	16	33	24	11	22	51	42	11	22	50	37
25	8	24	9	13	8	24	8	25	25	9	23	43	23	9	23	42	29	25	10	24	16	41	10	24	15	41	25	11	23	50	51	11	23	49	46
26	8	25	8	21	8	25	7	32	26	9	24	42	31	9	24	41	37	26	10	25	15	49	10	25	14	49	26	11	24	49	59	11	24	48	54
27	8	26	7	29	8	26	6	40	27	9	25	41	39	9	25	40	45	27	10	26	14	57	10	26	13	57	27	11	25	49	[illegible]	11	25	48	2
28	8	27	6	38	8	27	5	49	28	9	26	40	48	9	26	39	54	28	10	27	14	6	10	27	13	6	28	11	26	48	16	11	26	47	11
29	8	28	5	46	8	28	4	57	29	9	27	39	56	9	27	39	2	29	10	28	13	14	10	28	12	14	29	11	27	47	24	11	27	46	18
30	8	29	4	54	8	29	4	5	30	9	28	39	4	9	28	38	9	30	10	29	12	22	10	29	11	22	30	11	28	46	32	11	28	45	26
									31	9	29	38	13	9	29	37	18										31	11	29	45	41	11	29	44	35

TABLE V. *Of the Sun's Mean Longitude and Anomaly for Hours, Minutes, and Seconds.*

H	′	″	‴	H	°	′	″
′	″	‴	iv.	′	′	″	‴
″	‴	iv.	v.	″	″	‴	iv.
1	2	27	50	31	1	16	23
2	4	55	41	32	1	18	51
3	7	23	31	33	1	21	19
4	9	51	22	34	1	23	47
5	12	19	12	35	1	26	14
6	14	47	2	36	1	28	42
7	17	14	53	37	1	31	10
8	19	42	43	38	1	33	38
9	22	10	34	39	1	36	6
10	34	38	24	40	1	38	34
11	27	6	14	41	1	41	1
12	29	34	5	42	1	43	29
13	32	1	55	43	1	45	57
14	34	29	46	44	1	48	25
15	36	57	36	45	1	50	53
16	39	25	27	46	1	53	21
17	41	53	17	47	1	55	48
18	44	21	7	48	1	58	16
19	46	48	58	49	2	0	44
20	49	16	48	50	2	3	12
21	51	44	39	51	2	5	40
22	54	12	29	52	2	8	8
23	56	40	19	53	2	10	36
24	59	8	19	54	2	13	3
25	61	36	9	55	2	15	31
26	64	3	59	56	2	17	59
27	66	31	49	57	2	20	27
28	68	59	31	58	2	22	55
29	71	27	21	59	2	25	23
30	73	55	11	60	2	27	50

TABLE VI. *Equations of the Sun's Centre.*

Argument. Sun's Mean Anomaly.

Signs	— 0			— 1			— 2			— 3			— 4			— 5			
°	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°
0	0	0	0	0	56	47	1	39	6	1	55	37	1	41	12	0	58	53	30
1	0	1	59	0	58	30	1	40	7	1	55	39	1	40	12	0	57	7	29
2	0	3	57	1	0	12	1	41	6	1	55	38	1	39	10	0	55	19	28
3	0	5	56	1	1	53	1	42	3	1	55	36	1	38	6	0	53	30	27
4	0	7	54	1	3	33	1	42	59	1	55	31	1	37	0	0	51	40	26
5	0	9	52	1	5	12	1	43	52	1	55	24	1	35	52	0	49	49	25
6	0	11	50	1	6	50	1	44	44	1	55	15	1	34	43	0	47	57	24
7	0	13	48	1	8	27	1	45	34	1	55	3	1	33	32	0	46	5	23
8	0	15	46	1	10	2	1	46	22	1	54	50	1	32	19	0	44	11	22
9	0	17	43	1	11	36	1	47	8	1	54	35	1	31	4	0	42	16	21
10	0	19	40	1	13	9	1	47	53	1	54	17	1	29	47	0	40	21	20
11	0	21	37	1	14	41	1	48	35	1	53	57	1	28	29	0	38	25	19
12	0	23	33	1	16	11	1	49	15	1	53	36	1	27	9	0	36	28	18
13	0	25	29	1	17	40	1	49	54	1	53	12	1	25	48	0	34	30	17
14	0	27	25	1	19	8	1	50	30	1	52	46	1	24	25	0	32	32	16
15	0	29	20	1	20	34	1	51	5	1	52	18	1	23	0	0	30	33	15
16	0	31	15	1	21	59	1	51	37	1	51	48	1	21	34	0	28	33	14
17	0	33	9	1	23	22	1	52	8	1	51	15	1	20	6	0	26	33	13
18	0	35	2	1	24	44	1	52	36	1	50	41	1	18	36	0	24	33	12
19	0	36	55	1	26	5	1	53	3	1	50	5	1	17	5	0	22	32	11
20	0	38	47	1	27	24	1	53	27	1	49	26	1	15	33	0	20	30	10
21	0	40	39	1	28	41	1	53	50	1	48	46	1	13	59	0	18	28	9
22	0	42	30	1	29	57	1	54	10	1	48	3	1	12	24	0	16	26	8
23	0	44	20	1	31	11	1	54	28	1	47	19	1	10	47	0	14	24	7
24	0	46	9	1	32	24	1	54	44	1	46	32	1	9	9	0	12	21	6
25	0	47	57	1	33	35	1	54	58	1	45	44	1	7	29	0	10	18	5
26	0	49	45	1	34	45	1	55	10	1	44	53	1	5	49	0	8	14	4
27	0	51	32	1	35	53	1	55	20	1	44	1	1	4	7	0	6	11	3
28	0	53	18	1	36	59	1	55	28	1	43	7	1	2	24	0	4	7	2
29	0	55	3	1	38	3	1	55	34	1	42	10	1	0	39	0	2	4	1
30	0	56	47	1	39	6	1	55	37	1	41	12	0	58	53	0	0	0	0
Signs	+ 11			+ 10			+ 9			+ 8			+ 7			+ 6			

TABLE VII. *Logarithms of the Sun's Diſtance from the Earth.*

Argument. Sun's Mean Anomaly.

Signs °	0	1	2	3	4	5	°
0	5·007286	5·006347	5·003749	5·000124	4·996405	4·993620	30
1	5·007285	5·006284	5·003640	4·999995	4·996292	4·993555	29
2	5·007282	5·006220	5·003531	4·999867	4·996180	4·993491	28
3	5·007277	5·006154	5·003420	4·999739	4·996069	4·993429	27
4	5·007269	5·006087	5·003307	4·999611	4·995959	4·993369	26
5	5·007260	5·006018	5·003194	4·999483	4·995850	4·993311	25
6	5·007249	5·005946	5·003080	4·999354	4·995742	4·993255	24
7	5·007235	5·005872	5·002965	4·999227	4·995636	4·993201	23
8	5·007218	5·005797	5·002849	4·999099	4·995531	4·993150	22
9	5·007200	5·005720	5·002732	4·998971	4·995427	4·993102	21
10	5·007180	5·005642	5·002614	4·998844	4·995325	4·993055	20
11	5·007158	5·005562	5·002495	4·998717	4·995224	4·993009	19
12	5·007134	5·005480	5·002375	4·998590	4·995126	4·992966	18
13	5·007107	5·005397	5·002254	4·998463	4·995028	4·992926	17
14	5·007079	5·005312	5·002134	4·998336	4·994932	4·992888	16
15	5·007048	5·005225	5·002012	4·998210	4·994836	4·992852	15
16	5·007015	5·005136	5·001890	4·998084	4·994743	4·992818	14
17	5·006980	5·005047	5·001767	4·997960	4·994652	4·992786	13
18	5·006943	5·004956	5·001643	4·997837	4·994562	4·992757	12
19	5·006905	5·004863	5·001518	4·997714	4·994474	4·992731	11
20	5·006864	5·004768	5·001393	4·997591	4·994387	4·992706	10
21	5·006821	5·004672	5·001268	4·997468	4·994302	4·992683	9
22	5·006776	5·004575	5·001142	4·997347	4·994219	4·992663	8
23	5·006730	5·004477	5·001016	4·997226	4·994138	4·992646	7
24	5·006681	5·004377	5·000889	4·997106	4·994058	4·992631	6
25	5·006630	5·004275	5·000762	4·996987	4·993980	4·992618	5
26	5·006577	5·004173	5·000635	4·996868	4·993904	4·992607	4
27	5·006522	5·004069	5·000508	4·996750	4·993831	4·992599	3
28	5·006466	5·003963	5·000380	4·996634	4·993759	4·992593	2
29	5·006408	5·003857	5·000252	4·996519	4·993688	4·992590	1
30	5·006347	5·003749	5·000124	4·996405	4·993620	4·992589	0
Signs	11	10	9	8	7	6	

TABLE VIII. *The Sun's Declination to every Degree of his Longitude.*

Argument. Sun's Longitude.

Deg.	0 ♈ North. 6 ♎ South. ☉ Declin.			1 ♉ North. 7 ♏ South. ☉ Declin.			2 ♊ North. 8 ♐ South. ☉ Declin.			Deg.
	°	′	″	°	′	″	°	′	″	
0	0	0	0	11	29	5	20	10	25	30
1	0	23	53	11	50	6	20	22	57	29
2	0	47	47	12	10	56	20	35	7	28
3	1	11	39	12	31	34	20	46	55	27
4	1	35	30	12	51	59	20	58	20	26
5	1	59	20	13	12	12	21	9	21	25
6	2	23	8	13	32	12	21	19	59	24
7	2	46	54	13	51	58	21	30	13	23
8	3	10	37	14	11	30	21	40	3	22
9	3	34	17	14	30	48	21	49	29	21
10	3	57	54	14	49	52	21	58	30	20
11	4	21	27	15	8	40	22	7	6	19
12	4	44	57	15	27	13	22	15	17	18
13	5	8	22	15	45	30	22	23	3	17
14	5	31	42	16	3	31	22	30	24	16
15	5	54	57	16	21	16	22	37	18	15
16	6	18	6	16	38	44	22	43	47	14
17	6	41	9	16	55	55	22	49	50	13
18	7	4	6	17	12	48	22	55	27	12
19	7	26	57	17	29	23	23	0	38	11
20	7	49	41	17	45	40	23	5	22	10
21	8	12	17	18	1	38	23	9	39	9
22	8	34	45	18	17	18	23	13	29	8
23	8	57	5	18	32	38	23	16	53	7
24	9	19	17	18	47	38	23	19	50	6
25	9	41	19	19	2	18	23	22	20	5
26	10	3	12	19	16	37	23	24	22	4
27	10	24	56	19	30	35	23	25	57	3
28	10	46	30	19	44	13	23	27	5	2
29	11	7	53	19	57	30	23	27	46	1
30	11	29	5	20	10	25	23	28	0	0
D	☉ Declin. 11 ♓ South. 5 ♍ North.			☉ Declin. 10 ♒ South. 4 ♌ North.			☉ Declin. 9 ♑ South. 3 ♋ North.			D

TABLE IX. *Of the Sun's Apparent Semidiameter and Hourly Motion.*

Argument. Sun's Mean Anomaly.

Signs	0				1				2				3				4				5				
	Semidiam.		Hour.M.		Semidiam.		Hour.M.		Semidiam.		Hour.M.		Semidiam.		Hour.M.		Semidiam.		Hour.M.		Semidiam.		Hour.M.		
°	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	°
0	15	47	2	23	15	49	2	24	15	55	2	25	16	3	2	28	16	11	2	30	16	17	2	32	30
10	15	47	2	23	15	51	2	24	15	57	2	26	16	5	2	29	16	13	2	31	16	18	2	33	20
20	15	49	2	23	15	52	2	25	16	0	2	27	16	8	2	30	16	15	2	32	16	19	2	33	10
30	15	49	2	24	15	55	2	25	16	3	2	28	16	11	2	30	16	17	2	32	16	19	2	33	0
Signs	11				10				9				8				7				6				

TABLE X. *Of the Equation of Time fitted to each Degree of the Ecliptic. Place of the Apogee* 3^s 9°.

Argument. Sun's Longitude.

Deg.	♈ 0 + —	♉ 1	♊ 2 — +	♋ 3 +	♌ 4 +	♍ 5 + —	♎ 6 —	♏ 7 —	♐ 8 —	♑ 9 — +	♒ 10 +	♓ 11 +
	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s	m. s.	m. s.	m. s.	m. s.	m. s.
0	7 36	1 9	3 51	1 13	5 57	2 20	7 38	15 31	13 33	1 11	11 28	14 19
1	7 17	1 23	3 47	1 26	5 59	2 4	7 58	15 39	13 17	0 42	11 45	14 13
2	6 58	1 36	3 41	1 40	6 0	1 48	8 19	15 46	13 0	— 12	12 1	14 6
3	6 39	1 48	3 37	1 55	6 1	1 31	8 40	15 52	12 42	+ 17	12 17	13 59
4	6 20	2 0	3 32	2 7	6 1	1 14	9 1	15 57	12 23	0 46	12 32	13 51
5	6 1	2 11	3 26	2 20	6 0	0 56	9 21	16 2	12 4	1 16	12 46	13 43
6	5 42	2 22	3 19	2 33	5 59	0 38	9 41	16 6	11 44	1 45	12 59	13 34
7	5 24	2 32	3 12	2 45	5 57	0 20	10 1	16 9	11 23	2 14	13 12	13 24
8	5 5	2 42	3 4	2 58	5 54	+ 1	10 20	16 11	11 1	2 43	13 24	13 14
9	4 47	2 51	2 56	3 11	5 51	—18	10 39	16 13	10 39	3 11	13 35	13 3
10	4 28	3 0	2 47	3 23	5 47	0 37	10 57	16 13	10 16	3 39	13 45	12 51
11	4 9	3 8	2 38	3 35	5 42	0 57	11 15	16 13	9 53	4 7	13 54	12 39
12	3 50	3 16	2 29	3 46	5 37	1 17	11 33	16 12	9 29	4 35	14 2	12 27
13	3 32	3 23	2 19	3 58	5 31	1 38	11 51	16 10	9 5	5 2	14 9	12 14
14	3 13	3 30	2 8	4 9	5 24	1 58	12 8	16 7	8 40	5 29	14 16	12 0
15	2 55	3 36	1 57	4 19	5 17	2 19	12 25	16 4	8 14	5 56	14 22	11 46
16	2 37	3 41	1 46	4 29	5 9	2 40	12 41	16 0	7 48	6 22	14 27	11 31
17	2 19	3 46	1 35	4 39	5 1	3 1	12 57	15 55	7 22	6 48	14 31	11 16
18	2 1	3 50	1 23	4 48	4 52	3 22	13 12	15 49	6 55	7 13	14 35	11 1
19	1 43	3 53	1 11	4 57	4 43	3 44	13 27	15 42	6 28	7 37	14 38	10 46
20	1 26	3 56	0 59	5 5	4 33	4 5	13 42	15 35	6 0	8 1	14 40	10 30
21	1 9	3 58	0 46	5 13	4 22	4 26	13 56	15 26	5 32	8 24	14 41	10 14
22	0 52	4 0	0 34	5 20	4 11	4 47	14 9	15 17	5 4	8 47	14 42	9 58
23	0 36	4 1	0 21	5 27	3 50	5 9	14 21	15 7	4 36	9 9	14 41	9 41
24	0 20	4 1	— 8	5 33	3 46	5 30	14 33	14 56	4 8	9 31	14 40	9 24
25	+ 4	4 1	+ 5	5 39	3 33	5 32	14 44	14 44	3 39	9 53	14 39	9 6
26	—11	4 0	0 19	5 44	3 19	6 13	14 53	14 31	3 10	10 14	14 37	8 48
27	0 26	3 59	0 31	5 48	3 4	6 35	15 5	14 17	2 41	10 34	14 34	8 30
28	0 40	3 57	0 46	5 52	2 50	6 56	15 14	14 3	2 11	10 53	14 30	8 12
29	0 53	3 54	0 59	5 55	2 35	7 17	15 23	13 48	1 41	11 11	14 25	7 54
30	1 9	3 51	1 13	5 57	2 20	7 28	15 31	13 33	1 11	11 28	14 19	7 36

The equations with + are to be added to the apparent time to have the mean time ; thoſe with — are to be ſubtracted from the apparent for the mean time.

TABLE XI. *The Sun's Longitude for every Day in the Year, at Noon.*

Days.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	Octob.	Novem.	Decem.
	S. ° ′	S. ° ′	S. ° ′	S. ° ′	S. ° ′	S. ° ′	S. ° ′	S. ° ′	S. ° ′	S. ° ′	S. ° ′	S. ° ′
1	9 11 21	10 12 54	11 11 8	0 11 55	1 11 12	2 11 2	3 9 41	4 9 17	5 9 7	6 8 26	7 9 15	8 9 32
2	9 12 23	10 13 55	11 12 8	0 12 54	1 12 10	2 12 0	3 10 38	4 10 14	5 10 6	6 9 25	7 10 15	8 10 33
3	9 13 24	10 14 56	11 13 8	0 13 53	1 13 9	2 12 57	3 11 35	4 11 11	5 11 4	6 10 24	7 11 16	8 11 34
4	9 14 25	10 15 57	11 14 8	0 14 52	1 14 7	2 13 54	3 12 32	4 12 9	5 12 2	6 11 24	7 12 16	8 12 35
5	9 15 26	10 16 57	11 15 8	0 15 51	1 15 5	2 14 52	3 13 30	4 13 6	5 13 0	6 12 23	7 13 16	8 13 36
6	9 16 27	10 17 58	11 16 8	0 16 50	1 16 3	2 15 49	3 14 27	4 14 4	5 13 59	6 13 22	7 14 16	8 14 37
7	9 17 29	10 18 59	11 17 8	0 17 49	1 17 1	2 16 46	3 15 24	4 15 2	5 14 57	6 14 21	7 15 17	8 15 38
8	9 18 30	10 20 0	11 18 8	0 18 48	1 17 59	2 17 44	3 16 21	4 15 59	5 15 55	6 15 21	7 16 17	8 16 39
9	9 19 31	10 21 0	11 19 8	0 19 47	1 18 56	2 18 41	3 17 18	4 16 57	5 16 53	6 16 20	7 17 17	8 17 40
10	9 20 32	10 22 1	11 20 8	0 20 45	1 19 54	2 19 39	3 18 15	4 17 54	5 17 52	6 17 19	7 18 18	8 18 41
11	9 21 33	10 23 1	11 21 7	0 21 44	1 20 52	2 20 35	3 19 13	4 18 52	5 18 50	6 18 19	7 19 18	8 19 42
12	9 22 34	10 24 2	11 22 7	0 22 43	1 21 50	2 21 33	3 20 10	4 19 49	5 19 49	6 19 18	7 20 19	8 20 43
13	9 23 35	10 25 3	11 23 7	0 23 41	1 22 48	2 22 30	3 21 7	4 20 47	5 20 47	6 20 18	7 21 19	8 21 44
14	9 24 36	10 26 3	11 24 6	0 24 40	1 23 45	2 23 28	3 22 4	4 21 45	5 21 46	6 21 17	7 22 20	8 22 45
15	9 25 37	10 27 4	11 25 6	0 25 39	1 24 43	2 24 25	3 23 2	4 22 42	5 22 44	6 22 17	7 23 20	8 23 47
16	9 26 39	10 28 4	11 26 6	0 26 37	1 25 41	2 25 22	3 23 59	4 23 40	5 23 43	6 23 17	7 24 21	8 24 48
17	9 27 39	10 29 4	11 27 5	0 27 36	1 26 39	2 26 19	3 24 56	4 24 38	5 24 42	6 24 16	7 25 21	8 25 49
18	9 28 41	11 0 5	11 28 5	0 28 34	1 27 36	2 27 17	3 25 53	4 25 36	5 25 40	6 25 16	7 26 22	8 26 50
19	9 29 42	11 1 5	11 29 4	0 29 33	1 28 34	2 28 14	3 26 51	4 26 33	5 26 39	6 26 16	7 27 23	8 27 51
20	10 0 43	11 2 6	0 0 4	1 0 31	1 29 32	2 29 11	3 27 48	4 27 31	5 27 38	6 27 16	7 28 23	8 28 52
21	10 1 44	11 3 6	0 1 3	1 1 30	2 0 29	3 0 8	3 28 45	4 28 29	5 28 37	6 28 15	7 29 24	8 29 54
22	10 2 45	11 4 6	0 2 3	1 2 28	2 1 27	3 1 6	3 29 43	4 29 27	5 29 35	6 29 15	8 0 25	9 0 55
23	10 3 46	11 5 7	0 3 2	1 3 26	2 2 25	3 2 3	4 0 40	5 0 25	6 0 34	7 0 15	8 1 26	9 1 56
24	10 4 47	11 6 7	0 4 1	1 4 25	2 3 22	3 3 0	4 1 37	5 1 23	6 1 33	7 1 15	8 2 26	9 2 57
25	10 5 48	11 7 7	0 5 1	1 5 23	2 4 20	3 3 57	4 2 35	5 2 21	6 2 32	7 2 15	8 3 27	9 3 58
26	10 6 49	11 8 7	0 6 0	1 6 21	2 5 17	3 4 55	4 3 32	5 3 19	6 3 31	7 3 15	8 4 28	9 4 59
27	10 7 50	11 9 8	0 6 59	1 7 20	2 6 15	3 5 52	4 4 29	5 4 17	6 4 30	7 4 15	8 5 29	9 6 1
28	10 8 51	11 10 8	0 7 59	1 8 18	2 7 12	3 6 49	4 5 27	5 5 15	6 5 29	7 5 15	8 6 30	9 7 2
29	10 9 52		0 8 58	1 9 16	2 8 10	3 7 46	4 6 24	5 6 13	6 6 28	7 6 15	8 7 31	9 8 3
30	10 10 52		0 9 57	1 10 14	2 9 7	3 8 43	4 7 22	5 7 11	6 7 27	7 7 15	8 8 32	9 9 4
31	10 11 53		0 10 56		2 10 5		4 8 19	5 8 9		7 8 15		9 10 5

LUNAR TABLES.

TABLE I. *Of the Moon's Mean Motions in Julian Years.*

Years.	☽'s M. Long.				☽'s M. Ano.				☽'s ☊ Retrog.			
	s.	°	′	″	s.	°	′	″	s.	°	′	″
1	4	9	23	5	2	28	43	15	0	19	19	43
2	8	18	46	11	5	27	26	30	1	8	39	26
3	0	28	9	16	8	26	9	45	1	27	59	9
B 4	5	20	42	57	0	7	56	54	2	17	22	3
5	10	0	6	2	3	6	40	9	3	6	41	46
6	2	9	29	7	6	5	23	23	3	26	1	29
7	6	18	52	13	9	4	6	39	4	15	21	12
B 8	11	11	25	53	0	15	53	47	5	4	44	6
9	3	20	48	59	3	14	37	3	5	24	3	49
10	8	0	12	4	6	13	20	17	6	13	23	32
11	0	9	35	9	9	12	3	32	7	2	43	15
B 12	5	2	8	50	0	23	50	17	7	22	6	9
13	9	11	31	55	3	22	33	56	8	11	25	52
14	1	20	55	1	6	21	17	11	9	0	45	35
15	6	0	18	6	9	20	0	26	9	20	5	18
B 16	10	22	51	46	1	1	47	34	10	9	28	12
17	3	2	14	52	4	0	30	50	10	28	47	55
18	7	11	37	57	6	29	14	4	11	18	7	38
19	11	21	1	2	9	27	57	19	0	7	27	21
B 20	4	13	34	43	1	9	44	30	0	26	50	15
B 40	8	27	9	26	2	19	28	56	1	23	40	30
B 60	1	10	44	9	3	29	13	24	2	20	30	45
B 80	5	24	18	52	5	8	57	52	3	17	21	0
B 100	10	7	53	35	6	18	42	20	4	14	11	15
B 200	8	15	47	10	1	7	24	40	8	28	22	13
B 300	6	23	40	45	7	26	7	0	1	12	33	45
B 400	5	1	34	20	2	14	49	20	5	26	45	0
B 500	3	9	27	55	9	3	31	40	10	10	56	15

TABLE II. *The Moon's Mean Longitude and Anomaly for Current Years.*

A. D.	Mean Long.				Mean Anom.				Long. ☊.			
	s.	°	′	″	s.	°	′	″	s.	°	′	″
1761	7	1	8	8	10	12	34	50	2	7	33	33
1781	11	14	42	54	11	22	19	18	1	10	43	18
1791	7	14	54	59	6	5	39	35	6	27	19	46
1792	0	7	28	40	9	17	26	44	6	7	56	52
1793	4	16	51	45	0	16	9	59	5	18	37	9
1794	8	26	14	51	3	14	53	14	4	29	17	26
1795	1	5	37	57	6	13	36	29	4	9	57	43
B 1796	5	28	11	37	9	25	23	38	3	20	34	49
1797	10	7	34	43	0	24	6	53	3	1	15	6
1798	2	16	57	48	3	22	50	8	2	11	55	23
1799	6	26	20	54	6	21	33	23	1	22	35	40
1800	11	5	44	0	9	20	16	38	1	3	15	57
1801	3	15	7	5	0	18	59	52	0	13	56	14
1802	7	24	30	11	3	17	43	7	11	24	36	31
1803	0	3	53	16	6	16	26	22	11	5	16	48
B 1804	4	26	26	57	9	28	13	31	10	15	53	54
1805	9	5	50	2	0	26	56	46	9	26	34	11
1806	1	15	13	8	3	25	40	1	9	7	14	28
1807	5	24	36	14	6	24	23	16	8	17	54	45
B 1808	10	17	9	54	10	6	10	25	7	28	31	51
1809	2	26	33	0	1	4	53	40	7	9	12	8
1810	7	5	56	5	4	3	36	55	6	19	52	25
1811	11	15	19	11	7	2	20	9	6	0	32	42
B 1812	4	7	52	52	10	14	7	18	5	11	9	48
1813	8	17	15	57	1	12	50	33	4	21	50	5
1814	0	26	39	3	4	11	33	48	4	2	30	22
1815	5	6	2	8	7	10	17	3	3	13	10	39
B 1816	9	28	35	49	10	22	4	12	2	23	47	45
1817	2	7	58	55	1	20	47	27	2	4	28	2
1818	6	17	22	0	4	19	30	42	1	15	8	19
1819	10	26	45	6	7	18	13	57	0	25	48	36
B 1820	3	19	18	47	11	0	1	6	0	6	25	42
1821	7	28	41	54	1	28	44	21	11	17	5	59
1841	0	12	16	37	3	8	28	51	10	20	15	44

TABLE III. *The Moon's Mean Motions for Months and Days.*

Days.	January.										
	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	'	"	S.	°	'	"	°	'	"
1	0	13	10	35	0	13	3	54	0	3	11
2	0	26	21	10	0	26	7	48	0	6	21
3	1	9	31	45	1	9	11	42	0	9	32
4	1	22	42	20	1	22	15	36	0	12	43
5	2	5	52	55	2	5	19	30	0	15	53
6	2	19	3	30	2	18	23	24	0	19	4
7	3	2	14	5	3	1	27	18	0	22	14
8	3	15	24	40	3	14	31	12	0	25	25
9	3	28	35	15	3	27	35	6	0	28	36
10	4	11	45	50	4	10	39	0	0	31	46
11	4	24	56	25	4	23	42	53	0	34	57
12	5	8	7	0	5	6	46	47	0	38	8
13	5	21	17	35	5	19	50	41	0	41	18
14	6	4	28	10	6	2	54	35	0	14	29
15	6	17	38	45	6	15	58	29	0	47	40
16	7	0	49	20	6	29	2	23	0	50	50
17	7	13	59	55	7	12	6	17	0	54	1
18	7	27	10	30	7	25	10	11	0	57	11
19	8	10	21	5	8	8	14	5	1	0	22
20	8	23	31	41	8	21	17	59	1	3	33
21	9	6	42	16	9	4	41	53	1	6	43
22	9	19	52	51	9	17	25	47	1	9	54
23	10	3	3	26	10	0	29	41	1	13	5
24	10	16	14	1	10	13	33	35	1	16	15
25	10	29	24	36	10	26	37	29	1	19	26
26	11	12	35	11	11	9	41	23	1	22	37
27	11	25	45	46	11	22	45	17	1	25	47
28	0	8	56	21	0	5	49	11	1	28	58
29	0	22	6	56	0	18	53	5	1	32	8
30	1	5	17	31	1	1	56	59	1	35	19
31	1	18	28	6	1	15	0	53	1	38	30

Days.	February.										
	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	'	"	S.	°	'	"	°	'	"
1	2	1	38	41	1	28	4	47	1	41	40
2	2	14	49	16	2	11	8	41	1	44	51
3	2	27	59	51	2	24	12	35	1	48	2
4	3	11	10	26	3	7	16	29	1	51	12
5	3	24	21	1	3	20	20	22	1	54	23
6	4	7	31	36	4	3	24	16	1	57	34
7	4	20	42	11	4	16	28	10	2	0	44
8	5	3	52	46	4	29	32	4	2	3	55
9	5	17	3	21	5	12	35	58	2	7	5
10	6	0	13	56	5	25	39	52	2	10	16
11	6	13	24	31	6	8	43	46	2	13	27
12	6	26	35	6	6	21	47	40	2	16	37
13	7	9	45	41	7	4	51	34	2	19	48
14	7	22	56	16	7	17	55	28	2	22	59
15	8	6	6	51	8	0	59	22	2	26	9
16	8	19	17	26	8	14	3	16	2	29	20
17	9	2	28	1	8	27	7	10	2	32	31
18	9	15	38	36	9	10	11	4	2	35	41
19	9	28	49	11	9	23	14	58	2	38	52
20	10	11	59	46	10	6	18	52	2	42	3
21	10	25	10	21	10	19	22	46	2	45	13
22	11	8	20	56	11	2	26	40	2	48	24
23	11	21	31	31	11	15	30	34	2	51	34
24	0	4	42	7	11	28	34	28	2	54	45
25	0	17	52	42	0	11	38	22	2	57	56
26	1	1	3	17	0	24	42	16	3	1	6
27	1	14	13	52	1	7	46	10	3	4	17
28	1	27	24	27	1	20	50	4	3	7	28

Days.	March.										
	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	'	"	S.	°	'	"	°	'	"
1	2	10	35	2	2	3	53	57	3	10	38
2	2	23	45	37	2	16	57	51	3	13	49
3	3	6	56	12	3	0	1	45	3	17	0
4	3	20	6	47	3	13	5	39	3	20	10
5	4	3	17	22	3	26	9	33	3	23	21
6	4	16	27	57	4	9	13	27	3	26	31
7	4	29	38	32	4	22	17	21	3	29	42
8	5	12	49	7	5	5	21	15	3	32	53
9	5	25	59	42	5	18	25	9	3	36	3
10	6	9	10	17	6	1	29	3	3	39	14
11	6	22	20	52	6	14	32	57	3	42	25
12	7	5	31	27	6	27	36	51	3	45	35
13	7	18	42	2	7	10	40	45	3	48	46
14	8	1	52	37	7	23	44	39	3	51	57
15	8	15	3	12	8	6	48	33	3	55	7
16	8	28	13	47	8	19	52	27	3	58	18
17	9	11	24	22	9	2	56	21	4	1	28
18	9	24	34	57	9	16	0	15	4	4	39
19	10	7	45	32	9	29	4	9	4	7	50
20	10	20	56	7	10	12	8	3	4	11	0
21	11	4	6	42	10	25	11	57	4	14	11
22	11	17	17	17	11	8	15	51	4	17	22
23	0	0	27	52	11	21	19	45	4	20	32
24	0	13	38	27	0	4	23	39	4	23	43
25	0	26	49	2	0	17	27	32	4	26	34
26	1	9	59	37	1	0	31	26	4	30	4
27	1	23	10	12	1	13	35	20	4	33	15
28	2	6	20	47	1	26	39	14	4	36	25
29	2	19	31	22	2	9	43	8	4	39	36
30	3	2	41	57	2	22	47	2	4	42	47
31	3	15	52	33	3	5	50	56	4	45	57

In January and February of a biſſextile year, ſubtract 1 from the number of days.

TABLE III. *The Moon's Mean Motions for Months and Days.*

April.

Days.	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	′	″	S.	°	′	″	°	′	″
1	3	29	3	8	3	18	54	50	4	49	8
2	4	12	13	43	4	1	58	44	4	52	19
3	4	25	24	18	4	15	2	38	4	55	29
4	5	8	34	53	4	28	6	32	4	58	40
5	5	21	45	28	5	11	10	26	5	1	51
6	6	4	56	3	5	24	14	20	5	5	1
7	6	18	6	38	6	7	18	14	5	8	12
8	7	1	17	13	6	20	22	8	5	11	23
9	7	14	27	48	7	3	26	2	5	14	33
10	7	27	38	23	7	16	29	56	5	17	44
11	8	10	48	58	7	29	33	50	5	20	54
12	8	23	59	33	8	12	37	44	5	24	5
13	9	7	10	8	8	25	41	38	5	27	16
14	9	20	20	43	9	8	45	32	5	30	26
15	10	3	31	18	9	21	49	26	5	33	37
16	10	16	41	53	10	4	53	20	5	36	48
17	10	29	52	28	10	17	57	14	5	39	58
18	11	13	3	3	11	1	1	7	5	43	9
19	11	26	13	38	11	14	5	1	5	46	20
20	0	9	24	13	11	27	8	55	5	49	30
21	0	22	34	48	0	10	12	49	5	52	41
22	1	5	45	23	0	23	16	43	5	55	51
23	1	18	55	58	1	6	20	37	5	59	2
24	2	2	6	33	1	19	24	31	6	2	13
25	2	15	17	8	2	2	28	25	6	5	23
26	2	28	27	43	2	15	32	19	6	8	34
27	3	11	38	18	2	28	36	13	6	11	45
28	3	24	48	53	3	11	40	7	6	14	55
29	4	7	59	28	3	24	44	1	6	18	6
30	4	21	10	3	4	7	47	55	6	21	17

May.

Days.	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	′	″	S.	°	′	″	°	′	″
1	5	4	20	38	4	20	51	49	6	24	27
2	5	17	31	13	5	3	55	43	6	27	38
3	6	0	41	48	5	16	59	37	6	30	48
4	6	13	52	23	6	0	3	31	6	33	59
5	6	27	2	59	6	13	7	25	6	37	10
6	7	10	13	34	6	26	11	19	6	40	20
7	7	23	24	9	7	9	15	13	6	43	31
8	8	6	34	44	7	22	19	7	6	46	42
9	8	19	45	19	8	5	23	1	6	49	52
10	9	2	55	54	8	18	26	55	6	53	3
11	9	16	6	29	9	1	30	49	6	56	14
12	9	29	17	4	9	14	34	42	6	59	24
13	10	12	27	39	9	27	38	36	7	2	35
14	10	25	38	14	10	10	42	30	7	5	46
15	11	8	48	49	10	23	46	24	7	8	56
16	11	21	59	24	11	6	50	18	7	12	7
17	0	5	9	59	11	19	54	12	7	15	17
18	0	18	20	34	0	2	58	6	7	18	28
19	1	1	31	9	0	16	2	0	7	21	39
20	1	14	41	44	0	29	5	54	7	24	49
21	1	27	52	19	1	12	9	48	7	28	0
22	2	11	2	54	1	25	13	42	7	31	11
23	2	24	13	29	2	8	17	36	7	34	21
24	3	7	24	4	2	21	21	30	7	37	32
25	3	20	34	39	3	4	25	24	7	40	43
26	4	3	45	14	3	17	29	18	7	43	53
27	4	16	55	49	4	0	33	12	7	47	4
28	5	0	6	24	4	13	37	6	7	50	14
29	5	13	16	59	4	26	41	0	7	53	25
30	5	26	27	34	5	9	44	54	7	56	36
31	6	9	38	9	5	22	48	48	7	59	46

June.

Days.	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	′	″	S.	°	′	″	°	′	″
1	6	22	48	44	6	5	52	42	8	2	57
2	7	5	59	19	6	18	56	36	8	5	8
3	7	19	9	54	7	2	0	30	8	9	18
4	8	2	20	29	7	15	4	24	8	12	29
5	8	15	31	4	7	28	8	17	8	15	40
6	8	28	41	39	8	11	12	11	8	18	50
7	9	11	52	14	8	24	16	5	8	22	1
8	9	25	2	49	9	7	19	59	8	25	11
9	10	8	13	25	9	20	23	53	8	28	22
10	10	21	24	0	10	3	27	47	8	31	33
11	11	4	34	35	10	16	31	41	8	34	43
12	11	17	45	10	10	29	35	35	8	37	54
13	0	0	55	45	11	12	39	29	8	41	5
14	0	14	6	20	11	25	43	23	8	44	15
15	0	27	16	55	0	8	47	17	8	47	26
16	1	10	27	30	0	21	51	11	8	50	37
17	1	23	38	5	1	4	55	5	8	53	47
18	2	6	48	40	1	17	58	59	8	56	58
19	2	19	59	15	2	1	2	53	9	0	8
20	3	3	9	50	2	14	6	47	9	3	19
21	3	16	20	25	2	27	10	41	9	6	30
22	3	29	31	0	3	10	14	35	9	9	40
23	4	12	41	35	3	23	18	29	9	12	51
24	4	25	52	10	4	6	22	23	9	16	2
25	5	9	2	45	4	19	26	17	9	19	12
26	5	22	13	20	5	2	30	11	9	22	23
27	6	5	23	55	5	15	34	5	9	25	34
28	6	18	34	30	5	28	37	59	9	28	44
29	7	1	45	5	6	11	41	52	9	31	55
30	7	14	55	40	6	24	45	46	9	35	6

TABLE III. *The Moon's Mean Motions for Months and Days.*

Days.	July.										
	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	′	″	S.	°	′	″	°	′	″
1	7	28	6	15	7	7	49	40	9	38	16
2	8	11	16	50	7	20	53	34	9	41	27
3	8	24	27	25	8	3	57	28	9	44	37
4	9	7	38	0	8	17	1	22	9	47	48
5	9	20	48	35	9	0	5	16	9	50	59
6	10	3	59	10	9	13	9	10	9	54	9
7	10	17	9	45	9	26	13	4	9	57	20
8	11	0	20	20	10	9	16	58	10	0	31
9	11	13	30	55	10	22	20	52	10	3	41
10	11	26	41	30	11	5	24	46	10	6	52
11	0	9	52	5	11	18	28	40	10	10	3
12	0	23	2	40	0	1	32	34	10	13	13
13	1	6	13	15	0	14	36	28	10	16	24
14	1	19	23	51	0	27	40	21	10	19	34
15	2	2	34	26	1	10	44	16	10	22	45
16	2	15	45	1	1	23	48	10	10	25	56
17	2	28	55	36	2	6	52	4	10	29	6
18	3	12	6	11	2	19	55	58	10	32	17
19	3	25	16	46	3	2	59	52	10	35	28
20	4	8	27	21	3	16	3	46	10	38	38
21	4	21	37	56	3	29	7	40	10	41	49
22	5	4	48	31	4	12	11	34	10	45	0
23	5	17	59	6	4	25	15	27	10	48	10
24	6	1	9	41	5	8	19	21	10	51	21
25	6	14	20	16	5	21	23	15	10	54	32
26	6	27	30	51	6	4	27	9	10	57	42
27	7	10	41	26	6	17	31	3	11	0	53
28	7	23	52	1	7	0	34	57	11	4	3
29	8	7	2	36	7	13	38	51	11	7	14
30	8	20	13	11	7	26	42	45	11	10	25
31	9	3	23	46	8	9	46	39	11	13	35

Days.	Auguſt.										
	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	′	″	S.	°	′	″	°	′	″
1	9	16	34	21	8	22	50	33	11	16	46
2	9	29	44	56	9	5	54	27	11	19	57
3	10	12	55	31	9	18	58	21	11	23	7
4	10	26	6	6	10	2	2	15	11	26	18
5	11	9	16	41	10	15	6	9	11	29	29
6	11	22	27	16	10	28	10	3	11	32	39
7	0	5	37	51	11	11	13	57	11	35	50
8	0	18	48	26	11	24	17	51	11	39	0
9	1	1	59	1	0	7	21	45	11	42	11
10	1	15	9	36	0	20	25	39	11	45	22
11	1	28	20	11	1	3	29	33	11	48	32
12	2	11	30	46	1	16	33	27	11	51	43
13	2	24	41	21	1	29	37	21	11	54	54
14	3	7	51	56	2	12	41	15	11	58	4
15	3	21	2	31	2	25	45	9	12	1	15
16	4	4	13	6	3	8	49	3	12	4	26
17	4	17	23	41	3	21	52	56	12	7	36
18	5	0	34	17	4	4	56	50	12	10	47
19	5	13	44	52	4	18	0	44	12	13	57
20	5	26	55	27	5	1	4	38	12	17	8
21	6	10	6	2	5	14	8	32	12	20	19
22	6	23	16	37	5	27	12	26	12	23	29
23	7	6	27	12	6	10	16	20	12	26	40
24	7	19	37	47	6	23	20	14	12	29	51
25	8	2	48	22	7	6	24	8	12	33	1
26	8	15	58	57	7	19	28	2	12	36	12
27	8	29	9	32	8	2	31	56	12	39	23
28	9	12	20	7	8	15	35	50	12	42	33
29	9	25	30	42	8	28	39	44	12	45	44
30	10	8	41	17	9	11	43	38	12	48	54
31	10	21	51	52	9	24	47	32	12	52	5

Days.	September.										
	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	′	″	S.	°	′	″	°	′	″
1	11	5	2	27	10	7	51	26	12	55	16
2	11	18	13	2	10	20	55	20	12	58	26
3	0	1	23	37	11	3	59	14	13	1	37
4	0	14	34	12	11	17	3	8	13	4	48
5	0	27	44	47	0	0	7	2	13	7	58
6	1	10	55	22	0	13	10	56	13	11	9
7	1	24	5	57	0	26	14	50	13	14	20
8	2	7	16	32	1	9	18	44	13	17	30
9	2	20	27	7	1	22	22	38	13	20	41
10	3	3	37	42	2	5	26	31	13	23	52
11	3	16	48	17	2	18	30	26	13	27	2
12	3	29	58	52	3	1	34	19	13	30	13
13	4	13	9	27	3	14	38	13	13	33	23
14	4	26	20	2	3	27	42	7	13	36	34
15	5	9	30	37	4	10	46	1	13	39	45
16	5	22	41	12	4	23	49	55	13	42	55
17	6	5	51	47	5	6	53	49	13	46	6
18	6	19	2	22	5	19	57	43	13	49	17
19	7	2	12	57	6	3	1	37	13	52	27
20	7	15	23	32	6	16	5	31	13	55	38
21	7	28	34	7	6	29	9	25	13	58	49
22	8	11	44	43	7	12	13	19	14	1	59
23	8	24	55	18	7	25	17	13	14	5	10
24	9	8	5	53	8	8	21	7	14	8	20
25	9	21	16	28	8	21	25	1	14	11	31
26	10	4	27	3	9	4	28	55	14	14	42
27	10	17	37	38	9	17	32	49	14	17	52
28	11	0	48	13	10	0	36	43	14	21	3
29	11	13	58	48	10	13	40	37	14	24	14
30	11	27	9	23	10	26	44	31	14	27	24

TABLE III. *The Moon's Mean Motions for Months and Days.*

Days.	October.										
	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	′	″	S.	°	′	″	°	′	″
1	0	10	19	58	11	9	48	25	14	30	35
2	0	23	30	33	11	22	52	19	14	33	46
3	1	6	41	8	0	5	56	13	14	36	56
4	1	19	51	43	0	19	0	6	14	40	7
5	2	3	2	18	1	2	4	0	14	43	17
6	2	16	12	53	1	15	7	54	14	46	28
7	2	29	23	28	1	28	11	48	14	49	39
8	3	12	34	3	2	11	15	42	14	52	49
9	3	25	44	38	2	24	19	36	14	56	0
10	4	8	55	13	3	7	23	30	14	59	11
11	4	22	5	48	3	20	27	24	15	2	21
12	5	5	16	23	4	3	31	18	15	5	32
13	5	18	26	58	4	16	35	12	15	8	43
14	6	1	37	33	4	29	39	6	15	11	53
15	6	14	48	8	5	12	43	0	15	15	4
16	6	27	58	43	5	25	46	54	15	18	15
17	7	11	9	18	6	8	50	48	15	21	25
18	7	24	19	53	6	21	54	42	15	24	36
19	8	7	30	28	7	4	58	36	15	27	46
20	8	20	41	3	7	18	2	30	15	30	57
21	9	3	51	38	8	1	6	24	15	34	8
22	9	17	2	13	8	14	10	18	15	37	18
23	10	0	12	48	8	27	14	12	15	40	29
24	10	13	23	23	9	10	18	6	15	43	40
25	10	26	33	58	9	23	22	0	15	46	50
26	11	9	44	33	10	6	25	54	15	50	1
27	11	22	55	8	10	19	29	48	15	53	12
28	0	6	5	44	11	2	33	41	15	56	22
29	0	19	16	19	11	15	37	35	15	59	33
30	1	2	26	54	11	28	41	29	16	2	43
31	1	15	37	29	0	11	45	23	16	5	54

Days.	November.										
	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	′	″	S.	°	′	″	°	′	″
1	1	28	48	4	0	24	49	17	16	9	5
2	2	11	58	39	1	7	53	11	16	12	15
3	2	25	9	14	1	20	57	5	16	15	26
4	3	8	19	49	2	4	0	59	16	18	37
5	3	21	30	24	2	17	4	53	16	21	47
6	4	4	40	59	3	0	8	47	16	24	58
7	4	17	51	34	3	13	12	41	16	28	9
8	5	1	2	9	3	26	16	35	16	31	19
9	5	14	12	44	4	9	20	29	16	34	30
10	5	27	23	19	4	22	24	23	16	37	40
11	6	10	33	54	5	5	28	17	16	40	51
12	6	23	44	29	5	18	32	11	16	44	2
13	7	6	55	4	6	1	36	5	16	47	12
14	7	20	5	39	6	14	39	59	16	50	23
15	8	3	16	14	6	27	43	53	16	53	34
16	8	16	26	49	7	10	47	47	16	56	44
17	8	29	37	24	7	23	51	41	16	59	55
18	9	12	47	59	8	6	55	35	17	3	6
19	9	25	58	34	8	19	59	29	17	6	16
20	10	9	9	9	9	3	3	23	17	9	27
21	10	22	19	44	9	16	7	16	17	12	38
22	11	5	30	19	9	29	11	10	17	15	48
23	11	18	40	54	10	12	15	4	17	18	59
24	0	1	51	29	10	25	18	58	17	22	9
25	0	15	2	4	11	8	22	52	17	25	20
26	0	28	12	39	11	21	26	46	17	28	31
27	1	11	23	14	0	4	30	40	17	31	41
28	1	24	33	49	0	17	34	34	17	34	52
29	2	7	44	24	1	0	38	28	17	38	3
30	2	20	54	59	1	13	42	22	17	41	13

Days.	December.										
	M. Motion ☽.				Anomaly.				Mot. ☊.		
	S.	°	′	″	S.	°	′	″	°	′	″
1	3	4	5	34	1	26	46	16	17	44	24
2	3	17	16	10	2	9	50	10	17	47	35
3	4	0	26	45	2	22	54	4	17	50	45
4	4	13	37	20	3	5	57	58	17	53	56
5	4	26	47	55	3	19	1	52	17	57	6
6	5	9	58	30	4	2	5	46	18	0	17
7	5	23	9	5	4	15	9	40	18	3	28
8	6	6	19	40	4	28	13	34	18	6	38
9	6	19	30	15	5	11	17	28	18	9	49
10	7	2	40	50	5	24	21	22	18	13	0
11	7	15	51	25	6	7	25	16	18	16	10
12	7	29	2	0	6	20	29	10	18	19	21
13	8	12	12	35	7	3	33	4	18	22	32
14	8	25	23	10	7	16	36	58	18	25	42
15	9	8	33	45	7	29	40	51	18	28	53
16	9	21	44	20	8	12	44	45	18	32	3
17	10	4	54	55	8	25	48	39	18	35	14
18	10	18	5	30	9	8	52	33	18	38	25
19	11	1	16	5	9	21	56	27	18	41	35
20	11	14	26	40	10	5	0	21	18	44	46
21	11	27	37	15	10	18	4	15	18	47	56
22	0	10	47	50	11	1	8	9	18	51	7
23	0	23	58	25	11	14	12	3	18	54	18
24	1	7	9	0	11	27	15	57	18	57	29
25	1	20	19	35	0	10	19	51	19	0	39
26	2	3	30	10	0	23	23	45	19	3	50
27	2	16	40	45	1	6	27	39	19	7	0
28	2	29	51	20	1	19	31	33	19	10	11
29	3	13	1	55	2	2	35	27	19	13	22
30	3	26	12	30	2	15	39	21	19	16	32
31	4	9	23	5	2	28	43	15	19	19	43

TABLE IV. *The Moon's Mean Motions for Hours, Minutes, and Seconds.*

	M.Mo. ☽			Anomaly.			Mot. ☊			M.Mo. ☽			Anomaly.			Mot. ☊	
H.	°	′	″	°	′	″	′	″									
′	′	″	‴	′	″	‴	″	‴	′	′	″	‴	′	″	‴	″	‴
″	″	‴	⁗	″	‴	⁗	‴	⁗	″	″	‴	⁗	″	‴	⁗	‴	⁗
1	0	32	56	0	32	39	0	8	31	17	1	10	16	52	32	4	6
2	1	5	53	1	5	20	0	16	32	17	34	7	17	25	13	4	14
3	1	38	49	1	37	59	0	24	33	18	7	3	17	58	0	4	22
4	2	11	46	2	10	39	0	32	34	18	40	0	18	30	31	4	30
5	2	44	42	2	43	18	0	40	35	19	12	55	19	3	12	4	37
6	3	17	39	3	15	59	0	48	36	19	45	53	19	35	50	4	47
7	3	50	35	3	48	38	0	56	37	20	18	48	20	8	30	4	54
8	4	23	32	4	21	18	1	4	38	20	51	47	20	41	10	4	54
9	4	56	28	4	53	58	1	12	39	21	24	41	21	13	49	5	1
10	5	29	25	5	26	38	1	19	40	21	57	37	21	46	30	5	12
11	6	2	21	5	59	17	1	27	41	22	30	36	22	19	12	5	25
12	6	35	18	6	31	57	1	35	42	23	3	30	22	51	48	5	31
13	7	8	14	7	4	37	1	43	43	23	36	30	23	24	30	5	42
14	7	41	10	7	37	16	1	51	44	24	9	24	23	57	6	5	50
15	8	14	7	8	9	56	1	59	45	24	42	19	24	29	48	6	0
16	8	47	3	8	42	36	2	7	46	25	15	18	25	2	30	6	6
17	9	20	0	9	15	16	2	15	47	25	48	12	25	35	6	6	13
18	9	52	56	9	47	55	2	23	48	26	21	12	26	7	48	6	23
19	10	25	53	10	20	35	2	31	49	26	54	6	26	40	30	6	30
20	10	58	49	10	53	15	2	39	50	27	27	0	27	13	6	6	36
21	11	31	46	11	25	55	2	47	51	28	0	0	27	45	48	6	48
22	12	4	42	11	58	34	2	55	52	28	32	55	28	18	30	6	54
23	12	37	39	12	31	14	3	3	53	29	5	54	28	51	6	7	0
24	13	10	35	13	3	54	3	11	54	29	38	48	29	23	48	7	6
25	13	43	32	13	36	34	3	19	55	30	11	48	29	56	24	7	18
26	14	16	28	14	9	13	3	26	56	30	44	42	30	29	6	7	24
27	14	49	24	14	41	53	3	34	57	31	17	36	31	1	48	7	30
28	15	22	21	15	14	33	3	42	58	31	50	36	31	34	24	7	48
29	15	55	17	15	47	13	3	50	59	32	23	30	32	7	6	7	48
30	10	28	14	16	19	52	3	58	60	32	56	39	32	39	48	7	56

TABLES OF THE MOON'S EQUATIONS.

I. *Annual Equation of the Moon's Node.*

Argument. Sun's Mean Anomaly.

S.	+ 0		+ 1		+ 2		+ 3		+ 4		+ 5		S.
°	′	″	′	″	′	″	′	″	′	″	′	″	°
0	0	0	4	30	7	52	9	12	8	4	4	42	30
5	0	47	5	10	8	15	9	11	7	38	3	58	25
10	1	34	5	48	8	34	9	6	7	9	3	13	20
15	2	19	6	23	8	49	8	56	6	37	2	26	15
20	3	4	6	56	9	1	8	43	6	1	1	38	10
25	3	48	7	26	9	8	8	25	5	23	0	49	5
30	4	30	7	52	9	12	8	4	4	42	0	0	0
S.	— 11		— 10		— 9		— 8		— 7		— 6		S.

Annual Equation of Moon's Mean Anomaly.

Argument. Sun's Mean Anomaly.

S.	+ 0		+ 1		+ 2		+ 3		+ 4		+ 5		S.
°	′	″	′	″	′	″	′	″	′	″	′	″	°
0	0	0	10	37	18	33	21	42	19	1	11	5	30
5	1	51	12	11	19	27	21	39	18	1	9	23	25
10	3	40	13	41	20	13	21	27	16	53	7	36	20
15	5	29	15	4	20	49	21	5	15	37	5	45	15
20	7	15	16	21	21	16	20	34	14	13	3	52	10
25	8	58	17	31	21	34	19	52	12	42	1	56	5
30	10	37	18	33	21	42	19	1	11	5	0	0	0
S.	— 11		— 10		— 9		— 8		— 7		— 6		S.

I. *For the Moon's Longitude.*

Argument. Sun's Mean Anomaly.

S.	+ 0		+ 1		+ 2		+ 3		+ 4		+ 5		S.
°	′	″	′	″	′	″	′	″	′	″	′	″	°
0	0	0	5	27	9	31	11	9	9	47	5	42	30
5	0	57	6	15	9	59	11	8	9	16	4	49	25
10	1	53	7	1	10	23	11	1	8	41	3	54	20
15	2	49	7	44	10	41	10	50	8	2	2	58	15
20	3	43	8	23	10	55	10	34	7	19	1	59	10
25	4	36	8	59	11	4	10	13	6	32	1	0	5
30	5	27	9	31	11	9	9	47	5	42	0	0	0
S.	— 11		— 10		— 9		— 8		— 7		— 6		S.

II. *For the Moon's Longitude.*

Arg. II. 2 ☽ from ☉ + Arg. I.

S.	— 0	— 1	— 2	S.
S.	+ 6	+ 7	+ 8	S.
°	″	″	″	°
0	0	28	48	30
5	5	32	51	25
10	10	36	52	20
15	14	39	54	15
20	19	43	55	10
25	24	46	56	5
30	28	48	56	0
S.	+ 11	+ 10	+ 9	S.
S.	— 5	— 4	— 3	S.

III. *For the Moon's Longitude.*

Arg. III. 2 ☽ from ☉ — Arg. I.

S.	— 0	— 1	— 2	S.
S.	+ 6	+ 7	+ 8	S.
°	′ ″	′ ″	′ ″	°
0	0 0	0 38	1 5	30
5	0 7	0 43	1 8	25
10	0 13	0 48	1 11	20
15	0 19	0 53	1 13	15
20	0 26	0 58	1 14	10
25	0 32	1 2	1 15	5
30	0 38	1 5	1 15	0
S.	+ 11	+ 10	+ 9	S.
S.	— 5	— 4	— 3	S.

IV. *For the Moon's Longitude.*

Arg. IV. 2 ☽ from ☉ + ☽'s M. Anom.

S.	+ 0	+ 1	+ 2	S.
S.	— 6	— 7	— 8	S.
°	″	″	″	°
0	0	29	50	30
5	5	33	52	25
10	10	37	54	20
15	15	41	56	15
20	20	44	57	10
25	24	47	57	5
30	29	50	58	0
S.	— 11	— 10	— 9	S.
S.	+ 5	+ 4	+ 3	S.

V. *For the Moon's Longitude.* *Evection.*

Argument V. 2 ☽ from ☉ — ☽'s Mean Anomaly.

S.	— 0	— 1	— 2	— 3	— 4	— 5	S.
°	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	°
0	0 0 0	0 39 44	1 9 11	1 20 28	1 10 12	0 40 45	30
1	0 1 23	0 40 56	1 9 53	1 20 29	1 9 30	0 39 31	29
2	0 2 46	0 42 7	1 10 34	1 20 28	1 8 46	0 38 16	28
3	0 4 9	0 43 18	1 11 14	1 20 25	1 8 1	0 37 0	27
4	0 5 32	0 44 27	1 11 52	1 20 21	1 7 15	0 35 45	26
5	0 6 55	0 45 36	1 12 29	1 20 16	1 6 28	0 34 27	25
6	0 8 17	0 46 45	1 13 5	1 20 9	1 5 40	0 33 10	24
7	0 9 40	0 47 52	1 13 39	1 20 1	1 4 50	0 31 52	23
8	0 11 2	0 48 59	1 14 12	1 19 51	1 3 59	0 30 33	22
9	0 12 24	0 50 4	1 14 44	1 19 40	1 3 7	0 29 14	21
10	0 13 46	0 51 9	1 15 15	1 19 27	1 2 13	0 27 54	20
11	0 15 8	0 52 13	1 15 44	1 19 13	1 1 19	0 26 34	19
12	0 16 30	0 53 16	1 16 11	1 18 57	1 0 23	0 25 13	18
13	0 17 51	0 54 18	1 16 38	1 18 40	0 59 26	0 23 51	17
14	0 19 12	0 55 19	1 17 3	1 18 21	0 58 28	0 22 29	16
15	0 20 32	0 56 19	1 17 26	1 18 1	0 57 29	0 21 7	15
16	0 21 52	0 57 18	1 17 48	1 17 40	0 56 29	0 19 44	14
17	0 23 12	0 58 16	1 18 9	1 17 17	0 55 28	0 18 22	13
18	0 24 31	0 59 13	1 18 29	1 16 53	0 54 26	0 16 58	12
19	0 25 50	1 0 9	1 18 47	1 16 27	0 53 22	0 15 34	11
20	0 27 9	1 1 4	1 19 3	1 16 0	0 52 18	0 14 10	10
21	0 28 27	1 1 58	1 19 18	1 15 31	0 51 13	0 12 46	9
22	0 29 44	1 2 51	1 19 32	1 15 1	0 50 7	0 11 22	8
23	0 31 1	1 3 42	1 19 44	1 14 30	0 49 0	0 9 57	7
24	0 32 18	1 4 33	1 19 55	1 13 57	0 47 51	0 8 32	6
25	0 33 34	1 5 22	1 20 4	1 13 23	0 46 42	0 7 7	5
26	0 34 49	1 6 10	1 20 12	1 12 47	0 45 33	0 5 42	4
27	0 36 4	1 6 57	1 20 18	1 12 10	0 44 22	0 4 16	3
28	0 37 18	1 7 43	1 20 23	1 11 32	0 43 10	0 2 51	2
29	0 38 31	1 8 28	1 20 26	1 10 53	0 41 58	0 1 25	1
30	0 39 44	1 9 11	1 20 28	1 10 12	0 40 45	0 0 0	0
S.	+ 11	+ 10	+ 9	+ 8	+ 7	+ 6	S.

VI. *For the* ☽*'s Longitude.*

Argument.

Arg. V. + Arg. I.

S.	+ 0	+ 1	+ 2	S.
S.	— 6	— 7	— 8	S.
°	′ ″	′ ″	′ ″	°
0	0 0	1 2	1 47	30
5	0 11	1 11	1 52	25
10	0 21	1 19	1 56	20
15	0 32	1 27	1 59	15
20	0 42	1 35	2 2	10
25	0 52	1 41	2 3	5
30	1 2	1 47	2 3	0
S.	—11	—10	— 9	S.
S.	+ 5	+ 4	+ 3	S.

VII. *For the* ☽*'s Longitude.*

Argument.

Arg. V. — Arg. I.

S.	+ 0	+ 1	+ 2	S.
S.	— 6	— 7	— 8	S.
°	″	″	″	°
0	0	23	40	30
5	4	27	42	25
10	8	30	44	20
15	12	33	45	15
20	16	36	46	10
25	20	38	46	5
30	23	40	47	0
S.	—11	—10	— 9	S.
S.	+ 5	+ 4	+ 3	S.

VIII. *For the* ☽*'s Longitude.*

Argument.

☽'s Mean Anom. — Arg. I.

S.	+ 0	+ 1	+ 2	S.
S.	— 6	— 7	— 8	S.
°	″	″	″	°
0	0	21	36	30
5	4	24	38	25
10	7	27	40	20
15	11	30	41	15
20	14	32	41	10
25	18	34	42	5
30	21	36	42	0
S.	—11	—10	— 9	S.
S.	+ 5	+ 4	+ 3	S.

IX. *For the Moon's Longitude.*

Argument IX.

Mean Diſt. ☽ from ☉ — ☽'s Mean Anomaly.

S.	— 0	— 1	∓ 2	+ 3	+ 4	+ 5	S.
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	°
0	0 0	0 38	0 30	0 23	1 9	1 1	30
5	8	41	23	33	1 12	0 54	25
10	16	42	16	42	1 14	0 45	20
15	23	41	7	51	1 13	0 35	15
20	29	39	+ 3	58	1 11	23	10
25	34	35	13	1 5	1 7	12	5
30	38	30	23	1 9	1 1	0 0	0
S.	+ 11	+ 10	± 9	— 8	— 7	— 6	S.

X. *For the Moon's Longitude.*

Argument X.

Mean Long. ☊ — True Long. ☉.

S.	+ 0	+ 1	+ 2	S.
S.	+ 6	+ 7	+ 8	S.
°	″	′ ″	″	°
0	0	0 52	52	30
5	10	0 57	46	25
10	21	0 59	39	20
15	30	1 0	30	15
20	39	1 0	21	10
25	46	0 57	10	5
30	52	0 52	0	0
S.	—11	—10	— 9	S.
S.	— 5	— 4	— 3	S.

XI. *For the Moon's Longitude. Equation of the Centre.*

Argument XI. Moon's Correct Anomaly.

S.	— 0	— 1	— 2	— 3	— 4	— 5	S.
°	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	°
0	0 0 0	2 58 30	5 16 21	6 17 38	5 38 46	3 20 56	30
1	0 6 11	3 3 59	5 19 48	6 18 2	5 35 41	3 14 57	29
2	0 12 22	3 9 24	5 23 10	6 18 18	5 32 28	3 8 54	28
3	0 18 32	3 14 46	5 26 26	6 18 28	5 29 8	3 2 47	27
4	0 24 42	3 20 5	5 29 38	6 18 32	5 25 42	2 56 36	26
5	0 30 52	3 25 22	5 32 43	6 18 28	5 22 10	2 50 21	25
6	0 37 2	3 30 35	5 35 43	6 18 17	5 18 30	2 44 2	24
7	0 43 11	3 35 45	5 38 38	6 17 59	5 14 44	2 37 40	23
8	0 49 19	3 40 51	5 41 27	6 17 34	5 10 52	2 31 14	22
9	0 55 26	3 45 54	5 44 11	6 17 3	5 6 53	2 24 45	21
10	1 1 33	3 50 54	5 46 48	6 16 24	5 2 48	2 18 13	20
11	1 7 39	3 55 50	5 49 20	6 15 38	4 58 37	2 11 37	19
12	1 13 44	4 0 42	5 51 45	6 14 45	4 54 19	2 4 59	18
13	1 19 47	4 5 30	5 54 5	6 13 45	4 49 55	1 58 18	17
14	1 25 50	4 10 15	5 56 19	6 12 38	4 45 25	1 51 34	16
15	1 31 51	4 14 56	5 58 27	6 11 24	4 40 50	1 44 48	15
16	1 37 51	4 19 33	6 0 28	6 10 3	4 36 8	1 37 59	14
17	1 43 49	4 24 5	6 2 24	6 8 34	4 31 20	1 31 8	13
18	1 49 45	4 28 34	6 4 13	6 6 59	4 26 27	1 24 16	12
19	1 55 41	4 32 58	6 5 56	6 5 16	4 21 29	1 17 21	11
20	2 1 34	4 37 18	6 7 32	6 3 27	4 16 24	1 10 25	10
21	2 7 25	4 41 33	6 9 2	6 1 30	4 11 14	1 3 27	9
22	2 13 15	4 45 44	6 10 26	5 59 27	4 5 59	0 56 27	8
23	2 19 2	4 49 50	6 11 43	5 57 16	4 0 38	0 49 27	7
24	2 24 48	4 53 52	6 12 54	5 54 58	3 55 13	0 42 25	6
25	2 30 31	4 57 49	6 13 58	5 52 34	3 49 42	0 35 22	5
26	2 36 12	5 1 41	6 14 55	5 50 2	3 44 6	0 28 19	4
27	2 41 50	5 5 29	6 15 46	5 47 23	3 38 26	0 21 15	3
28	2 47 26	5 9 11	6 16 30	5 44 38	3 32 40	0 14 10	2
29	2 53 0	5 12 49	6 17 7	5 41 46	3 26 51	0 7 5	1
30	2 58 30	5 16 21	6 17 38	5 38 46	3 20 56	0 0 0	0
S.	+ 11	+ 10	+ 9	+ 8	+ 7	+ 6	S.

XII. *For the Moon's Longitude.* *Variation.*

Argument XII.

☽'s equated Long.—☉'s true Long.

S.	+ 0		+ 1		± 2		— 3		— 4		— 5		S.
°	′	″	′	″	′	″	′	″	′	″	′	″	°
0	0	0	30	9	29	6	2	2	32	28	31	55	30
1	1	14	30	43	28	26	3	16	33	3	31	15	29
2	2	27	31	15	27	44	4	30	33	36	30	33	28
3	3	40	31	44	27	0	5	43	34	6	29	48	27
4	4	53	32	11	26	13	6	57	34	35	29	2	26
5	6	6	32	36	25	25	8	10	35	1	28	13	25
6	7	18	32	58	24	35	9	22	35	24	27	22	24
7	8	30	33	18	23	43	10	34	35	45	26	29	23
8	9	41	33	35	22	49	11	46	36	3	25	35	22
9	10	51	33	49	21	53	12	56	36	19	24	38	21
10	12	0	34	1	20	56	14	6	36	32	23	40	20
11	13	9	34	11	19	57	15	15	36	43	22	40	19
12	14	16	34	18	18	57	16	23	36	51	21	39	18
13	15	23	34	22	17	55	17	29	36	57	20	35	17
14	16	28	34	23	16	52	18	35	36	59	19	30	16
15	17	31	34	22	15	47	19	39	37	0	18	24	15
16	18	34	34	19	14	41	20	42	36	57	17	17	14
17	19	35	34	13	13	34	21	44	36	53	16	8	13
18	20	35	34	4	12	26	22	44	36	45	14	58	12
19	21	33	33	53	11	17	23	43	36	35	13	47	11
20	22	30	33	39	10	8	24	40	36	22	12	36	10
21	23	24	33	23	8	57	25	35	36	6	11	23	9
22	24	17	33	4	7	45	26	29	35	48	10	9	8
23	25	8	32	43	6	34	27	21	35	28	8	54	7
24	25	57	32	19	5	21	28	10	35	5	7	39	6
25	26	45	31	53	4	8	28	59	34	39	6	24	5
26	27	30	31	24	2	54	29	45	34	11	5	7	4
27	28	13	30	53	1	41	30	29	33	41	3	51	3
28	28	54	30	19	+0	27	31	10	33	8	2	34	2
29	29	32	29	44	0	47	31	50	32	32	1	17	1
30	30	9	29	6	—2	2	32	28	31	55	0	0	0
S.	— 11		— 10		∓ 9		+ 8		+ 7		+ 6		S.

XIII. *For the* ☽*'s Longitude.*

Argument XIII.

Doub. eq. diſt. ☽ from ☊
— ☽ C. An.

S.	+ 0		+ 1		+ 2		S.
S.	— 6		— 7		— 8		S.
°	′	″	′	″	′	″	°
0	0	0	0	42	1	13	30
5	0	7	0	48	1	16	25
10	0	15	0	54	1	19	20
15	0	22	0	59	1	21	15
20	0	29	1	4	1	23	10
25	0	35	1	9	1	24	5
30	0	42	1	13	1	24	0
S.	—11		—10		— 9		S.
S.	+ 5		+ 4		+ 3		S.

Equation of the Equinoctial Points.

Argument.

Mean Longitude Moon's ☊.

S.	— 0	— 1	— 2	S.
S.	+ 6	+ 7	+ 8	S.
°	″	″	″	°
0	0	9	16	30
5	1	10	16	25
10	3	12	17	20
15	5	13	17	15
20	6	14	18	10
25	8	15	18	5
30	9	16	18	0
S.	+ 11	+ 10	+ 9	S.
S.	— 5	— 4	— 3	S.

XIV. *For the* ☽*'s Lon. Reduc.*

Argument XIV.

Long. ☽ in Orb. — C.
Long. ☊.

S.	— 0		— 1		— 2		S.
S.	— 6		— 7		— 8		S.
°	′	″	′	″	′	″	°
0	0	0	5	53	5	53	30
1	0	14	6	0	5	46	29
2	0	28	6	6	5	38	28
3	0	43	6	12	5	30	27
4	0	57	6	18	5	21	26
5	1	11	6	23	5	12	25
6	1	25	6	28	5	3	24
7	1	39	6	32	4	53	23
8	1	52	6	36	4	43	22
9	2	6	6	39	4	33	21
10	2	19	6	41	4	22	20
11	2	33	6	44	4	11	19
12	2	46	6	45	4	0	18
13	2	59	6	47	3	48	17
14	3	11	6	47	3	36	16
15	3	24	6	48	3	24	15
16	3	36	6	47	3	11	14
17	3	48	6	47	2	59	13
18	4	0	6	45	2	46	12
19	4	11	6	44	2	33	11
20	4	22	6	41	2	19	10
21	4	33	6	39	2	6	9
22	4	43	6	36	1	52	8
23	4	53	6	32	1	39	7
24	5	3	6	28	1	25	6
25	5	12	6	23	1	11	5
26	5	21	6	18	0	57	4
27	5	30	6	12	0	43	3
28	5	38	6	6	0	28	2
29	5	46	6	0	0	14	1
30	5	53	5	53	0	0	0
S.	+ 11		+ 10		+ 9		S.
S.	+ 5		+ 4		+ 3		S.

TABLES FOR FINDING THE MOON's LATITUDE.

I. *For the Moon's Latitude.*

Argument I.

Long. ☽ in Orbit—Cor. Long. ☊.

S.	+ 0			+ 1			+ 2			S.
S.	− 6			− 7			− 8			S.
°	°	′	″	°	′	″	°	′	″	°
0	0	0	0	2	34	18	4	27	23	30
1	0	5	23	2	38	56	4	30	2	29
2	0	10	46	2	43	32	4	32	37	28
3	0	16	9	2	48	5	4	35	6	27
4	0	21	31	2	52	35	4	37	31	26
5	0	26	53	2	57	1	4	39	50	25
6	0	32	15	3	1	24	4	42	4	24
7	0	37	36	3	5	44	4	44	14	23
8	0	42	56	3	10	1	4	46	17	22
9	0	48	16	3	14	14	4	48	16	21
10	0	53	35	3	18	23	4	50	9	20
11	0	58	52	3	22	29	4	51	58	19
12	1	4	9	3	26	32	4	53	40	18
13	1	9	24	3	30	30	4	55	18	17
14	1	14	39	3	34	25	4	56	50	16
15	1	19	51	3	38	16	4	58	16	15
16	1	25	3	3	42	2	4	59	38	14
17	1	30	13	3	45	45	5	0	53	13
18	1	35	21	3	49	24	5	2	3	12
19	1	40	27	3	52	58	5	3	8	11
20	1	45	32	3	56	28	5	4	7	10
21	1	50	35	3	59	54	5	5	0	9
22	1	55	35	4	3	16	5	5	48	8
23	2	0	34	4	6	33	5	6	31	7
24	2	5	30	4	9	45	5	7	7	6
25	2	10	25	4	12	53	5	7	38	5
26	2	15	16	4	15	57	5	8	4	4
27	2	20	6	4	18	55	5	8	23	3
28	2	24	52	4	21	49	5	8	38	2
29	2	29	36	4	24	38	5	8	46	1
30	2	34	18	4	27	23	5	8	49	0
S.	− 11			− 10			− 9			S.
S.	+ 5			+ 4			+ 3			S.

II. *For the Moon's Latitude.*

Argument II.

Double Diſt. ☽ in Orb. from ☉—Arg. I.

S.	+ 0		+ 1		+ 2		S.
S.	− 6		− 7		− 8		S.
°	′	″	′	″	′	″	°
0	0	0	4	24	7	38	30
1	0	9	4	32	7	42	29
2	0	18	4	40	7	46	28
3	0	28	4	48	7	51	27
4	0	37	4	55	7	55	26
5	0	46	5	3	7	59	25
6	0	55	5	11	8	3	24
7	1	4	5	18	8	6	23
8	1	14	5	25	8	10	22
9	1	23	5	32	8	13	21
10	1	32	5	40	8	16	20
11	1	41	5	47	8	20	19
12	1	50	5	54	8	22	18
13	1	59	6	0	8	25	17
14	2	8	6	7	8	28	16
15	2	17	6	14	8	30	15
16	2	26	6	20	8	33	14
17	2	34	6	26	8	35	13
18	2	43	6	33	8	37	12
19	2	52	6	39	8	39	11
20	3	1	6	45	8	40	10
21	3	9	6	51	8	42	9
22	3	18	6	56	8	43	8
23	3	26	7	2	8	44	7
24	3	35	7	7	8	45	6
25	3	43	7	13	8	46	5
26	3	52	7	18	8	47	4
27	4	0	7	23	8	48	3
28	4	8	7	28	8	48	2
29	4	16	7	33	8	48	1
30	4	24	7	38	8	48	0
S.	− 11		− 10		− 9		S.
S.	+ 5		+ 4		+ 3		S.

III. *For the Moon's Latitude.*

Argument.

Arg. I. — ☉ Mean Anom.

S.	— 0	— 1	— 2	S.
S.	+ 6	+ 7	+ 8	S.
°	"	"	"	°
0	0	9	15	30
10	3	11	16	20
20	6	13	17	10
30	9	15	18	0
S.	+ 11	+ 10	+ 9	S.
S.	— 5	— 4	— 3	S.

IV. *For the Moon's Latitude.*

Argument.

Arg. III. — ☽'s Mean An.

S.	— 0	— 1	— 2	S.
S.	+ 6	+ 7	+ 8	S.
°	"	"	"	°
0	0	13	22	30
10	4	16	24	20
20	9	19	25	10
30	13	22	25	0
S.	+ 11	+ 10	+ 9	S.
S.	— 5	— 4	— 3	S.

V. *For the Moon's Latitude.*

Argument.

Arg. IV. ☽'s Mean Anom.

S.	+ 0	+ 1	+ 2	S.
S.	— 6	— 7	— 8	S.
°	"	"	"	°
0	0	8	14	30
10	3	10	15	20
20	5	12	16	10
30	8	14	16	0
S.	— 11	— 10	— 9	S.
S.	+ 5	+ 4	+ 3	S.

FOR THE MOON'S EQUATORIAL PARALLAX.

I. Argument V. of Longitude, viz. 2 ☽ from ☉ — ☽'s Mean Anomaly.

S.	— 0	— 1	— 2	+ 3	+ 4	+ 5	S.
°	"	"	"	"	"	"	°
0	37	32	19	0	18	32	30
5	37	30	16	3	21	34	25
10	36	28	13	6	24	35	20
15	36	26	10	9	26	36	15
20	35	24	7	13	29	37	10
25	34	21	3	16	31	37	5
30	32	19	0	18	32	38	0
S.	— 11	— 10	— 9	+ 8	+ 7	+ 6	S.

II. Argument XI. of Longitude, viz. correct Anomaly of ☽.

S.	+ 0		+ 1		+ 2		+ 3		+ 4		+ 5		S.
°	'	"	'	"	'	"	'	"	'	"	'	"	°
0	54	13	54	33	55	32	57	1	58	39	59	59	30
5	54	14	54	41	55	46	57	17	58	55	60	8	25
10	54	15	54	49	55	59	57	34	59	10	60	15	20
15	54	18	54	58	56	14	57	51	59	24	60	21	15
20	54	22	55	9	56	29	58	7	59	36	60	25	10
25	54	27	55	20	56	45	58	24	59	48	60	28	5
30	54	33	55	32	57	1	58	39	59	59	60	29	0
S.	+ 11		+ 10		+ 9		+ 8		+ 7		+ 6		S.

III. Argument XII. of Longitude, viz. ☽'s equated Longitude — ☉ true Longitude.

S.	+ 0	± 1	− 2	− 3	± 4	+ 5	S.
°	″	″	″	″	″	″	°
0	25	12	14	26	13	14	30
5	25	8	17	25	8	18	25
10	24	+ 3	20	24	− 4	21	20
15	22	− 1	23	22	+ 0	24	15
20	19	5	24	19	5	25	10
25	16	10	25	16	10	27	5
30	12	14	26	13	14	27	0
S.	+ 11	± 10	− 9	− 8	∓ 7	+ 6	S.

TABLE for reducing the Equatorial to the Horizontal Parallax in any Latitude; and alſo the Reduction of the given Latitude.

Given Lat.	Reduction of Parallax.			Reduction of Lat.	
	′ 54	′ 57	′ 60		
°	″	″	″	′	″
6	0	0	0	3	6
12	1	1	1	6	4
18	1	1	2	8	57
24	2	2	3	11	6
30	4	4	4	12	56
36	5	5	5	14	12
42	6	7	7	14	51
48	8	8	9	14	51
54	9	10	10	14	12
60	10	11	12	12	56
66	12	12	13	12	6
72	13	13	14	8	56
78	13	14	15	6	4
84	14	15	16	3	6

The argument of this Table is the latitude of the place in the left hand column, and the equatorial parallax on the head; in the angle of meeting is the reduction of parallax; and in the ſame line in the right-hand column is the reduction of the latitude.

TABLE for finding the Moon's Diameter.

Argument. Equatorial Parallax.

Equat. Par.	☽'s Diam.	Equat. Par.	☽'s Diam.
′ ″	′ ″	′ ″	′ ″
54 0	29 26	58 0	31 37
54 10	29 31	58 10	31 42
54 20	29 37	58 20	31 47
54 30	29 42	58 30	31 53
54 40	29 48	58 40	31 58
54 50	29 53	58 50	32 4
55 0	29 58	59 0	32 9
55 10	30 4	59 10	32 15
55 20	30 9	59 20	32 20
55 30	30 15	59 30	32 26
55 40	30 20	59 40	32 31
55 50	30 26	59 50	32 36
56 0	30 31	60 0	32 42
56 10	30 37	60 10	32 47
56 20	30 42	60 20	32 53
56 30	30 47	60 30	32 58
56 40	30 53	60 40	33 4
56 50	30 58	60 50	33 9
57 0	31 4	61 0	33 15
57 10	31 9	61 10	33 20
57 20	31 15	61 20	33 26
57 30	31 20	61 30	33 31
57 40	31 26	61 40	33 36
57 50	31 31	61 50	33 42
58 0	31 37	62 0	33 47

TABLES FOR FINDING THE MOON's HOURLY MOTION IN LONGITUDE AND LATITUDE.

I. *For the Moon's Hourly Motion in Longitude.*

Argument V. of Long. viz. 2 ☽ from ☉ — ☽'s Mean Anomaly.

S.	− 0	− 1	− 2	+ 3	+ 4	+ 5	S.
°	″	″	″	″	″	″	°
0	41	36	21	0	21	37	30
5	41	34	18	3	24	38	25
10	41	32	15	7	27	40	20
15	40	30	11	10	30	41	15
20	39	27	8	14	32	42	10
25	38	24	4	17	35	42	5
30	36	21	0	21	37	43	0
S.	− 11	− 10	− 9	+ 8	+ 7	+ 6	S.

II. *For the Moon's Hourly Motion in Longitude.*

Argument XI. of Longitude, viz. the correct Anomaly of the ☽.

S.	+ 0		+ 1		+ 2		+ 3		+ 4		+ 5		S.
°	′	″	′	″	′	″	′	″	′	″	′	″	°
0	29	35	29	57	31	2	32	42	34	36	36	10	30
5	29	35	30	5	31	17	33	0	34	54	36	21	25
10	29	37	30	15	31	32	33	19	35	11	36	30	20
15	29	40	30	25	31	49	33	39	35	28	36	38	15
20	29	45	30	36	32	6	33	58	35	43	36	43	10
25	29	50	30	49	32	23	34	17	35	57	36	46	5
30	29	57	31	2	32	42	34	36	36	10	36	48	0
S.	+ 11		+ 10		+ 9		+ 8		+ 7		+ 6		S.

III. *For the Moon's Hourly Motion in Longitude.*

Argument XII. of Long. viz. ☽'s equat. Long. — ☉'s true Longitude.

S.	+ 0	± 1	− 2	− 3	∓ 4	+ 5	S.
°	″	″	″	″	″	″	°
0	40	19	21	40	20	21	30
5	39	13	26	39	13	27	25
10	37	+ 6	31	37	− 7	32	20
15	34	− 1	35	34	+ 0	36	15
20	30	8	38	30	7	39	10
25	25	15	39	25	14	41	5
30	19	21	40	20	21	42	0
S.	+ 11	± 10	− 9	− 8	∓ 7	+ 6	S.

I. *For the Moon's Hourly Motion in Latitude.*

Argument I. of Lat.

S.	+ 0		+ 1		+ 2		S.
S.	− 6		− 7		− 8		S.
°	′	″	′	″	′	″	°
0	2	58	2	34	1	29	30
5	2	57	2	26	1	15	25
10	2	55	2	16	1	1	20
15	2	52	2	6	0	46	15
20	2	47	1	54	0	31	10
25	2	41	1	42	0	15	5
30	2	34	1	29	0	0	0
S.	+ 11		+ 10		+ 9		S.
S.	− 5		− 4		− 3		S.

II. *For the Moon's Hourly Motion in Latitude.*

Argument II. of Lat.

S.	+ 0	+ 1	+ 2	S.
S.	− 6	− 7	− 8	S.
°	″	″	″	°
0	4	4	2	30
5	4	3	2	25
10	4	3	1	20
15	4	3	1	15
20	4	3	1	10
25	4	2	0	5
30	4	2	0	0
S.	+ 11	+ 10	+ 9	S.
S.	− 5	− 4	− 3	S.

Angle of the viſible Path of the Moon with the Ecliptic in Eclipſes.

Argument. Long. ☽ in her Orbit—Long. ☊.

0 Signs 6.

Horary Motion of the Moon from the Sun.

	27′	28′	29′	30′	31′	32′	33′	34′	35′	36′	
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	°
0	5 47	5 46	5 45	5 44	5 43	5 42	5 41	5 41	5 40	5 39	30
3	5 46	5 45	5 44	5 43	5 42	5 42	5 41	5 40	5 40	5 39	27
6	5 45	5 44	5 43	5 42	5 41	5 40	5 39	5 39	5 38	5 38	24
9	5 42	5 41	5 40	5 39	5 39	5 38	5 37	5 37	5 36	5 35	21
12	5 39	5 38	5 37	5 36	5 35	5 35	5 34	5 33	5 33	5 32	18
15	5 35	5 34	5 33	5 32	5 31	5 31	5 30	5 29	5 29	5 28	15

11 Signs 5.

Tables of the Mean Motions of the Moon from the Sun.

A.D.	S.	°	′	″
1761	9	20	47	17
1781	2	4	12	47
1791	10	4	49	47
B 1792	2	26	38	39
1793	7	6	16	4
1794	11	15	53	29
1795	3	25	30	54
B 1796	8	17	19	45
1797	0	26	57	10
1798	5	6	34	35
1799	9	16	12	0
1800	1	25	49	25
1801	6	5	26	49
1802	10	15	4	15
1803	2	24	41	39
B 1804	7	16	30	31
1805	11	26	7	56
1806	4	5	45	20
1807	8	15	22	45
B 1808	1	7	11	37
1809	5	16	49	3
1810	9	26	26	28
1811	2	6	3	54
B 1812	6	27	52	47
1813	11	7	30	12
1814	3	17	7	38
1815	7	26	45	3
B 1816	0	18	33	56
1817	4	28	11	22
1818	9	7	48	47
1819	1	17	26	13
B 1820	6	9	15	6
1821	10	18	52	32

Years complete.

	S.	°	′	″
1	4	9	37	24
2	8	19	14	49
3	0	28	52	14
B 4	5	20	41	6
5	10	0	18	30
6	2	9	55	54
7	6	19	33	19
B 8	11	11	22	10
9	3	20	59	36
10	8	0	37	0
11	0	10	14	24
B 12	5	2	3	16
13	9	11	40	40
14	1	21	18	5
15	6	0	55	30
B 16	10	22	44	21
17	3	2	21	46
18	7	11	59	10
19	11	21	36	35
B 20	4	13	25	26

Months.

	S.	°	′	″
January	0	0	0	0
Febr.	0	17	54	48
March	11	29	15	16
April	0	17	10	3
May	0	22	53	23
June	1	10	48	11
July	1	16	31	22
Auguſt	2	4	26	20
Sept.	2	22	21	8
Oct.	2	28	4	29
Nov.	3	15	59	16
Dec.	3	21	42	37

In months after February of biſſextile years ſubtract one day from the time found by the Tables.

TABLES of the Mean Motions of the Moon from the Sun.

Days.	S.	°	′	″
1	0	12	11	27
2	0	24	22	53
3	1	6	34	20
4	1	18	45	47
5	2	0	57	13
6	2	13	8	40
7	2	25	20	7
8	3	7	31	33
9	3	19	43	0
10	4	1	54	27
11	4	14	5	53
12	4	26	17	20
13	5	8	28	47
14	5	20	40	13
15	6	2	15	40
16	6	15	3	7
17	6	27	14	33
18	7	9	26	0
19	7	21	37	27
20	8	3	48	53
21	8	16	0	20
22	8	28	11	47
23	9	10	23	13
24	9	22	34	40
25	10	4	46	7
26	10	16	57	33
27	10	29	9	0
28	11	11	20	27
29	11	23	31	53
30	0	5	43	20
31	0	17	54	47

Hours and Minutes.							
H. ′ ″	° ′ ″	′ ″ ‴	″ ‴ ⁗	H. ′ ″	° ′ ″	′ ″ ‴	″ ‴ ⁗
1	0	30	29	31	15	44	48
2	1	0	57	32	16	15	17
3	1	31	26	33	16	45	45
4	2	1	54	34	17	16	13
5	2	32	23	35	17	46	42
6	3	2	52	36	18	17	10
7	3	33	20	37	18	47	39
8	4	3	49	38	19	18	7
9	4	34	18	39	19	48	36
10	5	4	46	40	20	19	5
11	5	35	15	41	20	49	33
12	6	5	43	42	21	20	2
13	6	36	12	43	21	50	31
14	7	6	41	44	22	20	59
15	7	37	9	45	22	51	28
16	8	7	38	46	23	21	56
17	8	38	6	47	23	52	25
18	9	8	35	48	24	22	54
19	9	39	4	49	24	53	22
20	10	9	32	50	25	23	51
21	10	40	1	51	25	54	19
22	11	10	30	52	26	24	48
23	11	40	59	53	26	55	17
24	12	11	28	54	27	25	45
25	12	41	56	55	27	56	14
26	13	12	25	56	28	26	43
27	13	42	54	57	28	57	12
28	14	13	22	58	29	27	40
29	14	43	51	59	29	58	9
30	15	14	19	60	30	28	37

	D.	h.	m.	″
A Lunation =	29	12	44	3
Half a lunation =	14	18	22	2

TABLE I. *Of Mean New Moons, &c. in March, from 1791 to 1821.*

	Mean New Moon in March.				Sun's Mean Anomaly.				Moon's Mean Anomaly.				Sun's Mean Dist. from the Moon's Node.			
A.D.	D.	H.	m.	s.	S.	°	′	″	S.	°	′	″	S.	°	′	″
1791	4	14	8	35	8	3	31	47	9	26	23	45	4	18	52	22
1792	22	11	41	15	8	21	53	59	9	2	0	52	5	27	35	24
1793	11	20	29	51	8	11	9	51	7	11	48	57	6	5	38	11
1794	30	18	2	32	8	29	32	3	6	17	26	4	7	14	21	13
1795	20	2	51	8	8	18	47	55	4	27	14	9	7	22	24	0
1796	8	11	39	44	8	8	3	47	3	7	2	14	8	0	26	47
1797	27	9	12	24	8	26	25	59	2	12	39	19	9	9	9	48
1798	16	18	1	1	8	15	41	51	0	22	27	25	9	17	12	35
1799	6	2	49	37	8	4	57	43	11	2	15	30	9	25	15	22
1800	25	0	22	17	8	23	19	55	10	7	52	36	11	3	58	23
1801	14	9	10	53	8	12	35	47	8	17	40	41	11	12	1	10
1802	3	17	59	29	8	1	51	39	6	27	28	46	11	20	3	57
1803	22	15	32	9	8	20	13	51	6	3	5	52	0	28	46	58
1804	11	0	20	45	8	9	29	43	4	12	53	57	1	6	49	45
1805	0	9	9	21	7	28	45	31	2	22	42	2	1	14	52	35
1806	19	6	42	1	8	17	7	43	1	28	19	8	2	23	35	36
1807	8	15	30	37	8	6	23	35	0	8	7	13	3	1	38	23
1808	26	13	3	17	8	24	45	47	11	13	44	19	4	10	21	24
1809	15	21	51	53	8	14	1	39	9	23	32	24	4	18	24	11
1810	5	6	40	29	8	3	17	31	8	3	20	29	4	26	26	58
1811	24	4	13	9	8	21	39	43	7	8	57	35	6	5	9	59
1812	12	13	1	45	8	10	55	35	5	18	45	40	6	13	12	46
1813	1	21	50	21	8	0	11	27	3	28	33	45	6	21	15	23
1814	20	19	23	1	8	18	33	39	3	4	10	51	7	29	58	24
1815	10	4	11	37	8	7	53	31	1	13	58	56	8	8	1	11
1816	28	1	44	17	8	26	15	43	0	19	36	2	9	16	44	12
1817	17	10	32	53	8	15	31	35	10	29	24	7	9	24	46	59
1818	6	19	21	29	8	4	47	27	9	9	12	12	10	2	49	46
1819	25	16	54	9	8	23	9	39	8	14	49	18	11	11	32	47
1820	14	1	42	45	8	12	25	31	6	24	37	23	11	19	35	34
1821	3	10	31	21	8	1	41	23	5	4	25	28	11	27	38	21

II. *Mean Anomalies, and Sun's Mean Diſtance from the Node, for 13½ Mean Lunations.*

	Mean Lunations.				Sun's Mean Anomaly.				Moon's Mean Anomaly.				Sun's Mean Diſt. from the Node.			
No	D.	H.	m.	s.	S.	°	′	″	S.	°	′	″	S.	°	′	″
1	29	12	44	3	0	29	6	19	0	25	49	0	1	0	40	14
2	59	1	28	6	1	28	12	39	1	21	38	1	2	1	20	28
3	88	14	12	9	2	27	18	58	2	17	27	1	3	2	0	42
4	118	2	56	12	3	26	25	17	3	13	16	2	4	2	40	56
5	147	15	40	15	4	25	31	37	4	9	5	2	5	3	21	10
6	177	4	24	18	5	24	37	56	5	4	54	3	6	4	1	24
7	206	17	8	21	6	23	44	15	6	0	43	3	7	4	41	38
8	236	5	52	24	7	22	50	35	6	26	32	3	8	5	21	53
9	265	18	36	27	8	21	56	54	7	22	21	4	9	6	2	6
10	295	7	20	30	9	21	3	14	8	18	10	4	10	6	42	20
11	324	20	4	33	10	20	9	33	9	13	59	5	11	7	22	34
12	354	8	48	36	11	19	15	52	10	9	48	5	0	8	2	47
13	383	21	32	40	0	18	22	12	11	5	37	6	1	8	43	1
½	14	18	22	2	0	14	33	10	6	12	54	30	0	15	20	7

III. *The Days of the Year reckoned from the beginning of March.*

Days.	March.	April.	May.	June.	July.	Aug.	Sept.	Octob.	Nov.	Dec.	Jan.	Feb.
1	1	32	62	93	123	154	185	215	246	276	307	338
2	2	33	63	94	124	155	186	216	247	277	308	339
3	3	34	64	95	125	156	187	217	248	278	309	340
4	4	35	65	96	126	157	188	218	249	279	310	341
5	5	36	66	97	127	158	189	219	250	280	311	342
6	6	37	67	98	128	159	190	220	251	281	312	343
7	7	38	68	99	129	160	191	221	252	282	313	344
8	8	39	69	100	130	161	192	222	253	283	314	345
9	9	40	70	101	131	162	193	223	254	284	315	346
10	10	41	71	102	132	163	194	224	255	285	316	347
11	11	42	72	103	133	164	195	225	256	286	317	348
12	12	43	73	104	134	165	196	226	257	287	318	349
13	13	44	74	105	135	166	197	227	258	288	319	350
14	14	45	75	106	136	167	198	228	259	289	320	351
15	15	46	76	107	137	168	199	229	260	290	321	352
16	16	47	77	108	138	169	200	230	261	291	322	353
17	17	48	78	109	139	170	201	231	262	292	323	354
18	18	49	79	110	140	171	202	232	263	293	324	355
19	19	50	80	111	141	172	203	233	264	294	325	356
20	20	51	81	112	142	173	204	234	265	295	326	357
21	21	52	82	113	143	174	205	235	266	296	327	358
22	22	53	83	114	144	175	206	236	267	297	328	359
23	23	54	84	115	145	176	207	237	268	298	329	360
24	24	55	85	116	146	177	208	238	269	299	330	361
25	25	56	86	117	147	178	209	239	270	300	331	362
26	26	57	87	118	148	179	210	240	271	301	332	363
27	27	58	88	119	149	180	211	241	272	302	333	364
28	28	59	89	120	150	181	212	242	273	303	334	365
29	29	60	90	121	151	182	213	243	274	304	335	366
30	30	61	91	122	152	183	214	244	275	305	336	
31	31		92		153	184		245		306	337	

IV. *The Annual or First Equation of the Mean to the true Syzygy.*

Argument. Sun's Mean Anomaly.

	S. 0			1			2			3			4			5			
°	H.	m.	s.	H.	m.	s.	H.	m.	s.	H.	m.	s.	H.	m.	s.	H.	m.	s.	°
0	0	0	0	2	3	12	3	35	0	4	10	53	3	39	30	2	7	45	30
1	0	4	18	2	6	55	3	37	10	4	10	57	3	37	19	2	3	55	29
2	0	8	35	2	10	36	3	39	18	4	10	55	3	35	6	2	0	1	28
3	0	12	51	2	14	14	3	41	23	4	10	49	3	32	50	1	56	5	27
4	0	17	8	2	17	52	3	43	26	4	10	39	3	30	30	1	52	6	26
5	0	21	24	2	21	27	3	45	25	4	10	24	3	28	5	1	48	4	25
6	0	25	39	2	25	9	3	47	19	4	10	4	3	25	35	1	41	1	24
7	0	28	55	2	28	29	3	49	7	4	9	39	3	23	0	1	39	56	23
8	0	34	11	2	31	57	3	50	50	4	9	10	3	20	20	1	35	49	22
9	0	38	26	2	35	22	3	52	29	4	8	37	3	17	35	1	31	41	21
10	0	42	39	2	38	44	3	54	4	4	7	59	3	14	49	1	27	31	20
11	0	46	52	2	42	3	3	55	35	4	7	16	3	11	59	1	23	19	19
12	0	51	4	2	45	18	3	57	2	4	6	29	3	9	6	1	19	5	18
13	0	55	17	2	48	30	3	58	27	4	5	37	3	6	10	1	14	49	17
14	0	59	27	2	51	40	3	59	49	4	4	41	3	3	10	1	10	32	16
15	1	3	36	2	54	48	4	1	7	4	3	40	3	0	7	1	6	15	15
16	1	7	45	2	57	53	4	2	18	4	2	35	2	57	0	1	1	56	14
17	1	11	53	3	0	54	4	3	23	4	1	26	2	53	49	0	57	36	13
18	1	16	0	3	3	51	4	4	22	4	0	12	2	50	36	0	53	15	12
19	1	20	6	3	6	45	4	5	18	3	58	52	2	47	18	0	48	52	11
20	1	24	10	3	9	36	4	6	10	3	57	27	2	43	57	0	44	28	10
21	1	28	12	3	12	24	4	6	58	3	55	59	2	40	33	0	40	2	9
22	1	32	12	3	15	9	4	7	41	3	54	26	2	37	6	0	35	36	8
23	1	36	10	3	17	51	4	8	21	3	52	49	2	33	35	0	31	10	7
24	1	40	6	3	20	30	4	8	57	3	51	9	2	30	2	0	26	44	6
25	1	44	1	3	23	5	4	9	29	3	49	26	2	26	26	0	22	17	5
26	1	47	54	3	25	36	4	9	55	3	47	38	2	22	47	0	17	50	4
27	1	51	46	3	28	3	4	10	16	3	45	44	2	19	5	0	13	23	3
28	1	55	37	3	30	26	4	10	33	3	43	45	2	15	20	0	8	56	2
29	1	59	26	3	32	45	4	10	45	3	41	40	2	11	35	0	4	29	1
30	2	3	12	3	35	0	4	10	53	3	39	30	2	7	45	0	0	0	0
	S. 11			10			9			8			7			6			

+

V. *Equation of the Moon's Mean Anomaly.*

Argument. Sun's Mean Anomaly.

	S. 0			1			2			3			4			5			
°	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°
0	0	0	0	0	46	45	1	21	32	1	35	1	1	23	4	0	48	19	30
1	0	1	37	0	48	10	1	22	21	1	35	2	1	22	14	0	46	51	29
2	0	3	13	0	49	34	1	23	10	1	35	1	1	21	24	0	45	23	28
3	0	4	52	0	50	53	1	23	57	1	35	0	1	20	32	0	43	54	27
4	0	6	28	0	52	19	1	24	41	1	34	57	1	19	38	0	42	24	26
5	0	8	6	0	53	40	1	25	24	1	34	50	1	18	42	0	40	53	25
6	0	9	42	0	55	0	1	26	6	1	34	43	1	17	45	0	39	21	24
7	0	11	20	0	56	21	1	26	48	1	34	33	1	16	48	0	37	49	23
8	0	12	56	0	57	38	1	27	28	1	34	22	1	15	47	0	36	15	22
9	0	14	33	0	58	56	1	28	6	1	34	9	1	14	44	0	34	40	21
10	0	16	10	1	0	13	1	28	43	1	33	53	1	13	41	0	33	5	20
11	0	17	47	1	1	29	1	29	17	1	33	37	1	12	37	0	31	31	19
12	0	19	23	1	2	43	1	29	51	1	33	20	1	11	33	0	29	54	18
13	0	20	59	1	3	56	1	30	22	1	33	0	1	10	26	0	28	18	17
14	0	22	35	1	5	8	1	30	50	1	32	38	1	9	17	0	26	40	16
15	0	24	10	1	6	18	1	31	19	1	32	14	1	8	8	0	25	3	15
16	0	25	45	1	7	27	1	31	45	1	31	50	1	6	58	0	23	23	14
17	0	27	19	1	8	36	1	32	12	1	31	23	1	5	46	0	21	45	13
18	0	28	52	1	9	42	1	32	34	1	30	55	1	4	32	0	20	7	12
19	0	30	25	1	10	49	1	32	57	1	30	25	1	3	19	0	18	28	11
20	0	31	57	1	11	54	1	33	17	1	29	54	1	2	1	0	16	48	10
21	0	33	29	1	12	58	1	33	36	1	29	20	1	0	45	0	15	8	9
22	0	35	2	1	14	1	1	33	52	1	28	45	0	59	26	0	13	28	8
23	0	36	32	1	15	1	1	34	6	1	28	9	0	58	7	0	11	48	7
24	0	38	1	1	16	0	1	34	18	1	27	30	0	56	45	0	10	7	6
25	0	39	29	1	16	59	1	34	30	1	26	50	0	55	23	0	8	20	5
26	0	40	59	1	17	57	1	34	40	1	26	27	0	54	1	0	6	44	4
27	0	42	26	1	18	52	1	34	48	1	25	5	0	52	37	0	5	3	3
28	0	43	54	1	19	47	1	34	54	1	24	39	0	51	12	0	3	21	2
29	0	45	19	1	20	40	1	34	58	1	23	52	0	49	45	0	1	40	1
30	0	46	45	1	21	32	1	35	1	1	23	4	0	48	19	0	0	0	0
	S. 11			10			9			8			7			6			

+

VI. *The Second Equation of the Mean to the True Syzygy.*

Argument. Moon's equated Anomaly.

+

	S. 0	1	2	3	4	5	
°	H. m. s.	H. m. s.	H. m. s.	H. m. s.	H. m. s.	H. m. s.	°
0	0 0 0	5 12 48	8 47 8	9 46 44	8 8 59	4 34 33	30
1	0 10 58	5 21 56	8 51 45	9 45 3	8 3 12	4 26 1	29
2	0 21 56	5 30 57	8 56 10	9 45 12	7 57 23	4 17 25	28
3	0 32 54	5 39 51	9 0 25	9 44 11	7 51 33	4 8 47	27
4	0 43 52	5 48 37	9 4 31	9 42 59	7 45 46	4 0 7	26
5	0 54 50	5 57 17	9 8 25	9 41 36	7 39 46	3 51 23	25
6	1 5 48	6 5 51	9 12 9	9 40 3	7 33 36	3 42 32	24
7	1 16 46	6 14 19	9 15 43	9 38 19	7 27 22	3 33 38	23
8	1 27 44	6 22 41	9 19 5	9 36 24	7 21 2	3 24 42	22
9	1 38 40	6 30 57	9 22 14	9 34 18	7 14 30	3 15 44	21
10	1 49 33	6 39 4	9 25 12	9 32 1	7 7 50	3 6 45	20
11	2 0 23	6 47 0	9 27 54	9 29 33	7 1 2	2 57 43	19
12	2 11 10	6 54 46	9 30 32	9 26 54	6 54 8	2 48 39	18
13	2 21 54	7 2 24	9 32 58	9 24 4	6 47 9	2 39 34	17
14	2 32 34	7 9 52	9 35 12	9 21 3	6 40 6	2 30 28	16
15	2 43 9	7 17 9	9 37 14	9 17 51	6 32 56	2 21 19	15
16	2 53 38	7 24 19	9 39 8	9 14 28	6 25 40	2 12 8	14
17	3 4 3	7 31 18	9 40 51	9 10 54	6 18 18	2 2 53	13
18	3 14 24	7 38 9	9 42 21	9 7 9	6 10 49	1 53 36	12
19	3 24 42	7 44 51	9 43 42	9 3 13	6 3 16	1 44 16	11
20	3 34 58	7 51 24	9 44 53	8 59 6	5 55 38	1 34 54	10
21	3 45 11	7 57 45	9 45 52	8 54 50	5 47 54	1 25 31	9
22	3 55 21	8 3 56	9 46 38	8 50 24	5 40 4	1 16 7	8
23	4 5 26	8 9 57	9 47 13	8 45 48	5 32 9	1 6 41	7
24	4 15 26	8 15 46	9 47 36	8 41 2	5 24 9	0 57 13	6
25	4 25 20	8 21 24	9 47 49	8 36 6	5 16 5	0 47 44	5
26	4 35 6	8 26 53	9 47 54	8 31 0	5 7 56	0 38 13	4
27	4 44 42	8 32 11	9 47 46	8 25 44	4 59 42	0 28 41	3
28	4 54 11	8 37 19	9 47 33	8 20 18	4 51 15	0 19 8	2
29	5 3 33	8 42 18	9 47 14	8 14 33	4 43 2	0 9 34	1
30	5 12 48	8 47 8	9 46 44	8 8 59	4 34 33	0 0 0	0
	S. 11	10	9	8	7	6	

VII. *The third Equation of the Mean to the True Syzygy.*

Argument. ☉'s Mean Anom. — ☽'s Mean Anom.

	S. —0	— 1	— 2	
	S. +6	+ 7	+ 8	
°	m. s.	m. s.	m. s.	°
0	0 0	2 22	4 12	30
1	0 5	2 26	4 15	29
2	0 10	2 30	4 18	28
3	0 15	2 34	4 21	27
4	0 20	2 38	4 24	26
5	0 25	2 42	4 27	25
6	0 30	2 46	4 30	24
7	0 35	2 50	4 32	23
8	0 40	2 54	4 34	22
9	0 45	2 58	4 36	21
10	0 50	3 2	4 38	20
11	0 55	3 6	4 40	19
12	1 0	3 10	4 42	18
13	1 5	3 14	4 44	17
14	1 10	3 18	4 46	16
15	1 15	3 22	4 48	15
16	1 20	3 26	4 50	14
17	1 25	3 30	4 51	13
18	1 30	3 34	4 52	12
19	1 35	3 38	4 53	11
20	1 40	3 42	4 54	10
21	1 45	3 45	4 55	9
22	1 49	3 48	4 56	8
23	1 52	3 51	4 57	7
24	1 56	3 54	4 57	6
25	2 0	3 57	4 57	5
26	2 4	4 0	4 58	4
27	2 9	4 3	4 58	3
28	2 13	4 6	4 58	2
29	2 18	4 9	4 58	1
30	2 22	4 12	4 58	0
	S. —5	— 4	—3 S	
	S. +11	+10	+9 S.	

VIII. *The fourth Equation of the Mean to the True Syzygy.*

Argument. ☉'s Mean Diſt. from ☽'s ☊.

	+ 0	+ 1	+ 2	
	+ 6	+ 7	+ 8	
°	m. s.	m. s.	m. s.	°
0	0 0	1 22	1 22	30
1	0 4	1 23	1 21	29
2	0 7	1 24	1 20	28
3	0 10	1 25	1 18	27
4	0 13	1 26	1 16	26
5	0 16	1 27	1 14	25
6	0 20	1 28	1 12	24
7	0 23	1 29	1 10	23
8	0 26	1 30	1 8	22
9	0 29	1 31	1 6	21
10	0 32	1 32	1 3	20
11	0 35	1 33	1 0	19
12	0 38	1 33	0 57	18
13	0 41	1 34	0 54	17
14	0 44	1 34	0 51	16
15	0 47	1 34	0 49	15
16	0 50	1 34	0 45	14
17	0 52	1 34	0 41	13
18	0 54	1 34	0 37	12
19	0 57	1 33	0 34	11
20	1 0	1 33	0 31	10
21	1 2	1 32	0 28	9
22	1 5	1 31	0 25	8
23	1 8	1 30	0 22	7
24	1 10	1 29	0 19	6
25	1 12	1 28	0 16	5
26	1 14	1 27	0 13	4
27	1 16	1 26	0 10	3
28	1 18	1 25	0 6	2
29	1 20	1 24	0 3	1
30	1 22	1 22	0 0	0
	S.—5	— 4	—3	
	S.—11	—10	—9 S.	

Table of Logiſtical Logarithms.

	0	1	2	3	4	5	6	7		0	1	2	3	4	5	6	7
	0	60	120	180	240	300	360	420		0	60	120	180	240	300	360	420
0	00000	17782	14771	13010	11761	10792	10000	9331	30	20792	16021	13802	12341	11249	10378	9652	9031
1	35563	17710	14735	12986	11743	10777	9988	9320	31	20649	15973	13773	12320	11233	10365	9641	9021
2	32553	17639	14699	12962	11725	10763	9976	9310	32	20512	15925	13745	12300	11217	10352	9630	9012
3	30792	17570	14664	12939	11707	10749	9964	9300	33	20378	15878	13716	12279	11201	10339	9619	9002
4	29542	17501	14629	12915	11689	10734	9952	9289	34	20248	15832	13688	12259	11186	10326	9608	8992
5	28573	17434	14594	12891	11671	10720	9940	9279	35	20122	15786	13660	12239	11170	10313	9597	8983
6	27782	17368	14559	12868	11654	10706	9928	9269	36	20000	15740	13632	12218	11154	10300	9586	8973
7	27112	17302	14525	12845	11636	10692	9916	9259	37	19881	15695	13604	12198	11138	10287	9575	8964
8	26532	17238	14491	12821	11619	10678	9905	9249	38	19765	15651	13576	12178	11123	10274	9564	8954
9	26021	17175	14457	12798	11601	10663	9893	9238	39	19652	15607	13549	12159	11107	10261	9553	8945
10	25563	17112	14424	12775	11584	10649	9881	9228	40	19542	15563	13522	12139	11091	10248	9542	8935
11	25149	17050	14390	12753	11566	10635	9869	9218	41	19435	15520	13495	12119	11076	10235	9532	8926
12	24771	16990	14357	12730	11549	10621	9858	9208	42	19331	15477	13468	12099	11061	10223	9521	8917
13	24424	16930	14325	12707	11532	10608	9846	9198	43	19228	15435	13441	12080	11045	10210	9510	8907
14	24102	16871	14292	12685	11515	10594	9834	9188	44	19128	15393	13415	12061	11030	10197	9499	8898
15	23802	16812	14260	12663	11498	10580	9823	9178	45	19031	15351	13388	12041	11015	10185	9488	8888
16	23522	16755	14228	12640	11481	10566	9811	9168	46	18935	15310	13362	12022	10999	10172	9478	8879
17	23259	16698	14196	12618	11464	10552	9800	9158	47	18842	15269	13336	12003	10984	10160	9467	8870
18	23010	16642	14165	12596	11447	10539	9788	9148	48	18751	15229	13310	11984	10969	10147	9456	8861
19	22775	16587	14133	12574	11430	10525	9777	9138	49	18661	15189	13284	11965	10954	10135	9446	8851
20	22553	16532	14102	12553	11413	10512	9765	9128	50	18573	15149	13259	11946	10939	10122	9435	8842
21	22341	16478	14071	12531	11397	10498	9754	9119	51	18487	15110	13233	11927	10924	10110	9425	8833
22	22139	16425	14040	12510	11380	10484	9742	9109	52	18403	15071	13208	11908	10909	10098	9414	8824
23	21946	16372	14010	12488	11363	10471	9731	9099	53	18320	15032	13183	11889	10894	10085	9404	8814
24	21761	16320	13979	12467	11347	10458	9720	9089	54	18239	14994	13158	11871	10880	10073	9393	8805
25	21584	16269	13949	12445	11331	10444	9708	9079	55	18159	14956	13133	11852	10865	10061	9383	8796
26	21413	16218	13919	12424	11314	10431	9697	9070	56	18081	14918	13108	11834	10850	10049	9372	8787
27	21249	16168	13890	12403	11298	10418	9686	9060	57	18004	14881	13083	11816	10835	10036	9362	8778
28	21091	16118	13860	12382	11282	10404	9675	9050	58	17929	14844	13059	11797	10821	10024	9351	8769
29	20939	16069	13831	12362	11266	10391	9664	9041	59	17855	14808	13034	11779	10806	10012	9341	8760
30	20792	16021	13802	12341	11249	10378	9652	9031	60	17782	14771	13010	11761	10792	10000	9331	8752

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	8	9	10	11	12	13	14	15	16		8	9	10	11	12	13	14	15	16
	480	540	600	660	720	780	840	900	960		480	540	600	660	720	780	840	900	960
0	8751	8239	7782	7368	6990	6642	6320	6021	5740	30	8487	8004	7570	7175	6812	6478	6168	5878	5607
1	8742	8231	7774	7361	6984	6637	6315	6016	5736	31	8479	7997	7563	7168	6807	6473	6163	5874	5602
2	8733	8223	7767	7354	6978	6631	6310	6011	5731	32	8470	7989	7556	7162	6801	6467	6158	5869	5598
3	8724	8215	7760	7348	6972	6625	6305	6006	5727	33	8462	7981	7549	7156	6795	6462	6153	5864	5594
4	8715	8207	7753	7341	6966	6620	6300	6001	5722	34	8453	7974	7542	7149	6789	6457	6148	5860	5589
5	8706	8199	7745	7335	6960	6614	6294	5997	5718	35	8445	7966	7535	7143	6784	6451	6143	5855	5585
6	8697	8191	7738	7328	6954	6609	6289	5992	5713	36	8437	7959	7528	7137	6778	6446	6138	5850	5580
7	8688	8183	7731	7322	6948	6603	6284	5987	5709	37	8428	7951	7522	7131	6772	6441	6133	5846	5576
8	8679	8175	7724	7315	6942	6598	6279	5982	5704	38	8420	7944	7515	7124	6766	6435	6128	5841	5572
9	8670	8167	7717	7309	6936	6592	6274	5977	5700	39	8411	7936	7508	7118	6761	6430	6123	5836	5567
10	8661	8159	7710	7302	6930	6587	6269	5973	5695	40	8403	7929	7501	7112	6755	6425	6118	5832	5563
11	8652	8152	7703	7296	6924	6581	6264	5968	5691	41	8395	7921	7494	7106	6749	6420	6113	5827	5559
12	8643	8144	7696	7289	6918	6576	6259	5963	5686	42	8386	7914	7488	7100	6743	6414	6108	5823	5554
13	8635	8136	7688	7283	6912	6570	6254	5958	5682	43	8378	7906	7481	7093	6738	6409	6103	5818	5550
14	8626	8128	7681	7276	6906	6565	6248	5954	5677	44	8370	7899	7474	7087	6732	6404	6099	5813	5546
15	8617	8120	7674	7270	6900	6559	6243	5949	5673	45	8361	7891	7467	7081	6726	6398	6094	5809	5541
16	8608	8112	7667	7264	6894	6554	6238	5944	5669	46	8353	7884	7461	7075	6721	6393	6089	5804	5537
17	8599	8104	7660	7257	6888	6548	6233	5939	5664	47	8345	7877	7454	7069	6715	6388	6084	5800	5533
18	8591	8097	7653	7251	6882	6543	6228	5935	5660	48	8337	7869	7447	7063	6709	6383	6079	5795	5528
19	8582	8089	7646	7244	6877	6538	6223	5930	5655	49	8328	7862	7441	7057	6704	6377	6074	5790	5524
20	8573	8081	7639	7238	6871	6532	6218	5925	5651	50	8320	7855	7434	7050	6698	6372	6069	5786	5520
21	8565	8073	7632	7232	6865	6527	6213	5920	5646	51	8312	7847	7427	7044	6692	6367	6064	5781	5516
22	8556	8066	7625	7225	6859	6521	6208	5916	5642	52	8304	7840	7421	7038	6687	6362	6059	5777	5511
23	8547	8058	7618	7219	6853	6516	6203	5911	5637	53	8296	7832	7414	7032	6681	6357	6055	5772	5507
24	8539	8050	7611	7212	6847	6510	6198	5906	5633	54	8288	7825	7407	7026	6676	6351	6050	5768	5503
25	8530	8043	7604	7206	6841	6505	6193	5902	5629	55	8279	7818	7401	7020	6670	6346	6045	5763	5498
26	8522	8035	7597	7200	6836	6500	6188	5897	5624	56	8271	7811	7394	7014	6664	6341	6040	5758	5494
27	8513	8027	7590	7193	6830	6494	6183	5892	5620	57	8263	7803	7387	7008	6659	6336	6035	5754	5490
28	8504	8020	7583	7187	6824	6489	6178	5888	5615	58	8255	7796	7381	7002	6653	6331	6030	5749	5486
29	8496	8012	7577	7181	6818	6484	6173	5883	5611	59	8247	7789	7374	6996	6648	6325	6025	5745	5481
30	8487	8004	7570	7175	6812	6478	6168	5878	5607	60	8239	7782	7368	6990	6642	6320	6021	5740	5477

TABLE of Logiſtical Logarithms.

	17	18	19	20	21	22	23	24	25		17	18	19	20	21	22	23	24	25
	1020	1080	1140	1200	1260	1320	1380	1440	1500		1020	1080	1140	1200	1260	1320	1380	1440	1500
0	5477	5229	4994	4771	4559	4357	4164	3979	3802	30	5351	5110	4881	4664	4457	4260	4071	3890	3716
1	5473	5225	4990	4768	4556	4354	4161	3976	3799	31	5347	5106	4877	4660	4454	4256	4068	3887	3713
2	5469	5221	4986	4764	4552	4351	4158	3973	3796	32	5343	5102	4874	4657	4450	4253	4065	3884	3710
3	5464	5217	4983	4760	4549	4347	4155	3970	3793	33	5339	5098	4870	4653	4447	4250	4062	3881	3708
4	5460	5213	4979	4757	4546	4344	4152	3967	3791	34	5335	5094	4866	4650	4444	4247	4059	3878	3705
5	5456	5209	4975	4753	4542	4341	4149	3964	3788	35	5331	5090	4863	4646	4440	4244	4055	3875	3702
6	5452	5205	4971	4750	4539	4338	4145	3961	3785	36	5326	5086	4859	4643	4437	4240	4052	3872	3699
7	5447	5201	4967	4746	4535	4334	4142	3958	3782	37	5322	5082	4855	4639	4434	4237	4049	3869	3696
8	5443	5197	4964	4742	4532	4331	4139	3955	3779	38	5318	5079	4852	4636	4430	4234	4046	3866	3693
9	5439	5193	4960	4739	4528	4328	4136	3952	3776	39	5314	5075	4848	4632	4427	4231	4043	3863	3691
10	5435	5189	4956	4735	4525	4325	4133	3949	3773	40	5310	5071	4844	4629	4424	4228	4040	3860	3688
11	5430	5185	4952	4732	4522	4321	4130	3946	3770	41	5306	5067	4841	4625	4420	4224	4037	3857	3685
12	5426	5181	4949	4728	4518	4318	4127	3943	3768	42	5302	5063	4837	4622	4417	4221	4034	3855	3682
13	5422	5177	4945	4724	4515	4315	4124	3940	3765	43	5298	5059	4833	4618	4414	4218	4031	3852	3679
14	5418	5173	4941	4721	4511	4311	4120	3937	3762	44	5294	5055	4830	4615	4410	4215	4028	3849	3677
15	5414	5169	4937	4717	4508	4308	4117	3934	3759	45	5290	5051	4826	4611	4407	4212	4025	3846	3674
16	5409	5165	4933	4714	4505	4305	4114	3931	3756	46	5285	5048	4822	4608	4404	4209	4022	3843	3671
17	5405	5161	4930	4710	4501	4302	4111	3928	3753	47	5281	5044	4819	4604	4400	4205	4019	3840	3668
18	5401	5157	4926	4707	4498	4298	4108	3925	3750	48	5277	5040	4815	4601	4397	4202	4016	3837	3665
19	5397	5153	4922	4703	4494	4295	4105	3922	3747	49	5273	5036	4811	4597	4394	4199	4013	3834	3663
20	5393	5149	4918	4699	4491	4292	4102	3919	3745	50	5269	5032	4808	4594	4390	4196	4010	3831	3660
21	5389	5145	4915	4696	4488	4289	4099	3917	3742	51	5265	5028	4804	4590	4387	4193	4007	3828	3657
22	5384	5141	4911	4692	4484	4285	4096	3914	3739	52	5261	5025	4800	4587	4384	4189	4004	3825	3654
23	5380	5137	4907	4689	4481	4282	4092	3911	3736	53	5257	5021	4797	4584	4380	4186	4001	3822	3651
24	5376	5133	4903	4685	4477	4279	4089	3908	3733	54	5253	5017	4793	4580	4377	4183	3998	3820	3649
25	5372	5129	4900	4682	4474	4276	4086	3905	3730	55	5249	5013	4789	4577	4374	4180	3995	3817	3646
26	5368	5125	4896	4678	4471	4273	4083	3902	3727	56	5245	5009	4786	4573	4370	4177	3991	3814	3643
27	5364	5122	4892	4675	4467	4269	4080	3899	3725	57	5241	5005	4782	4570	4367	4174	3988	3811	3640
28	5359	5118	4889	4671	4464	4266	4077	3896	3722	58	5237	5002	4778	4566	4364	4171	3985	3808	3637
29	5355	5114	4885	4668	4460	4263	4074	3893	3719	59	5233	4998	4775	4563	4361	4167	3982	3805	3635
30	5351	5110	4881	4664	4457	4260	4071	3890	3716	60	5229	4994	4771	4559	4357	4164	3979	3802	3632

TABLE of Logiſtical Logarithms.

	26	27	28	29	30	31	32	33	34		26	27	28	29	30	31	32	33	34
	1560	1620	1680	1740	1800	1860	1920	1980	2040		1560	1620	1680	1740	1800	1860	1920	1980	2040
0	3632	3468	3310	3158	3010	2868	2730	2596	2467	30	3549	3388	3233	3083	2939	2798	2663	2531	2403
1	3629	3465	3307	3155	3008	2866	2728	2594	2465	31	3546	3386	3231	3081	2936	2796	2660	2529	2401
2	3626	3463	3305	3153	3005	2863	2725	2592	2462	32	3544	3383	3228	3078	2934	2794	2658	2527	2399
3	3623	3460	3302	3150	3003	2861	2723	2590	2460	33	3541	3380	3225	3076	2931	2792	2656	2525	2397
4	3621	3457	3300	3148	3001	2859	2721	2588	2458	34	3538	3378	3223	3073	2929	2789	2654	2522	2395
5	3618	3454	3297	3145	2998	2856	2719	2585	2456	35	3535	3375	3220	3071	2927	2787	2652	2520	2393
6	3615	3452	3294	3143	2996	2854	2716	2583	2454	36	3533	3372	3218	3069	2924	2785	2649	2518	2391
7	3612	3449	3292	3140	2993	2852	2714	2581	2452	37	3530	3370	3215	3066	2922	2782	2647	2516	2389
8	3610	3446	3289	3138	2991	2849	2712	2579	2450	38	3527	3367	3213	3064	2920	2780	2645	2514	2387
9	3607	3444	3287	3135	2989	2847	2710	2577	2448	39	3525	3365	3210	3061	2917	2778	2643	2512	2384
10	3604	3441	3284	3133	2986	2845	2707	2574	2445	40	3522	3362	3208	3059	2915	2775	2640	2510	2382
11	3601	3438	3282	3130	2984	2842	2705	2572	2443	41	3519	3359	3205	3056	2912	2773	2638	2507	2380
12	3598	3436	3279	3128	2981	2840	2703	2570	2441	42	3516	3357	3203	3054	2910	2771	2636	2505	2378
13	3596	3433	3276	3125	2979	2838	2701	2568	2439	43	3514	3354	3200	3052	2908	2769	2634	2503	2376
14	3593	3431	3274	3123	2977	2835	2698	2566	2437	44	3511	3351	3198	3049	2905	2766	2632	2501	2374
15	3590	3428	3271	3120	2974	2833	2696	2564	2435	45	3508	3349	3195	3047	2903	2764	2629	2499	2372
16	3587	3425	3269	3118	2972	2831	2694	2561	2433	46	3506	3346	3193	3044	2901	2762	2627	2497	2370
17	3585	3423	3266	3115	2969	2828	2692	2559	2431	47	3503	3344	3190	3042	2898	2760	2625	2494	2368
18	3582	3420	3264	3113	2967	2826	2689	2557	2429	48	3500	3341	3188	3039	2896	2757	2623	2492	2366
19	3579	3417	3261	3110	2965	2824	2687	2555	2426	49	3497	3338	3185	3037	2894	2755	2621	2490	2364
20	3576	3415	3259	3108	2962	2821	2685	2553	2424	50	3495	3336	3183	3034	2891	2753	2618	2488	2362
21	3574	3412	3256	3105	2960	2819	2683	2551	2422	51	3492	3333	3180	3032	2889	2750	2616	2486	2359
22	3571	3409	3253	3103	2958	2817	2681	2548	2420	52	3489	3331	3178	3030	2887	2748	2614	2484	2357
23	3568	3407	3251	3101	2955	2815	2678	2546	2418	53	3487	3328	3175	3027	2884	2746	2612	2482	2355
24	3565	3404	3248	3098	2953	2812	2676	2544	2416	54	3484	3325	3173	3025	2882	2744	2610	2480	2353
25	3563	3401	3246	3096	2950	2810	2674	2542	2414	55	3481	3323	3170	3022	2880	2741	2607	2477	2351
26	3560	3399	3243	3093	2948	2808	2672	2540	2412	56	3479	3320	3168	3020	2877	2739	2605	2475	2349
27	3557	3396	3241	3091	2946	2805	2669	2538	2410	57	3476	3318	3165	3018	2875	2737	2603	2473	2347
28	3555	3393	3238	3088	2943	2803	2667	2535	2408	58	3473	3315	3163	3015	2873	2735	2601	2471	2345
29	3552	3391	3236	3086	2941	2801	2665	2533	2405	59	3471	3313	3160	3013	2870	2732	2599	2469	2343
30	3549	3388	3233	3083	2939	2798	2663	2531	2403	60	3468	3310	3158	3010	2868	2730	2596	2467	2341

Table of Logiſtical Logarithms.

	35	36	37	38	39	40	41	42	43		35	36	37	38	39	40	41	42	43
	2100	2160	2220	2280	2340	2400	2460	2520	2580		2100	2160	2220	2280	2340	2400	2460	2520	2580
0	2341	2218	2099	1984	1871	1761	1654	1549	1447	30	2279	2159	2041	1927	1816	1707	1601	1498	1397
1	2339	2216	2098	1982	1869	1759	1652	1547	1445	31	2277	2157	2039	1925	1814	1705	1599	1496	1395
2	2337	2214	2096	1980	1867	1757	1650	1546	1443	32	2275	2155	2037	1923	1812	1703	1598	1494	1393
3	2335	2212	2094	1978	1865	1755	1648	1544	1442	33	2273	2153	2035	1921	1810	1702	1596	1493	1392
4	2333	2210	2092	1976	1863	1754	1647	1542	1440	34	2271	2151	2033	1919	1808	1700	1594	1491	1390
5	2331	2208	2090	1974	1862	1752	1645	1540	1438	35	2269	2149	2032	1918	1806	1698	1592	1489	1388
6	2328	2206	2088	1972	1860	1750	1643	1539	1437	36	2267	2147	2030	1916	1805	1696	1591	1487	1387
7	2326	2204	2086	1970	1858	1748	1641	1537	1435	37	2265	2145	2028	1914	1803	1694	1589	1486	1385
8	2324	2202	2084	1968	1856	1746	1640	1535	1433	38	2263	2143	2026	1912	1801	1693	1587	1484	1383
9	2322	2200	2082	1967	1854	1745	1638	1534	1432	39	2261	2141	2024	1910	1799	1691	1585	1482	1382
10	2320	2198	2080	1965	1852	1743	1636	1532	1430	40	2259	2139	2022	1908	1797	1689	1584	1481	1380
11	2318	2196	2078	1963	1850	1741	1634	1530	1428	41	2257	2137	2020	1906	1795	1687	1582	1479	1378
12	2316	2194	2076	1961	1849	1739	1633	1528	1427	42	2255	2135	2018	1904	1794	1686	1580	1477	1377
13	2314	2192	2074	1959	1847	1737	1631	1527	1425	43	2253	2133	2016	1903	1792	1684	1578	1476	1375
14	2312	2190	2072	1957	1845	1736	1629	1525	1423	44	2251	2131	2014	1901	1790	1682	1577	1474	1373
15	2310	2188	2070	1955	1843	1734	1627	1523	1422	45	2249	2129	2012	1899	1788	1680	1575	1472	1372
16	2308	2186	2068	1953	1841	1732	1626	1522	1420	46	2247	2127	2010	1897	1786	1678	1573	1470	1370
17	2306	2184	2066	1951	1839	1730	1624	1520	1418	47	2245	2125	2009	1895	1785	1677	1571	1469	1368
18	2304	2182	2064	1950	1838	1728	1622	1518	1417	48	2243	2123	2007	1893	1783	1675	1570	1467	1367
19	2302	2180	2062	1948	1836	1727	1620	1516	1415	49	2241	2121	2005	1891	1781	1673	1568	1465	1365
20	2300	2178	2061	1946	1834	1725	1619	1515	1413	50	2239	2119	2003	1889	1779	1671	1566	1464	1363
21	2298	2176	2059	1944	1832	1723	1617	1513	1412	51	2237	3117	2001	1888	1777	1670	1565	1462	1362
22	2296	2174	2057	1942	1830	1721	1615	1511	1410	52	2235	2115	1999	1886	1775	1668	1563	1460	1360
23	2294	2172	2055	1940	1828	1719	1613	1510	1408	53	2233	2113	1997	1884	1774	1666	1561	1459	1359
24	2291	2170	2053	1938	1827	1718	1612	1508	1407	54	2231	2111	1995	1882	1772	1664	1559	1457	1357
25	2289	2169	2051	1936	1825	1716	1610	1506	1405	55	2229	2109	1993	1880	1770	1663	1558	1455	1355
26	2287	2167	2049	1934	1823	1714	1608	1504	1403	56	2227	2107	1991	1878	1768	1661	1556	1454	1354
27	2285	2165	2047	1933	1821	1712	1606	1503	1402	57	2225	2105	1989	1876	1766	1659	1554	1452	1352
28	2283	2163	2045	1931	1819	1711	1605	1501	1400	58	2223	2103	1987	1875	1765	1657	1552	1450	1350
29	2281	2161	2043	1929	1817	1709	1603	1499	1398	59	2220	2101	1986	1873	1763	1655	1551	1449	1349
30	2279	2159	2041	1927	1816	1707	1601	1498	1397	60	2218	2099	1984	1871	1761	1654	1549	1447	1347

Table of Logiſtical Logarithms.

	44	45	46	47	48	49	50	51	52		44	45	46	47	48	49	50	51	52
	2640	2700	2760	2820	2880	2940	3000	3060	3120		2640	2700	2760	2820	2880	2940	3000	3060	3120
0	1347	1249	1154	1061	969	880	792	706	621	30	1298	1201	1107	1015	924	835	749	663	580
1	1345	1248	1152	1059	968	878	790	704	620	31	1296	1200	1105	1013	923	834	747	662	579
2	1344	1246	1151	1057	966	877	789	703	619	32	1295	1198	1104	1012	921	833	746	661	577
3	1342	1245	1149	1056	965	875	787	702	617	33	1293	1197	1102	1010	920	831	744	659	576
4	1340	1243	1148	1054	963	874	786	700	616	34	1291	1195	1101	1008	918	830	743	658	574
5	1339	1241	1146	1053	962	872	785	699	615	35	1290	1193	1099	1007	917	828	741	656	573
6	1337	1240	1145	1051	960	871	783	697	613	36	1288	1192	1098	1005	915	827	740	655	572
7	1335	1238	1143	1050	959	869	782	696	612	37	1287	1190	1096	1004	914	825	739	654	570
8	1334	1237	1141	1048	957	868	780	694	610	38	1285	1189	1095	1002	912	824	737	652	569
9	1332	1235	1140	1047	956	866	779	693	609	39	1283	1187	1093	1001	911	822	736	651	568
10	1331	1233	1138	1045	954	865	777	692	608	40	1282	1186	1091	999	909	821	734	649	566
11	1329	1232	1137	1044	953	863	776	690	606	41	1280	1184	1090	998	908	819	733	648	565
12	1327	1230	1135	1042	951	862	774	689	605	42	1278	1182	1088	996	906	818	731	647	563
13	1326	1229	1134	1041	950	860	773	687	603	43	1277	1181	1087	995	905	816	730	645	562
14	1324	1227	1132	1039	948	859	772	686	602	44	1275	1179	1085	993	903	815	729	644	561
15	1322	1225	1130	1037	947	857	770	685	601	45	1274	1178	1084	992	902	814	727	642	559
16	1321	1224	1129	1036	945	856	769	683	599	46	1272	1176	1082	990	900	812	726	641	558
17	1319	1222	1127	1034	944	855	767	682	598	47	1270	1174	1081	989	899	811	724	640	557
18	1317	1221	1126	1033	942	853	766	680	596	48	1269	1173	1079	987	897	809	723	638	555
19	1316	1219	1124	1031	941	852	764	679	595	49	1267	1171	1078	986	896	808	721	637	554
20	1314	1217	1123	1030	939	850	763	678	594	50	1266	1170	1076	984	894	806	720	635	552
21	1313	1216	1121	1028	938	849	762	676	592	51	1264	1168	1074	983	893	805	719	634	551
22	1311	1214	1119	1027	936	847	760	675	591	52	1262	1167	1073	981	89	803	717	633	550
23	1309	1213	1118	1025	935	846	759	673	590	53	1261	1165	1071	980	890	802	716	631	548
24	1308	1211	1116	1024	933	844	757	672	588	54	1259	1163	1070	978	888	801	714	630	547
25	1306	1209	1115	1022	932	843	756	670	587	55	1257	1162	1068	977	887	799	713	628	546
26	1304	1208	1113	1021	930	841	754	669	585	56	1256	1160	1067	975	885	798	711	627	544
27	1303	1206	1112	1019	929	840	753	668	584	57	1254	1159	1065	974	884	796	710	626	543
28	1301	1205	1110	1018	927	838	751	666	583	58	1253	1157	1064	972	883	795	709	624	541
29	1300	1203	1109	1016	926	837	750	665	581	59	1251	1156	1062	971	881	793	707	623	540
30	1298	1201	1107	1015	924	835	749	663	580	60	1249	1154	1061	969	880	792	706	621	539

Table of Logiſtical Logarithms.

	53	54	55	56	57	58	59		53	54	55	56	57	58	59
	3180	3240	3300	3360	3420	3480	3540		3180	3240	3300	3360	3420	3480	3540
0	539	458	378	300	223	147	73	30	498	418	339	261	185	110	36
1	537	456	377	298	221	146	72	31	497	416	337	260	184	109	35
2	536	455	375	297	220	145	71	32	495	415	336	258	182	107	34
3	535	454	374	296	219	143	69	33	494	414	335	257	181	106	33
4	533	452	373	294	218	142	68	34	493	412	333	256	180	105	31
5	532	451	371	293	216	141	67	35	491	411	332	255	179	104	30
6	531	450	370	292	215	140	66	36	490	410	331	253	177	103	29
7	529	448	369	291	214	139	64	37	489	408	329	252	176	101	28
8	528	447	367	289	213	137	63	38	487	407	328	251	175	100	27
9	526	446	366	288	211	136	62	39	486	406	327	250	174	99	25
10	525	444	365	287	210	135	61	40	484	404	326	248	172	98	24
11	524	443	363	285	209	134	60	41	483	403	324	247	171	96	23
12	522	442	362	284	208	132	58	42	482	402	323	246	170	95	22
13	521	440	361	283	206	131	57	43	480	400	322	244	169	94	21
14	520	439	359	282	205	130	56	44	479	399	320	243	167	93	19
15	518	438	358	280	204	129	55	45	478	398	319	242	166	91	18
16	517	436	357	279	202	127	53	46	476	396	318	241	165	90	17
17	516	435	356	278	201	126	52	47	475	395	316	239	163	89	16
18	514	434	354	276	200	125	51	48	474	394	315	238	162	88	15
19	513	432	353	275	199	124	50	49	472	392	314	237	161	87	13
20	512	431	352	274	197	122	49	50	471	391	313	235	160	85	12
21	510	430	350	273	196	121	47	51	470	390	311	234	158	84	11
22	509	428	349	271	195	120	46	52	468	388	310	233	157	83	10
23	507	427	348	270	194	119	45	53	467	387	309	232	156	82	8
24	506	426	346	269	192	117	44	54	466	386	307	230	155	80	7
25	505	424	345	267	191	116	42	55	464	384	306	229	153	79	6
26	503	423	344	266	190	115	41	56	463	383	305	228	152	78	5
27	502	422	342	265	189	114	40	57	462	382	304	227	151	77	4
28	501	420	341	264	187	112	39	58	460	381	302	225	150	75	2
29	499	419	340	262	186	111	38	59	459	379	301	224	148	74	1
30	498	418	339	261	185	110	36	60	458	378	300	223	147	73	0

APPENDIX.

Of the FACTITIOUS AIRS, *and the* FIRST PRINCIPLES *of* CHEMISTRY.

CHAP. I.

Of CHEMICAL ATTRACTIONS.

DEF. I. CHEMISTRY is that ſcience by means of which are aſcertained the nature and properties of bodies.

The methods of obtaining this knowledge are *analyſis* and *ſyntheſis*: but there being no criteria by which we can diſtinguiſh the primary elements, there can be no certainty, in any inſtance, of a complete analyſis.

DEF. II. That power which tends continually to bring principles together which are diſunited, and which retains with more or leſs energy, thoſe which are already in a ſtate of combination, is called the attraction of affinity.

It is impoſſible to produce any change in nature without interrupting or modifying this attractive power.

DEF. III. Affinity of aggregation is that which exiſts between two principles of the ſame nature.

Two drops of water which come together form an aggregate. Each drop may be called an integrant part.

An aggregate differs from a heap, becauſe the integrant parts of the latter have no perceptible adheſion to each other, as a heap of ſand, corn, &c. They both differ from a mixture, the conſtituent parts of which are of a different nature, as gunpowder.

The affinity of aggregation is ſtronger, the nearer the parts approach each other; conſequently whatever tends to ſeparate theſe parts diminiſhes their affinity. This effect is produced by heat; thus, melted metals have no conſiſtence. Hence the *matter of heat,* which is called by chemiſts, *caloric,* produces an effect oppoſite to attraction.

There are four kinds of aggregates, ariſing from the different degrees of force of this attraction, acting between the conſtituent principles of bodies;—theſe are the hard;—the ſoft;—the fluid;—and the aeriform aggregates; and of theſe there are ſeveral varieties. Water appears in all theſe four ſtates of aggregation.

DEF. IV. When the very minute parts of one ſubſtance unite with thoſe of another, ſo intimately as to form a body which has properties different from thoſe of either of them, the union is called the attraction of *compoſition.* The parts into which a body is divided by decompoſition, are called component parts, or principles.

Sand and ſalt of tartar expoſed to a ſtrong heat combine, and form a compound called glaſs.

Bodies of different kinds exert a tendency, or attraction upon each other, by virtue of which all the changes of compoſition and decompoſition are effected. The affinity of compoſition exhibits invariable laws in all the phenomena it cauſes.

1. It acts only between the conſtituent parts of bodies. Two bodies placed near each other do not unite; but, if they be divided and mixed, a combination may ariſe. Let ſome common ſalt be triturated with litharge: or ſome ſal ammoniac with lime. The energy of the affinity of compoſition is always proportioned to the degree of the diviſion of bodies.

2. The affinity of compoſition is in the inverſe ratio of that of aggregation. The difficulty of decompoſing a body is greater in the proportion as its conſtituent principles are united by a greater force. Gaſes and vapours tend to combination, becauſe their aggregation is weak: and nature, which is conſtantly renewing the productions of this globe, never combines ſolid with ſolid; but by reducing every thing into the form of gas, breaks the impediments of aggregation, and theſe gaſes uniting form ſolids in their turn.

3. This attraction takes place between bodies of a diſſimilar nature. The more diſſimilar they are, the greater is the force with which they combine. The intimate combination of acids and alkalis;[*] of acids and metals; of alkalis and oils, are in proof of this. It was formerly ſuppoſed that a certain ſimilarity exiſted between ſubſtances ſubject to this mutual attraction; and on this error the term *affinity* was founded.

4. Several bodies, ſometimes three, in other caſes four, combine in conſequence of this attraction.

5. One of the ſubſtances that are to enter into combination, at leaſt, muſt be in a fluid ſtate.

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[*] As it is not intended to enter at large into the principles of chemiſtry, it will be ſufficient in this place briefly to explain the nature and properties of *alkalis* and *acids*, which are the agents moſt frequently employed in chemical experiments.

Firſt. Of Alkalis.—It is agreed to call every ſubſtance an alkali which is characteriſed by *all* the following properties. (1) An acid burning urinous taſte. (2) The property of turning ſyrup of violets green. (3) The virtue of forming glaſs when fuſed with flinty ſubſtances. (4) The faculty of rendering oils miſcible with water;—of efferveſcing with certain acids;—and of forming neutral ſalts with all of them.

Alkalis are divided into the *fixed* and *volatile*. This diſtinction is eſtabliſhed upon the ſmell of theſe ſubſtances: the former are not volatized by any heat, and emit no characteriſtic ſmell; but the latter are eaſily reduced to vapour, and emit a very penetrating odour.

Fixed alkalis are of two kinds, the one is called *vegetable* alkali, or pot-aſh; the other *mineral* alkali, or ſoda.

1. Vegetable alkali may be extracted from various ſubſtances; as (1) from the lixivium of wood-aſhes, which is obtained by waſhing the aſhes, and concentrating the diſſolution in iron boilers. Hence wood-aſhes are employed by laundreſſes and bleachers; the uſe of which is, that the alkali may combine with the fat ſubſtances, and render them ſoluble in water. (2) The lees of wine are almoſt totally converted into very pure alkali by combuſtion. (3) The combuſtion of tartar of wine likewiſe affords an alkali of conſiderable purity, and is known by the name of ſalt of tartar, which is commonly employed in medical uſes.—Vegetable alkali attracts the humidity of the air, and is thus reſolved into a liquor.

2. Mineral alkali is ſo called, becauſe it forms the baſis of marine ſalt: and it is obtained from marine plants by combuſtion. It differs from the vegetable becauſe (1) it is leſs cauſtic. (2) It does not attract the humidity of the air, but effloreſces in it. (3) It cryſtallizes in rhomboidal octahedrons. (4) It forms different products with the ſame baſes. (5) It is more proper for vitrification. Fixed alkalis eaſily combine with ſulphur; either by the fuſion of equal parts of alkali and ſulphur; or by digeſting the liquid alkali upon ſulphur; theſe ſolutions are called livers of ſulphur.

Volatile alkali is chiefly obtained from the diſtillation of all the parts of animal ſubſtances. *Horns* are employed in preference, becauſe they are reſolved almoſt entirely into oil and volatile alkali. The putrefaction of all animal ſubſtances likewiſe produces volatile alkali.

Volatile alkali is known by its ſtrong penetrating ſmell. It is eaſily reducible into the ſtate of gas, and preſerves this form at the temperature of the atmoſphere. This gas may be obtained by decompoſing ſal ammoniac by quick-lime, and receiving the product over mercury.

(1) Alkaline gas kills animals, and corrodes the ſkin. (2) It is improper for combuſtion. (3) It is lighter than atmoſpheric air.

Secondly. Of Acids.—Subſtances either in a liquid or concrete form, which have a ſour taſte, are called acids. They are extracted from various bodies, mineral, vegetable, and animal; on which account they are diſtinguiſhed into mineral, vegetable, and animal acids. Thoſe of the firſt claſs are, the vitriolic, nitrous, and marine acids. The principal vegetable acids are the acetous, the vinous, the eſſential acid of vegetables, and that obtained from oils and reſins. The animal acids are principally the phoſphoric; the acid of ants and other inſects; and the acids of healthy and diſeaſed ſtomachs.

The properties which belong to all acids in general, by which they are diſtinguiſhed from the other claſſes of bodies are, (1) they have a great affinity to water, with which they ſo eaſily combine, that moſt of them appear in a liquid form, on account of the water mixed with them. (2) They may be combined with ſpirits of wine. (3) With abſorbent earths. (4) With alkalis. (5) With metallic ſubſtances; but every metal will not combine with every acid indiſcriminately. (6) When they are much concentrated they may be combined with oils.

The following are principal effects ariſing from theſe combinations. (1) When concentrated acids are properly mixed with pounded ice, or ſnow, they produce cold. (2) They diſſolve ſolid animal ſubſtances. (3) They coagulate moſt of the animal fluids. (4) They change the blue vegetable colours to red, ſuch as the colour of turnſole, ſyrup of violets, &c. which are uſed to aſcertain the preſence of acids; excepting the vitriolic acid, which deſtroys theſe colours. (5) They reſiſt fermentation. (6) They require a greater degree of cold than water, to freeze them. (7) When they are combined with various of the above mentioned ſubſtances, different elaſtic fluids are produced.

A ſolid is diſſolved in a liquid, which is called its *menſtruum*, in conſequence of a mutual attraction equal on both ſides between the parts of the ſolid and the parts of the liquid.

6. The compound which reſults from the combination of two or more bodies, is of a different temperature from thoſe of the ingredients; which fact can, frequently, be diſcovered by means only of the thermometer.

7. Two or more bodies united, in conſequence of the attraction of compoſition, form a compound, having properties very different from thoſe which each of the bodies poſſeſſed previouſly to their union.

Thus, bodies which poſſeſs remarkable ſapidity, form a compound that is nearly inſipid; or that differs materially in quality and degree from thoſe which the bodies ſeparately poſſeſſed. Others inconſiderably ſapid form a compound that is highly corroſive.

In conſequence of chemical combination, the forms of bodies are affected. Bodies which ſeparately are not ſuſceptible of cryſtallization, form a compound which aſſumes a regular figure. Others vice verſa.

The ſame cauſe affects the conſiſtence of bodies. Thus two fluids, in conſequence of chemical combination, appear under a ſolid form.

In the compound, the capacity of emitting odours, is different from that of the ingredients. Bodies which are remarkably odorous, form a compound that is almoſt without ſcent. Others vice verſa.

Their diſpoſition to fuſibility is altered. Thus bodies, which do not admit eaſily of fuſion, become, in a ſtate of chemical combination, extremely fuſible. See table of heat, p. 126. Art. lead, bismuth, and tin.

8. The attraction of compoſition is meaſured by the difficulty with which a combination, formed between the two bodies, is deſtroyed.

9. Every ſubſtance has its peculiar affinities with the various other ſubſtances preſented to it. If all bodies had the ſame degree of affinity with each other, no change could take place amongſt them; we ſhould not be able to diſplace any principle by preſenting one body to another. It is in conſequence of this difference in the affinities that all chemical decompoſitions are effected: all the operations of nature and art are founded upon it. Hence ariſe the terms *ſimple* affinity; *double* affinity, &c. (1) Two principles united, and then ſeparated by means of a third, afford an example of ſimple affinity: it conſiſts in diſplacing one principle by the addition of a third. The body diſengaged is called the *precipitate*;—the ſubſtance uſed to ſeparate the compound, is called the *precipitant*. An alkali precipitates metals from their ſolutions. Sometimes the new compound itſelf is precipitated, as when the vitriolic acid is poured on a ſolution of the muriate of lime. At other times the diſengaged body, and the new compound, are precipitated together, as when Epſom ſalt is diſſolved in water, and precipitated by means of lime water. (2) It often happens that the compound of two principles cannot be deſtroyed either by a third, or a fourth body ſeparately applied: but if theſe two bodies be united, and then brought into contact with the ſame compound, a decompoſition will take place. This phenomenon conſtitutes the double affinity. Vitriolated tartar is not completely decompoſed by nitric acid, or by lime, when either of theſe principles is ſeparately preſented; but if the nitric acid be combined with lime, this nitrate of lime will decompoſe the vitriolated tartar. Here the affinity of the vitriolic acid with the alkali is weakened by its affinity to the lime. (3) There are caſes in which two bodies, having no perceptible affinity to each other, obtain a diſpoſition to unite by the intervention of a third; this is called the affinity of an *intermedium*. An alkali is the intermedium of union between oil and water; hence the theory of waſhing, &c.

10. The phenomena of cryſtallization are reſolvable into this attraction. Bodies put on a cryſtalline form, in conſequence of their particles joining together at the proper ſurfaces, by means of a tendency which they have to unite in this manner, preferably to any other. In order to diſpoſe a ſubſtance to cryſtallization, it is neceſſary to reduce it to the moſt complete ſtate of diviſion; which may be effected by ſolution, or by an operation purely mechanical. Solution may be effected by water, as in the caſe of ſalts; or by fire, as in the caſe of metals: the ſolution is not complete until a degree of heat is applied, of ſufficient intenſity, to convert them into a ſtate of gas.

Nature and art preſent cryſtals both regular and irregular. In order to procure regular and well formed cryſtals by art, three circumſtances are required, *time*, *ſpace*, and *repoſe*. *Time* cauſes the abundant fluid to be diſſipated, and brings the integral parts nearer to each other. *Space* is a neceſſary condition to prevent the operations of nature from being reſtrained. A *ſtate of repoſe* is neceſſary to obtain very regular forms; without this condition the cryſtallization obtained will be confuſed and indeterminate.

A very ſingular property is obſervable in ſalts, which may be referred to cryſtallization, but is likewiſe, in ſome meaſure, remote from it, becauſe it does not depend upon the ſame cauſes. This is the property of riſing along the ſides of the veſſels which contain the ſolution. It is known by the name of ſaline vegetation; and depends on the concurrence of air and light. The effect may be determined, at pleaſure, towards any part of the veſſel, by managing and directing the action of theſe two agents.

CHAP. II.

Of FIRE.

The principal agent employed by nature to balance the power of attraction, is fire. By the natural effect of attraction we ſhould poſſeſs none but ſolid and compact bodies : but the caloric, unequally diſperſed in bodies, tends to deſtroy this adheſion of particles. Hence the various ſubſtances that compoſe this globe are ſubject, on the one hand, to a general law which tends to bring them together : and on the other, to a powerful agent which tends to remove them from each other : upon the reſpective energy of theſe two forces, the conſiſtence of all bodies depends. When the attraction prevails, they are in a ſolid ſtate ; when the caloric is moſt powerful, they are in a ſtate of gas ; and the liquid form appears to be the point of equilibrium between theſe two powers.

On this ſubject two things are to be conſidered, viz. *heat* and *light*.

FIRST. Concerning heat, which in common language has a double ſignification. It is uſed indiſcriminately to expreſs a ſenſation of the mind ; and an unknown principle, which is the exciting cauſe of that ſenſation. Philoſophers uſe it in the latter ſenſe. The effects of heat are known and meaſured (1) by the peculiar ſenſations which it excites in animals : and when conſidered as exciting thoſe ſenſations, it is called *ſenſible* heat. (2) Heat is known by the effect it produces on the thermometer. This is called the *temperature* of heat in bodies. (3) It is found by experiment that, in bodies of different kinds, the quantities of abſolute heat may be unequal, though the temperatures and weights be the ſame.

SECT. I.—*Of the* TRANSMISSION *of* HEAT.

I. Heat has a conſtant tendency to diffuſe itſelf over all bodies, till they are brought to the ſame temperature.

Thus it is found, by the thermometer, that if two bodies, of different temperatures, are mixed together, or placed contiguous, the heat paſſes from one to the other till their temperatures become equal ; and that all *inanimate* bodies, when heated, and placed in a cold medium, continually loſe heat, till they are brought to the ſtate of the ſurrounding medium.

1. In homogeneous bodies of the ſame temperature, the quantity of heat is as the quantity of matter.

2. The temperature of two equal homogeneous bodies, immediately after mixture, is the arithmetical mean between their reſpective temperatures previouſly to mixture.

3. In a mixture of homogeneous bodies of unequal weights, the exceſs of the hotter body is divided between them in the proportion to their quantities of matter.

This rule does not hold good in heterogeneous bodies.

EXP. If equal *bulks* of mercury and water be mixed together, and the water be the hotter body ; the water, in order to bring about a common temperature, gives out only $\frac{1}{3}$ of its exceſs, which raiſes the mercury $\frac{2}{3}$ of the ſame ſum : if mercury be the hotter body, it communicates $\frac{2}{3}$ of its exceſs while the water is raiſed $\frac{1}{3}$ of that ſum. Hence it follows that the heat neceſſary to raiſe a given *bulk* of water to any degree of temperature, is to that neceſſary to raiſe an equal *bulk* of mercury to the ſame, in the ratio of $\frac{2}{3} : \frac{1}{3}$, or of 2 : 1. And the ſpecific gravity of mercury being to that of water as 14 : 1, it follows, that the heat neceſſary to raiſe a given *weight* of water to a certain temperature, will be to that neceſſary to raiſe an equal *weight* of mercury to the ſame temperature, in the ratio of 28 : 1. Conſequently, the comparative heat of equal *bulks* of water and mercury is as 2 : 1 ; but in equal *weights*, it is as 28 : 1.

Hence we diſcover the quantities of abſolute heat contained in different bodies, which, when compared with the abſolute heat of water, as a ſtandard, is called their comparative heat. On this principle tables of the comparative heat of bodies are conſtructed. See Dr. Crawford's valuable work on animal heat.

DEF. V. The capacity of a body for heat is its diſpoſition to require, or to give out more or leſs heat in undergoing equal changes of temperature.

The body, therefore, which has the leſs capacity for containing heat, has its temperature more increaſed by the addition of a given quantity of abſolute heat than that which has a greater.

II. The capacity for heat, in any body, is found to vary under the different ſtates of aggregation in which it is found. It is leaſt in ſolid bodies, greater in liquid, and greateſt of all in aeriform ſubſtances.

The capacity for containing heat may continue unchanged while the abſolute heat is varied without end. If a pound of ice be ſuppoſed to retain its ſolid form, the quantity of its abſolute heat will be altered by every increaſe or diminution of its ſenſible heat; but as long as its form continues the ſame, its capacity for receiving heat is not affected by an alteration of temperature.

When bodies chemically combine, the capacity of the compound is not made up of the capacity of the principles.

SECT. II.—*Of the* Action *of* Heat *upon* Bodies.

Heat expands bodies according to all their dimenſions.

This is known by experiment, to which however there are exceptions, but not ſuch as to afford a juſt objection to the general fact; becauſe, in thoſe inſtances, by the action of heat, a fluid is extricated that previouſly ſeparated the particles from each other. Bodies are expanded leaſt in the ſolid ſtate, more in the liquid ſtate, and moſt in an aeriform one. In ſome bodies, as mercury, linſeed oil, and rectified ſpirits, the expanſions, within certain limits, are proportional to the quantities of abſolute heat communicated to them: theſe limits are near the points of congelation and evaporation. Thus the quantity of heat required to raiſe a body four degrees in temperature by the mercurial thermometer, is double that which is required to raiſe it two degrees, four times of that required to raiſe it one degree, and ſo on in proportion.

The tranſmiſſion of heat, and its power of expanding bodies in all their dimenſions, are the principles upon which thermometers are conſtructed. See p. 125.

The objections to the univerſality of this law, are, (1) water contracts till it has ſunk to within 8°. of the freezing point; it then dilates. This difficulty may be reſolved by the laws of cryſtallization, which will explain the phenomena of the burſting of water-pipes, looſe pavements, &c. (2) Caſt-iron expands as it cools. This objection is obviated on the ſame principle as the laſt, viz. the diſpoſition of its particles to unite, ſo as to give the maſs a certain peculiar configuration. (3) The contraction of clay, upon which depend Mr. Wedgewood's thermometers, ariſes from the action of the heat on the moiſture which is expelled, and not upon the particles of the clay themſelves.

SECT. III.—*Of* Heat *as the Cauſe of the ſeveral Changes in the Forms of Bodies.*

I. Heat is contained in conſiderable quantities in all bodies, when at the common temperature of the atmoſphere.

In the year 1780, at Glaſgow, the thermometer ſunk 25°. below the beginning of Fahrenheit's ſcale. In the deſerts of Siberia, during an intenſe froſt, the mercury was found congealed in thermometers expoſed to the atmoſphere, and a quantity of that fluid, in an open bowl, placed in a ſimilar ſituation, at the ſame time, became ſolid. Experiments made at Hudſon's Bay ſhew that the freezing point of mercury is very nearly 40°. below the zero of Fahrenheit. Hence it follows, that the atmoſphere in Siberia muſt have been cooled to minus 40°. It is alſo manifeſt, that heat is contained in conſiderable quantities in all bodies, when at the common temperature of the atmoſphere.

II. Heat changes the forms of bodies.

It converts ſolids into liquids; and liquids into vapours, or into permanently elaſtic fluids. The liquid, and vaporous forms of bodies, are brought about in the ſame way with expanſion, being, in fact, only higher degrees of it. A different principle ſeems to take place in the aeriform ſtate: the particles appear to be thrown into certain ſpheres, in which they are fixed by ſome other force than that of heat. Theſe they do not quit in any degree of cold that has yet been produced.

Hence all bodies in nature, are either ſolid, liquid, or in a ſtate of elaſtic aeriform vapour, according to the degree of heat to which they are expoſed.

III. Every body which paſſes from the ſolid to the liquid ſtate abſorbs a portion of heat which is no longer ſenſible to the thermometer, but exiſts in a true ſtate of combination; and is called *latent* heat.

Exp. 1. If a pound of water at 32°. be mixed with an equal quantity of the ſame fluid at 172°. the temperature of the mixture will be 102°. which is the arithmetical mean between the heat of the warm water and of the cold.

2. If a pound of ice at 32°. be mixed with a pound of water at 172°. the temperature of the mixture will be 32°.—In the firſt experiment a quantity of heat, which raiſed the thermometer 70°. paſſed from the warm water into the cold, by which the temperature of the cold water was increaſed 70°. But in the laſt experiment, a quantity which raiſed the thermometer 140°. paſſed from the warm water into the ice, in conſequence of which the ice was melted, but its ſenſible heat was not increaſed. Whence it follows, that in the melting of ice, 140°. of heat are abſorbed, which produce no effect upon the thermometer.

3. A thermometer plunged into a veſſel filled with pounded ice, will deſcend to 32°. If, then, the veſſel be immerſed in boiling water, the thermometer will not riſe during the whole time of the liquefaction of the ice.

4. Dr. Black found that a piece of ice, ſuſpended in a warm room, took upwards of five hours to liquefy, during which time a ſtream of cold air continued to flow from it. Had the heat which was communicated to it, by the ſucceſſive portions of air which paſſed over it, not been abſorbed, but employed in making it ſenſibly hotter, its temperature would, at the end of the time, have exceeded that of red hot iron.

Hence the reaſon why ice and ſnow continue in their ſolid ſtate ſo long after the temperature of the atmoſphere has riſen conſiderably above the freezing point.

Cold is produced in the diſſolution of all cryſtalliſed ſalts. Mr. Walker, an ingenious gentleman of Oxford, in the years 1787 and 1788, communicated to the Royal Society the reſult of many accurate experiments on this ſubject. The following mixtures produced the greateſt degrees of cold.

5. Three parts of ſtrong fuming nitrous acid diluted with water in the proportion of 2 to 1 by weight, well mixed and cooled to the temperature of the air; 4 parts of Glauber's ſalts; nitrous ammoniac 3½ parts; each reduced ſeparately to fine powder. The Glauber's ſalt is to be added to the diluted acid, the mixture well ſtirred, and immediately afterward the powdered ammoniac, again ſtirring the mixture. In theſe kind of experiments the ſalts ſhould be procured as dry and tranſparent as poſſible.

6. Eighteen ounces of concentrated nitrous acid diluted with water in proportion of 2 to 1: Glauber's ſalt 24 ounces; ſal ammoniac 12 ounces. On adding the Glauber's ſalt to the diluted acid, the thermometer fell from 50°. to —1°. and on adding the ammoniac it fell —9°. that is full 60°.

7. Mr. Walker effected the congelation of quickſilver by a combination of this mixture with ſnow and ice. When he began his experiment, the temperature of the mercury was 45°. ſo that, the freezing point of that metal being —39°. there was produced 84°. of cold. This gentleman has, however, frozen mercury without the aid of either ice or ſnow.

8. The greateſt degree of cold which Mr. Walker has produced was by repeating an experiment firſt made by Mr. Lowitz of Peterſburg. By inſtantaneouſly mixing four parts of fixed ſal ammoniac, reduced to powder with three parts of light, dry, and freſh ſnow, the thermometer was ſunk —48°. but upon mixing the ſame articles previouſly cooled by art to —40°. the thermometer ſunk to 63°. below 0.

Equal parts of ſal ammoniac and nitre in powder, make a cheap and convenient compoſition for producing cold by a ſolution in water; it will freeze water and cream at Midſummer. See Phil. Tranſ. 1787, 1788.

IV. The heat which liquids abſorb, when they acquire a fluid form, is again ſeparated from them when they return to a ſolid ſtate.

EXP. I. If a pound of water at 32°. be mixed with an equal quantity of ice at 4°. nearly ⅕ of the water will be frozen, and the temperature of the mixture will be 32°. In this experiment the ice is raiſed from 4°. to the freezing point: it is therefore evident, that by the congelation of nearly ⅕ of a pound of water, a quantity of heat is *evolved*, ſufficient to raiſe a pound of ice 28°.

2. Fahrenheit expoſed ſome water in a beer glaſs, perfectly at reſt, to an atmoſphere that was 10° below the freezing point. The water ſunk down to the temperature of the air, and retained its fluid form. Upon agitating the veſſel, the water immediately congealed into a ſpongy kind of ſubſtance, in which that which remained fluid became entangled, and the whole roſe to the freezing point. Here it is evident, that the portion which became ſolid gave off a quantity of heat, ſufficient to raiſe the whole to the freezing point.

V. All bodies, by paſſing from the fluid ſtate, to the aeriform, abſorb heat, which is again given out when they recover their liquid ſtate.

EXP. I. If a veſſel of water be taken at 45°. or 50°. and heat be applied, it will reach the boiling point in ¼ of an hour. If the communication of ſenſible heat were all that was neceſſary to evaporation, the whole might be expected *immediately* to evaporate; a conſiderable time, however, is neceſſary to bring this about, which muſt be owing to the great heat that it requires to aſſume the vaporous ſtate.

2. A veſſel containing a few ounces of water at 54°. was expoſed to ſuch heat as made it boil in 4½ minutes, but it took more than 20 minutes wholly to evaporate. The heat was aſcertained to flow into it in the ſame proportion during the laſt 15½ as during the 4½ firſt minutes, but was neither ſenſible in the water, the veſſel, or the vapour: it was then abſorbed as neceſſary to the vaporous form.

3. A bottle cloſely corked was raiſed ſeveral degrees above the boiling point; but being ſuddenly opened, a ſmall portion of it only flew off in the form of vapour, and the reſidue ſunk immediately to the boiling point.

4. Water in Papin's digeſter was heated up to 400°. the lid being ſuddenly removed, a great quantity of ſteam ruſhed out with much violence, carrying ſome of the water along with it; but the greater part of the water remained in the veſſel, and ſunk down all at once to 212.

5. Water and other liquors uſed for drink, in hot climates, are put into porous veſſels, and being wrapped up in wet cloths, are expoſed to the ſun, or to a current of warm air, by which means the encloſed liquor is preſently cooled.

6. A thermometer taken out of water, and expoſed to air, always deſcends. It afterwards riſes till it has acquired the temperature of the atmoſphere; but the time of deſcending is leſs than that which it employs to riſe again. It never riſes to the common temperature till its bulb is dry.

7. A thermometer ſuſpended in the receiver of an air-pump, deſcends two or three degrees during the time of exhauſtion, and afterwards riſes to the temperature of the vacuum.

8. A thermometer plunged in alkohol, in the receiver of an air-pump, deſcends, and the lower in proportion as the bubbles are ſtronger which iſſue from the alkohol. If it be withdrawn from this liquor, and ſuſpended wet beneath the receiver, it falls ſeveral degrees while the air is exhauſting.

9. If the bulb of a thermometer be wrapped in fine linen, and kept moiſt by ether, and the evaporation be facilitated by the agitation of the air, the thermometer will deſcend to the freezing point.

Hence perſpiration produces a certain degree of cold. Workmen employed in glaſs-houſes, foundaries, &c. live in a medium much hotter than their own bodies, the natural temperature of which is equalized by perſpiration.

Hence almoſt all fevers end in perſpiration, thereby carrying off the matter of heat.

From the conſideration of the foregoing experiments, particularly the 2d and 9th, Dr. Darwin was led to ſuſpect, that elaſtic fluids, if *mechanically expanded*, would attract or abſorb heat from bodies in their vicinity: and that if *mechanically condenſed*, the fluid matter of heat would be preſſed out of them, and diffuſe itſelf among adjacent bodies.

To aſcertain theſe facts he made the following experiments.

EXP. I. The blaſt from an air-gun was repeatedly thrown on the bulb of a thermometer, and it uniformly ſunk the mercury about 2°. The air-gun, before it was diſcharged, was left in the vicinity of the thermometer, in order that it might loſe the heat acquired in the act of charging.

2. A thermometer was ſo applied to the receiver of the air-gun that a continued ſtream of air might paſs over the bulb at the time of its expanſion, which ſunk the thermometer from 5°. to 7°. This experiment was repeated four times.

During the act of condenſing the air into the receiver, there was conſiderable heat generated in that end of the ſyringe next the air-globe, and the globe itſelf became hotter than could have been expected from its contact with the ſyringe. In exploding an air-gun, the ſtream of air always becomes viſible, owing to the cold, then produced, precipitating the vapour which it contained.

3. A thermometer was placed in the receiver of an air-pump, and the air being haſtily exhauſted, the mercury ſunk 2°. or 3°. and after ſome minutes regained its previous height. This appears to ariſe from the abſorption of heat from the mercury by the expanſion of the air. Both during the exhauſtion, and re-admiſſion of the air, a ſteam was regularly condenſed on the inſide of the glaſs which in a few minutes was abſorbed. The ſteam muſt have been precipitated, by being deprived of its heat by the expanded air.

4. Similar to theſe experiments is the phenomenon obſerved in what is called the fountain of Hiero, conſtructed on a very large ſcale, in the mines of Hungary. In this machine the air is compreſſed by a column of water 260 feet high: a ſtop-cock being opened, the air ruſhes out with great violence, and in conſequence of its previous condenſation becomes very much expanded, and the moiſture it contained is not only precipitated, as in the exhauſted receiver, but falls down in a ſhower of ſnow with icicles adhering to the cock.

Hence Dr. Darwin concludes, that in all circumſtances of the mechanical expanſion of air, the air becomes capable of abſorbing the caloric from bodies in contact with it. This principle the doctor extends (1) to account for the continual changes of the atmoſphere. (2) To account for the coldneſs of the higher regions of the atmoſphere; and for that degree of cold which is experienced at the tops of high mountains, and which renders them regions of perpetual froſt and ſnow. The *Andes*, almoſt under the line, reſts its baſe on burning ſand; about its middle height is a moſt pleaſant temperate climate, covering an extenſive plain, on which is built the city of Quito;—while its forehead is encircled with eternal ſnow, coeval, perhaps, with the elevation of the mountain. See Phil. Tranſ. 1787.

SECONDLY. Concerning light; the phyſical properties of which having been amply diſcuſſed in a former part of this work, we ſhall only notice, in this place, a few of thoſe operations of nature which ſeem to depend upon its influence. Vegetation cannot proceed without light. Plants deprived of this fluid become pale; and when placed in ſituations in which the light can come to them from one part only, they are obſerved to incline towards that part, ſhewing, thereby, the neceſſity which they have for this beneficial fluid. Without the influence of light, vegetables would exhibit but one lifeleſs colour; on this principle, celery, endive, and other plants, are bleached. Vegetables are not only indebted to the light for their colour, but likewiſe for their ſmell, taſte, combuſtibility, and the reſinous principle which equally depend upon it. Hence aromatic ſubſtances, reſins, volatile oils, &c. are the inheritance of ſouthern climates, where the light is more pure, conſtant, and intenſe. Hence alſo the nations of the eaſt obtain from the wood, bark, or roots of trees, many of the moſt valuable colouring matters both for permanency and luſtre which all the ingenuity of European dyers has never been able to imitate. Vegetables, expoſed to the ſun's rays, emit vital air in conſiderable quantities; and may, perhaps, be conſidered as the apparatus adapted to take from the rays of the ſun, or from the fluid matter of light, thoſe principles on which animal life chiefly depends.

Of the influence which ſolar rays have upon colour, animals afford inſtances, ſuch as worms and grubs, which live in the earth or in wood, and are of a whitiſh colour. Birds and flying inſects of the night are likewiſe diſtinguiſhable from thoſe of the day by the want of brilliancy of colour: and the difference is equally marked between thoſe of northern and ſouthern climates.

Light diſengages vital air from ſeveral fluids, ſuch as the nitric and oxigenated marine acid. It reduces the calxes of gold, ſilver, &c. Light, likewiſe, determines the phenomena of vegetation exhibited by ſaline ſolutions. See the concluſion of Chap. I.

Hence then we may conclude, with M. Lavoiſier, that organization, ſenſation, ſpontaneous motion, and life, exiſt only at or very near the ſurface of the earth, and in places to which light is acceſſible.

C H A P. III.

Of PERMANENTLY ELASTIC FLUIDS, *or* AIRS.

SECT. I.—*Of Airs in general, and of the Apparatus uſed in this Department of Science.*

The different airs with which we are now acquainted are generally denominated *gaſes*. By the French chemiſts theſe gaſes are conſidered as differing only from liquids and ſolids by their containing a greater quantity of heat; ſo that to reduce a ſubſtance to a ſtate of gas conſiſts only in diſſolving it in the matter of heat or caloric.

Caloric combines with various bodies, with greater or leſs facility; ſeveral, at the common temperature of the atmoſphere, are conſtantly in the ſtate of gas;—others paſs to that ſtate at ſome degrees higher, which are called evaporable ſubſtances.

The various ſtates under which bodies preſent themſelves to our eyes depend, as has been ſhewn, almoſt wholly upon the different degrees of caloric with which theſe ſame bodies are combined. All bodies are capable of paſſing to a gaſeous ſtate, by means of caloric, the quantity of which is governed, (1) By the affinity of aggregation which oppoſes a new combination. (2) By the weight of the conſtituent particles which render their volatilization more or leſs difficult. (3) By the attraction between the caloric and ſolid body.

All bodies, whether ſolid or liquid, when they are volatilized by heat, appear either in a ſtate of *vapour*, or in that of *gas*. In the former caſe, theſe ſubſtances quickly loſe the caloric which raiſed them to that ſtate. But in the latter, the combination with the volatilized ſubſtance is ſuch that the ordinary temperature of the atmoſphere is inſufficient to overcome the union.

It has been obſerved, that the ſame body becomes ſolid, or fluid, or aeriform, according to the quantity of caloric by which it is penetrated; or according as the repulſive force exerted by the caloric is equal to, ſtronger, or weaker, than the attraction of the particles of the body it acts upon. If, however, theſe two powers only exiſted, bodies would become liquid at an indiviſible degree of the thermometer, and would almoſt inſtantaneouſly paſs from the ſolid ſtate of aggregation to that of aeriform elaſticity. Thus water, at the moment when it ceaſed to be ice, would begin to boil, and would be transformed into an aeriform fluid. That this does not happen, muſt depend on the action of ſome third power, which is no other than the preſſure of the atmoſphere, which cauſes the water to remain in a liquid ſtate till it be raiſed to the temperature of 212°. the quantity of caloric in the lower temperatures being inſufficient to overcome the preſſure of the atmoſphere.

Whence it appears, that without this atmoſpheric preſſure we ſhould have no permanent liquid, nor even any aeriform fluids, becauſe the moment the force of attraction was overcome by the repulſive power of the caloric, the particles would ſeparate themſelves indefinitely, having nothing to give limits to their expanſion, unleſs their own gravity might collect them together. The truth of theſe poſitions will be made evident by the following experiment. Having filled a ſmall narrow glaſs veſſel with ether, and having properly ſecured it from evaporation by a double piece of wet bladder; place it under the receiver of an air-pump, and exhauſt the air; then, by means of a ſharp pointed wire, let the bladder be burſt. The ether will immediately boil with great violence, and be changed into an elaſtic aeriform fluid, which fills the receiver. If the quantity of ether be ſufficient to leave a few drops in the phial after the evaporation is finiſhed, the elaſtic fluid produced will ſuſtain the mercury in the barometer attached to the air-pump, at eight or ten inches in winter, and from twenty to twenty-five in ſummer. If a thermometer be introduced into the phial, containing the ether, it will deſcend conſiderably during the evaporation.

In this experiment the weight of the atmoſphere, which, in its ordinary ſtate, preſſes on the ſurface of the ether, is taken away, and the effects, reſulting from this removal, evidently prove, that, in the common temperature of the earth, ether would always exiſt in an aeriform ſtate, but for the preſſure of the atmoſphere.

The ſame experiment ſucceeds with all evaporable fluids, ſuch as alkohol, water, and even mercury; with this difference, that the atmoſphere formed in the receiver by the alkohol only ſupports the attached barometer about one inch in winter, and about four or five in ſummer: that formed by water, in the ſame ſituation, raiſes the mercury only a few lines, and that by quickſilver but a few fractions of a line.

The method of making experiments with permanently elaſtic fluids, or gaſes, though ſimple, requires ſome deſcription. We live immerſed in an atmoſphere not greatly differing in denſity from theſe fluids, which are not, for that reaſon, ſufficiently ponderous to be detained in open veſſels by their weight. Their remarkable levity, however, affords a method of confining them by means of other denſer fluids. Dr. Prieſtley, whoſe labours in this department of ſcience have ſo far exceeded thoſe of his predeceſſors, and contemporaries, both in extent and importance, that he has, with great juſtice, been conſidered the father of this branch of natural philoſophy, makes uſe of the following apparatus.

A, repreſents a wooden tub, K, K, is a ſhelf fixed in the tub. [Plate 13. Fig. 17.] When this apparatus is uſed, the tub is to be filled with water to ſuch an height, as to riſe about one inch above the upper ſurface of the ſhelf. B, G, F, are glaſs jars, inverted with their mouths downwards, reſting upon the ſhelf. If theſe, or any other veſſels, open only at one end, be plunged under water, and inverted after they are filled, they will remain full, provided their mouths be kept immerſed. For in this caſe the water is ſuſtained by the preſſure of the atmoſphere on the ſame principle as the mercury in the barometer.

If common air, or any other fluid reſembling common air in lightneſs and elaſticity, be ſuffered to enter theſe veſſels, it muſt, from its levity, riſe to the upper part, and the water will ſubſide. Let a bottle, cup, &c. in that ſtate which is uſually called empty, be plunged into the water with its mouth downwards, and but little water will enter, owing to the elaſticity of the included air; if the veſſel be now turned up, it immediately fills, and the air eſcaping riſes in one or more bubbles to the ſurface. Suppoſe this operation to be performed under one of the jars which are filled with water: the air will aſcend as before, but, inſtead of eſcaping, it will be detained in the upper part of the jar. When the receiving veſſel has a narrow neck, the air may be poured through a glaſs funnel.

C is a glaſs veſſel whoſe bottom is blown very thin, that it may ſupport the heat of a candle, or ſpirits of wine lamp, ſuddenly applied, without cracking. [Plate 13. Fig. 17.] In its neck is fitted a tube D, curved nearly in the ſhape of an S. This kind of veſſel is very uſeful in various chemical operations, for which reaſon it will be convenient to have them of ſeveral ſizes. In the figure the veſſel C is repreſented as containing a fluid in the act of combining with a ſubſtance that gives out air, which paſſes through the tube into the jar B, under whoſe mouth the other extremity of the tube is placed by means of a ſmall hole in the ſhelf.

At E is a ſmall retort of glaſs, or earthen ware, whoſe neck being plunged in the water, beneath the jar F, is ſuppoſed to emit the elaſtic fluid, extricated from the contents of the retort, which is received in the jar.

In order to expel air from ſolid ſubſtances by means of heat, a gun-barrel, with the touch-hole ſcrewed up and rivetted, may be uſed inſtead of an iron retort. The ſubject may be put into the chamber of the barrel, and the reſt of the bore may be filled with dry ſand that has been well burned, to expel whatever air it might have contained. The ſtem of a tobacco-pipe, or a ſmall glaſs tube, being luted in the orifice of the barrel, the other extremity muſt be put into the fire that the heat may expel the contents. This air will of courſe paſs through the tube, and may be received under an inverted veſſel in the manner already deſcribed.

A more accurate method of procuring air from ſeveral ſubſtances, by means of heat, is to put them, if they will bear it, into phials of quickſilver, with the mouths inverted in the ſame, and then throw the focus of a burning lens, or mirror, upon them.

Many kinds of air eaſily combine with water, and therefore require to be treated in an apparatus in which mercury is made uſe of inſtead of water.

When trial is to be made of any kind of air, whether it be fit for maintaining combuſtion, the air may be put in a long narrow glaſs veſſel, whoſe mouth being carefully covered may be turned upward. A bit of wax candle being faſtened to a wire, which is ſo bended as that the flame of the candle may be uppermoſt, is to be let down into the veſſel, which muſt be kept covered till the inſtant of plunging the lighted candle in the air.

Where the change of dimenſions, which follows from the mixture of ſeveral kinds of air is to be aſcertained, a graduated narrow veſſel AB may be made uſe of. [Plate 13. Fig. 16.] The graduations may be made by pouring in ſucceſſive equal meaſures of water into this veſſel, and marking its ſurface at each addition. The meaſure may be afterwards uſed for the different kinds of air, and the change of dimenſions will be ſhewn by the riſe or fall of the mercury, or water in the tube AB. The purity of common air being determinable by the diminution produced by the addition of nitrous air, (hereafter to be deſcribed) theſe tubes have been called *eudiometers*. For the conſtruction of ſeveral other kind of eudiometers, ſee Cavallo on Air.

The proceſs of impregnating water with any aerial fluid with which it will combine, does not require any particular apparatus, but may be performed with utenſils every where to be met with. Water may be impregnated with fixed air in the following manner. A quart bottle C is filled with water, and inverted into the baſon F, which likewiſe contains ſome water. [Plate 13. Fig. 18.] The inverſion is eaſily made, without the loſs of any of the contents of the bottle, by covering the orifice with a card to be withdrawn after immerſion. A is a half pint phial, into which ſome broken pieces of marble or chalk are put; and upon them is poured as much vitriolic acid, diluted with water, as may fill the bottle about two-thirds. B is a bladder, whoſe neck is tied faſt round a perforated cork of a tapering figure. After the efferveſcence has begun, the cork is to be thruſt into the neck of the phial A, the bladder being previouſly emptied by preſſure. Fixed air will eſcape from the chalk, and inflate the bladder. When this laſt is full, it muſt be diſengaged from the bottle, and the bended tube E muſt be thruſt into the orifice of its cork. The aperture of the tube being then placed beneath the mouth of the bottle C, it is eaſy to diſcharge the aerial contents of the bladder of preſſure into this laſt. By agitating of the bottle C, without withdrawing its neck out of the water, the abſorption takes place in a few ſeconds. Two or three repetitions of this proceſs, impregnates the water ſo fully that it will abſorb no more in this kind of way. The Pyrmont water is of this kind.

SECT. II.—*Of* INFLAMMABLE AIR.

This gas is, by the French chemiſts, termed hydrogenous gas, from the ſuppoſition, that its baſe, which they call hydrogene, is one of the conſtituent principles of water. Its property of burning, in conjunction with reſpirable air, has cauſed it to be diſtinguiſhed by the name of inflammable air. It may be extracted by means of heat, from all bodies in which it is a conſtituent part, but the pureſt is ſaid to be that which is afforded by the decompoſition of water. For this purpoſe, diluted vitriolic acid is poured upon iron, orzinc, the water which ſerves as a vehicle for the acid is decompoſed on the metal, with which its oxigene* combines, while the inflammable air eſcapes. Water may be decompoſed by throwing it upon iron ſtrongly heated ; and inflammable air is obtained by cauſing water to paſs through an iron tube ignited to whiteneſs. It ſhould be obſerved, that the two parties into which chemiſts are divided do not agree in their theories on theſe facts ; the one, with Dr. Prieſtley at their head, contends that the air comes from the metal ; the other, that it is really a conſtituent part of the water. Inflammable air may be obtained in conſiderable quantities from all ponds, and other ſtagnant waters. The method is as follows :—Fill a wide-mouthed bottle, (a common bottle with a funnel will anſwer the ſame end) and keep it inverted in the pond ; then ſtir, with a ſtick, the mud at the bottom juſt under the inverted bottle, ſo as to let the bubbles of air which come out of the mud enter the bottle, which air is inflammable. The bottle muſt be well ſtopped, or corked, while under water, after it may be conveyed to any diſtance : it will be the ſafeſt way to carry it in an inverted poſition, on account of the levity of the air.

The properties of inflammable gas, are, 1. It has a diſagreeable ſtinking odour. When it is extracted over mercury, it has ſcarcely any ſmell. It contains half its weight of water, and loſes its ſmell the moment it is deprived of this additional ſubſtance. The volume of gas is $\frac{1}{8}$th larger when received over water, than when received over mercury. Hence it appears, that the offenſive ſmell of this gas ariſes only from the water it holds in ſolution.

2. Inflammable air is not proper for reſpiration. Birds, ſucceſſively placed in a veſſel of this gas, died without producing the ſmalleſt change in the gas itſelf. From ſome experiments made by M. Chaptal, he concludes, that inflammable air, though unfit for the purpoſes of reſpiration, is not a poiſon : for he reſpired ſeveral times the ſame volume of this air, without injury, and without any change in the gas. He conſiders the lungs as an organ which is nouriſhed by the air, digeſts that which is preſented to it, retaining the beneficial, and rejecting the noxious parts.

3. Inflammable air is not combuſtible alone. If a phial filled with this gas be reverſed, and a lighted taper be preſented to it, the gas will burn at the ſurface where it is in contact with the atmoſpheric air, but the candle is extinguiſhed the moment it is plunged lower. Hence the reaſon of the following phenomenon :—If, in hot climates, the mud at the bottom of a pond be well ſtirred, and immediately after, a lighted candle is brought to the ſurface of the water, the inflammable air taking fire, a flame is inſtantly ſpread over the ſurface of the pond. Hence alſo ſeveral meteors ſeen in the atmoſphere ;—ignes fatui ;—falling ſtars, &c. have been ſuſpected to be the effects of inflammable air fired by electricity.

4. Inflammable air is about 12 times lighter than common air ; but its ſpecific gravity varies according to its purity, and the ſubſtances from which it is obtained. This levity has induced ſome philoſophers to preſume that inflammable air ought to arrive at, and occupy the ſuperior parts of our atmoſphere. Upon this ſuppoſition, the moſt brilliant conjectures have been made reſpecting the influence which a ſtratum of this gas, predominating over the reſt of the atmoſphere, ought to produce in meteorology. This continual loſs of matter is not agreeable to the wiſe economy of nature. Beſides, this gas, during its aſcent in the air, combines with other bodies, eſpecially the purer parts of the atmoſphere, and water and other products are the reſult.

Hence the theory of air balloons. In order that a balloon may riſe in the atmoſphere, it is ſufficient that the weight of the balloon itſelf, and the air which it encloſes, ſhould be leſs than that of an equal bulk of atmoſpheric air, and then, by a well known property of fluids, it muſt riſe till its weight is in equilibrio with an equal volume of ſurrounding air.

5. Inflammable air exhibits various characters, according to its degree of purity, and the nature of the ſubſtances with which it is mixed. That afforded by vegetables contains aerial acid, or fixed air, and oil. That of marſhes is mixed with a greater or leſs quantity of aerial acid. Its colour varies alſo according to its mixtures.

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* See the next Section.

6. It will diſſolve ſulphur, and thus form *hepatic* gas ; the principal properties of this gas, are, (1.) It is improper for reſpiration. (2.) It renders white metals black. (3.) It imparts a green colour to the ſyrup of violets. (4.) It burns with a blue flame, and depoſits ſulphur in combuſtion. (5.) It mixes with the vital air of the atmoſphere, and forms water, while the ſulphur is precipitated. (6.) It impregnates water, and is ſparingly ſoluble in the fluid.

7. The air which burns at the ſurface of certain ſprings, and forms what is known by the name of burning ſprings, conſiſts of inflammable air holding phoſphorus in ſolution.

SECT. III.—*Of* DEPHLOGISTICATED *or* VITAL AIR.

This air is called, in the new nomenclature of Lavoiſier, oxigene, or oxigenous gas. It was firſt diſcovered by Dr. Prieſtley on the firſt of Auguſt, 1774, and called by him dephlogiſticated air. A few months after, and without any communication with Dr. Prieſtley, it was diſcovered by Mr. Scheele, of Sweden, who called it empyreal air.

There is no place yet diſcovered upon the ſurface of the earth, where vital air is found naturally like inflammable air ; but it may be obtained from ſeveral ſubſtances, and by various proceſſes. Pure vital air may be obtained by the expoſure of nitre to a ſtrong degree of heat. About 1200 cubic inches of this air may be obtained from a pound of this ſalt. All the metallic calxes, when moiſtened with nitrous acid and heated, yield vital air. An ounce of red precipitate affords about a pint meaſure of this gas. All the mineral acids have vital air for their baſe, and ſome of them yield it freely. Vegetables, expoſed to the light of the ſun, emit vital air, and the quantity thus obtained is in proportion to the vigour of the plant, and the brilliancy of the light.

Exp. Encloſe three or four cabbage-leaves, taken freſh from the plant, in a glaſs receiver filled with water, and inverted in a veſſel of the ſame fluid, and let the whole be expoſed to the rays of the ſun. Small bubbles of air will be formed on the leaves, and detaching themſelves, riſe to the upper part of the veſſel and diſplace the water.

Vegetation inceſſantly repairs the conſumption of vital air. The plant abſorbs the atmoſpherical azote,* and emits vital air. Man, on the contrary, is kept alive by the vital air, and emits the azote. Hence it appears, that the animal and vegetable kingdoms labour for each other, and that by this reciprocity of ſervices the atmoſphere is continually repaired, and an equilibrium maintained between its conſtituent principles.

I. The influence of the ſolar light has the property of decompoſing certain ſubſtances, and thereby diſengaging this gas.

Exp. A bottle of oxigenated muriatic acid expoſed to the light of the ſun, ſuffers all its ſuperabundant oxigene to eſcape. The ſame acid expoſed to the ſun in a bottle wrapped in black paper, ſuffers no change. The nitric acid likewiſe affords vital air, when expoſed to the light of the ſun, whereas heat alone volatilizes it without decompoſition. The muriate, or marine ſalt of ſilver, placed under water, and expoſed to the ſun, emits vital air. So alſo will red precipitate.

II. Vital air may be obtained by diſengaging it from its baſes by means of the vitriolic acid.

Exp. Take the phial C, into which put an ounce or two of manganeſe, formed into a liquid paſte with vitriolic acid. [Plate 13. Fig. 17.] Then place it under the ſhelf, as in the figure, and apply a lighted wax taper, or live coal, to the lower part of the phial, and vital air will be immediately diſengaged.

This air poſſeſſes properties which vary according to the ſeveral ſubſtances from which it is obtained. That which is got from the calx of mercury, almoſt always holds a ſmall quantity of that metal in ſolution. Mr. Chaptal was witneſs to its having produced a ſpeedy ſalivation on two perſons who uſed it for diſorders of the lungs.

That which is extracted from plants is thought not to be equally pure with that afforded by the metallic calxes. I have, however, ſeen vital air, extracted by Dr. Ingenhouz, from freſh gathered cabbage-leaves, with which he performed ſeveral brilliant experiments. The Doctor threw away the air which was generated during the firſt 15 or 20 minutes. That which was afterwards produced, poſſeſſed very great purity ; but from whatever ſubſtance it is obtained, its general properties are the following :

1. The ſpecific gravity of vital air, is to that of atmoſpherical as 11 to 10.

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* See Section VI.

2. It is the only fluid proper for combuſtion, which was the reaſon why Mr. Scheele called it the *air of fire.* The four following principles are conſidered as inconteſtable reſults of known facts. (1.) Combuſtion never takes place without vital air. (2.) In every combuſtion there is an abſorption of vital air. If phoſphorus, or ſulphur, be burned in very pure vital air, the whole of it will be abſorbed. When the combuſtion is effected in a mixture of ſeveral gaſes, the oxigene alone is abſorbed, and the others remain unchanged.

Exp. The combuſtion of ſulphur, or phoſphorus, may be thus effected: AB is a bell-glaſs filled with vital air, ſtanding in the veſſel CD, which contains mercury. [Plate 13. Fig. 20.] F is a ſmall ſtand, upon the top of which, E, may be placed ſome ſulphur, or phoſphorus, conveyed under the glaſs through the mercury. By means of the lens G, about 3 inches in diameter, the ſubſtance is eaſily inflamed by the heat from the rays of the ſun. During the combuſtion, there is a conſiderable abſorption of vital air; and when phoſphorus is the ſubſtance uſed, the whole inſide of the bell-glaſs becomes covered with light flakes of concrete phoſphoric acid.

The combuſtion of phoſphorus ſucceeds equally well in atmoſpheric air, only that it goes on much ſlower, and the abſorption is but about $\frac{1}{5}$th part of the gas employed. (3.) There is an increaſe of weight in the products of combuſtion equal to the weight of the vital air abſorbed.

Exp. During the calcination of metals, an abſorption of vital air takes place, and an increaſe of weight in the metal, equal to the weight of air abſorbed. Thus, if the metal to be calcined be placed in a ſmall porcelain cup on the ſtand FE, under the glaſs receiver AB, filled with air, in the baſon of mercury DC, and the focus of the burning-glaſs G be made to fall upon the metal; in a few minutes calcination takes place, a part of the vital air is combined with the metal, and a proportional diminution of the volume of air is produced; that which remains is azotic gas, mixed with a ſmall quantity of vital air. [Plate 13. Fig. 20.]

When pure vital air is uſed, its weight may be aſcertained, and alſo the weight of the metal both before and after calcination, and it will be found that during the experiment, the metal has increaſed in weight equal to the quantity of air abſorbed.

The experiment of Dr. Ingenhouz, deſcribed below, is a true calcination of iron. (4.) In all combuſtion there is a diſengagement of heat and light; for in moſt caſes of this kind, the oxigenous gas becomes fixed and concrete. It therefore abandons the caloric which maintained it in the aeriform ſtate, and this caloric, ſet at liberty, produces heat, and endeavours to combine with the ſubſtances neareſt at hand.

This diſengagement of heat is, therefore, a conſtant effect in all caſes wherein vital air is fixed in bodies; and it follows from this principle: (1.) That heat is moſt eminently reſident in the oxigenous gas which maintained combuſtion. (2.) That the more oxigene is abſorbed in a given time, the ſtronger will be the heat. (3.) The method of obtaining the moſt violent heat conſiſts in burning bodies in the pureſt air. (4.) That fire and heat muſt be more intenſe in proportion as the air is more condenſed. (5.) That currents of air are neceſſary to expedite and maintain combuſtion.

Hence the theory of Argand's lamp. This lamp, by means of a thin circular wick, through the middle of which a current of air is introduced by a funnel, produces a very thin flame, and conſequently expoſes a very large ſurface of the oily vapour to the contact of the air. As there is, however, a ſtrong attraction between the particles of fire, there would be danger of the flame uniting from all the ſides of the lamp, at a certain height above the funnel, and ſo forming a conical flame, was it not that this effect is prevented by a tube of glaſs with which the flame is ſurrounded, and which, when warmed, counteracts the attraction that the different ſides of the circular flame would have for each other, and ſo preſerves the current of air free and without interruption.

Exp. 1. A profeſſor of Gottingen, ſoldered the blade of a knife to a watch ſpring by means of oxigenous gas. M. Lavoiſier has ſubjected almoſt all known bodies to the action of fire maintained by vital air alone, and produced effects ſuperior to thoſe of the burning glaſs.

2. The following is Dr. Ingenhouz's elegant experiment upon the combuſtion of iron. Take a piece of very fine iron wire twiſted into a ſpiral BC; fix one of its extremities B, into a cork A, adapted to the neck of the bottle DEFG, and fix to the other extremity of the wire C a ſmall morſel of tinder. [Plate 13. Fig. 19.] Then fill the bottle with vital air, and lighting the tinder, introduce it quickly into the bottle, which you ſtop with the cork A. The inſtant the tinder comes into contact with the vital air, it begins to burn with great intenſity, and communicating the inflammation to the iron wire, it alſo takes fire and burns rapidly, throwing out brilliant ſparks, which fall to the bottom of the phial in round globules. This experiment I have frequently ſeen performed by Dr. Ingenhouz, with the vital air obtained from cabbage-leaves.

3. The light of glow-worms is ſo bright in oxigenous gas, that a ſingle inſect affords light ſufficient to read a print of a very ſmall character.

In the act of combuſtion there are three ſtates in which bodies appear; viz. ignition, inflammation, and detonation. (1.) *Ignition,* which takes place when the combuſtible body is not in an aeriform ſtate, nor ſuſceptible of aſſuming that ſtate, by the ſimple heat of combuſtion. This happens when well made charcoal is burned. (2.) When the combuſtible body, in the form of vapour, or gas, is preſented to pure air, the reſult is *flame* ; and the flame is more conſiderable in proportion as the combuſtible body is more volatile. The flame of a candle is kept up by the volatilization of the tallow, or wax. (3.) *Detonation* is a ſpeedy and rapid inflammation which occaſions a noiſe by the inſtantaneous formation of a vacuum. Moſt detonations are produced by the mixture of pure and inflammable airs. A very ſtrong detonation may be produced by a mixture of one part of vital air, and two of inflammable.

Oxigenous gas is the only gas proper for reſpiration, which entitles it to the name of *vital* air. It has been long known that animals cannot live without the help of air ; but the phenomena of reſpiration were but imperfectly underſtood till the preſent age. The following principles may be conſidered as well-eſtabliſhed facts :

1. No animal can live without the aſſiſtance of air. This fact is univerſally admitted.

2. All animals do not require the ſame purity in the air. Birds, men, and the greateſt part of quadrupeds, require a very pure air ; but thoſe animals which live in the earth, or which hide themſelves in a ſtate of ſtupefaction during the winter, can ſubſiſt by means of a leſs pure air.

3. The manner of reſpiring air is different in different animals. Amphibious animals reſpire by means of lungs ; fiſhes come from time to time to inhale the air at the ſurface of the water ; when they fill their veſicle, and digeſt it afterwards at their eaſe. Reſpiration is effected in inſects by means of trachæ diſtributed along the body. Theſe inſects exhibit ſeveral very evident points of analogy with vegetables. (1.) Their reſpiratory organs are formed in the ſame manner, being diſpoſed through the whole body of the vegetable and animal. (2.) Inſects do not require a great degree of purity in the air, and plants are nouriſhed with atmoſpherical azote. (3.) They both tranſpire vital air. The abbé Fontana diſcovered ſeveral inſects in ſtagnant waters, which when expoſed to the ſun afforded vital air. Dr. Ingenhouz found the green matter, which is formed in ſtagnant waters, to be a maſs of animalcula, which, when expoſed to the ſun, afforded a large quantity of vital air. (4.) Inſects afford, by chemical analyſes, principles ſimilar to thoſe of plants, ſuch as reſines, volatile oils, &c.

Animals with lungs, only reſpire by virtue of the vital air which ſurrounds them : any gas deprived of this mixture becomes immediately unfit for reſpiration. From a number of experiments it appears, (1.) That an animal lives longer in vital than in atmoſpherical air. (2.) That an animal can live in the air in which another died. (3.) That independently of the nature of the air, reſpect muſt be had to the conſtitution of the animals. (4.) That there is either an abſorption of air, or the production of a new kind of gas, which is abſorbed by the water as it riſes.

EXP. Theſe facts may be aſcertained by the following method : In the bell-glaſs AB, filled with vital air, and ſtanding in a veſſel of water, or mercury, DC, introduce a ſmall animal, as a mouſe, bird, &c. which may be done without injury to the animal. [Plate 13. Fig. 20.] Hence may be aſcertained the effects of different kinds of air upon the creatures immerſed ; the quantity of air abſorbed by reſpiration, &c. &c.

III. It remains now to enquire, what are the changes produced by reſpiration. 1. In the air. 2. In the blood.

1. The air emitted by expiration, is a mixture of azotic gas, carbonic acid, and vital air. If the air which iſſues from the lungs be made to paſs through lime-water, it renders it turbid ; if it is received through tincture of turnſole, it reddens it : and if a pure alkali be ſubſtituted inſtead of the tincture of turnſole, it becomes efferveſcent. Frugivorous animals vitiate the air leſs than carnivorous animals. A portion of air is abſorbed in reſpiration, which, by the experiments of M. de la Metherie, has proved to be about 360 cubic inches in an hour. This fact affords a proof of the facility with which air is vitiated by reſpiration, when it is not renewed, and ſhews why the air of crowded places is ſo unwholeſome.

2. The firſt effect which the air appears to produce upon the blood, is that of giving it a vermillion colour. The air which returns from the lungs is of a higher colour. Hence ariſes the great intenſity of the colour of arterial compared with venous blood. Dr. Prieſtley cauſed the blood of a ſheep to paſs ſucceſſively into vital air, common air, azotic air, &c. and he found that the blackeſt parts aſſumed a red colour in reſpirable air, and that the intenſity of this colour was in proportion to the quantity of vital air preſent. The reſult of many experiments is, that the vermillion colour of blood is the effect of the contact, abſorption, and combination of pure air with the blood.

The ſecond effect of reſpiration is, to eſtabliſh a real focus of heat in the lungs. The heat in each claſs of animals is proportioned to the magnitude of the lungs ; for there is an abſorption of vital air in reſpiration. Reſpiration then may be conſidered as an operation, by means of which vital air paſſes continually from the gaſeous to the concrete ſtate; it muſt, therefore, at each inſtant abandon the heat which held it in ſolution, and in the ſtate of gas. This heat, produced at every inſpiration, muſt be proportioned to the volume and activity of the lungs; to the purity of the air; and the rapidity of the inſpirations.

The neceſſity of vital air in reſpiration, enables us to lay down poſitive principles for purifying the corrupted air of any given place, which may be done, (1.) By correcting the vitiated air by means of ſubſtances which are capable of ſeizing the noxious principle. (2.) By ſubſtituting freſh air, by means of ventilators, &c. (3.) By introducing a new quantity of vital air into the vitiated atmoſphere.

SECT. IV.—*Of* AERIAL ACID, *or* FIXED AIR.

The acid which is known by the names of *fixed air*, *aerial*, or *carbonic acid*, is found in great quantities in nature, and in three different ſtates. To Dr. Black we are indebted for the principal diſcoveries reſpecting this gas.

I. It is found in a ſtate of gas.—Fixed air being conſiderably heavier than common air, occupies the lower parts of ſuch caverns, &c. as contain materials from which it is produced by decompoſition. On account of its direful effects, it has been called by miners the *choke damp*. In this ſtate it is found in the Grotto del Cano, near Naples, which has been long famous on account of a ſtratum of fixed air which covers its bottom, into which, if a dog, or other animal that holds its head near the ground, be thruſt, it is immediately ſuffocated, though a man in the erect poſture, may venture into the cave with ſafety: hence the origin of its name. This gas is alſo found upon the ſurface of the lake Averno, in Italy; on the ſurface of ſeveral ſprings, and in ſeveral ſubterraneous places, ſuch as tombs, cellars, &c. It is diſengaged alſo in this form by the decompoſition of vegetables heaped together; by the putrefaction of animal ſubſtances; it is alſo emitted in large quantities by bodies in a ſtate of vinous fermentation; and on account of its ſpecific gravity, always occupies the upper part of the veſſels in which the fermenting proceſs is going on.

EXP. 1. A lighted paper, or candle, dipped into fixed air, is immediately extinguiſhed; and the ſmoke renders its ſurface viſible, which may be thrown into waves by agitation.

2. Water immerſed in this air, and briſkly agitated, ſoon becomes impregnated, and obtains the taſte of Pyrmont water.

3. This gas may be poured from one veſſel to another, in which operation a very ſingular phenomenon may be obſerved: if a candle, or ſmall animal, be placed in a deep veſſel, the former is extinguiſhed, and the latter expires in a few ſeconds after the fixed air is poured upon them, though the eye cannot perceive any thing that is tranſferred from one veſſel to the other.

When fixed air exiſts in a ſtate of gas, it may be collected; (1.) By filling a bottle with water, and emptying it into the atmoſphere of this gas. (2.) By expoſing lime-water, for which it has a great attraction, in its atmoſphere, the gas precipitates the lime, and combines with the water, from which it may afterwards be extracted.

From the eagerneſs with which lime-water imbibes fixed air, it has been recommended as uſeful in purifying the foul air in hoſpitals. The method conſiſts ſimply in placing at different diſtances in the hoſpital wards, veſſels with lime-water for the purpoſe of abſorbing the fixed air.

The following experiment has been lately ſuggeſted as the beſt means of aſcertaining the quantity of fixed air in any given place.

EXP. Take two phials; let the one be filled with common water, the other with lime-water. At the place where you wiſh to examine the purity of the air, empty the phial of common water, then, filling it half full of lime-water, and corking it, ſhake the phial for ſome time: the quantity of ſediment ſhews the proportion of fixed air.

II. Fixed air exiſts in a ſtate of mixture, as in certain mineral waters. In this ſtate it may be obtained, (1.) By agitating the liquid which contains it in a bottle, to which is affixed a moiſtened bladder. (2.) By diſtillation of the ſame fluid. (3.) By precipitating the fixed air by means of lime-water, weighing the precipitate, and deducting $\frac{1}{3}\frac{3}{2}$ parts for the proportion of fixed air; it having been deduced by analyſis, that 32 parts of chalk contain 17 parts of lime, 2 of water, and 13 of fixed air.

III. Fixed air is found in a ſtate of combination. It compoſes about one-third of the weight of lime-ſtone, marble, and ſome other calcareous earths.

The artificial methods of obtaining fixed air from thoſe ſubſtances in which it exiſts in a ſtate of combination, are principally three. (1.) By fermentation. Dr. Hales obtained 639 cubic inches of fixed air from 42 cubic inches of beer. Mr. Cavendiſh found that out of 100 parts of dry ſugar, 57 were converted into fixed air. (2.) Fixed air may be obtained by the ſimple application of heat. Dr. Prieſtley, in order to obſerve the proportion of fixed air contained in different ſorts of wine, filled phials with ſeveral ſpecies of it, and then plunged them into a veſſel of water ſet over the fire to boil, receiving the gas after the uſual method, into a receiver filled with, and inverted in a baſon of quickſilver. (3.) Fixed air in the ſtate of combination, is obtained by the ſuperior affinity of ſome other acid, as the vitriolic ; for moſt of the acids have a ſtronger action on bodies than this.

Exp. 1. Put ſome chalk or marble groſsly powdered into the bottle C, and pour upon it ſome vitriolic acid diluted with water, in proportion of about 4 parts of water to one of acid. [Plate 13. Fig. 17.] The mixture will effervеſce, heat will be produced, and fixed air will be copiouſly emitted, which paſſing through the bended tube will go into the bottle B.

2. Mr. Cavallo gives the following method of obtaining fixed air in conſiderable quantities. In the bottle C, mix equal parts of brown ſugar, and good yeaſt of beer, to which add about twice the bulk of water. This mixture expoſed to a degree of heat not exceeding 90°. will yield fixed air plentifully.

It is certain that fixed air is an acid ; for (1.) The tincture of turnſole, agitated in a bottle with this gas, becomes red. (2.) Ammoniac, or volatile alkali, poured into a veſſel filled with this gas, becomes neutralized. (3.) Water impregnated with this gas, is ſtrongly ſub-acid. (4.) It neutralizes alkalies, and cauſes them to cryſtallize.

The properties of fixed air, are,

1. It is unfit for reſpiration. It is this gas which produces the many accidents at the opening of cellars, in places where wine, cider, or beer are ſuffered to ferment. Birds plunged into the aerial acid gas, ſuddenly periſh. The famous lake of Averno, where Virgil placed the entrance of hell, exhales ſo large a quantity of fixed air, that birds cannot fly over it with impunity. Frogs will live in fixed air from 40 to 60 minutes, by ſuſpending their reſpiration. Inſects become torpid after remaining a certain time in this air, but they reſume their livelineſs the moment they are expoſed to the free atmoſphere.

2. Fixed air is improper for vegetation. Dr. Prieſtley having kept the roots of ſeveral plants in water impregnated with fixed air, obſerved that they all periſhed. Plants that grow in water ſlightly acidulated with this gas, emit a large quantity of vital air ; for, in this caſe, the acid is decompoſed, the carbonaceous principle combines with the vegetable, while the vital air eſcapes. *Fungi*, formed in ſubterraneous places, are almoſt wholly reſolved into acid ; but if theſe vegetables be gradually expoſed to the action of the light, the proportion of the acid diminiſhes, while that of the coaly principle augments, and the vegetable becomes coloured.

3. Fixed air is eaſily diſſolvable in water. Water impregnated with this gas poſſeſſes very valuable medicinal qualities. A method of facilitating this mixture has been already mentioned ; but the moſt approved apparatus for the purpoſe, is that invented by Dr. Nooth, and improved by Parker. The natural acidulous mineral waters do not differ from theſe, excepting in conſequence of their holding other principles in ſolution, and they may be perfectly imitated when their analyſis is known.

4. Fixed air is heavier than the common atmoſpheric air in the proportion of about 68 to 45. This weight cauſes it to occupy the loweſt ſituations, and gives it the property which has been deſcribed, of being poured, like other fluids, from one veſſel to another.

5. It reſiſts putrefaction. Hence it is that ſubſtances in which this gas abounds, are very powerful remedies in all putrid diſeaſes. Sir John Pringle ſuppoſed that the frequent uſe of ſugar, and freſh vegetables, which at this time make up a conſiderable part of the diet of the European nations, prevents thoſe putrid diſeaſes and plagues which were formerly more frequent. Dr. Macbride aſcribes the preſervation of the body from putrefaction, in a great meaſure to the fixed air, which in the ordinary proceſs of digeſtion is diſengaged from the aliment, and incorporates with the fluids of the body.

S E C T. V.—*Of* NITROUS AIR.

Nitrous air, though in ſome meaſure known to Dr. Hales, was properly diſcovered by Dr. Prieſtley. It is a permanently claſtic fluid, never found naturally like the fixed and inflammable airs, but is entirely artificial. This fluid is diſengaged by the action of the diluted nitrous acid on a great number of combuſtible bodies.

All metallic ſubſtances, excepting gold, plaitna, and regulus of antimony, when mixed with diluted nitrous acid, yield nitrous air. Silver, copper, braſs, iron, mercury, biſmuth, or nickel, give out this air in conſiderable quantities. Some of them, as mercury, require the aid of heat ; as the flame of a candle, in order to produce the claſtic fluid.

Exp. 1. Nitrous air is obtained by putting ſome ſmall pieces or filings of copper, braſs, &c. in the bottle C together with ſome diluted nitrous acid ; the proportion of acid to water muſt be as 1 to 2 or 3. [Plate 13. Fig. 17.] The ſmell of nitrous gas is very penetrating and offenſive ; it occaſions a red ſmoke when it comes into contact with the air. 2.

2. Silver, copper, and iron, diſſolved in equal quantities of diluted nitrous acid, yield nitrous gas in the proportions of 6, $6\frac{1}{4}$, and 8. The ſame proportions muſt be obſerved in regard to the water and acid, in order that the diſſolution of the metal may be effected with equal rapidity, ſilver requiring the leaſt water, and iron the moſt. As nitrous air is chiefly uſed to aſcertain the purity of reſpirable air, it is of conſequence that it ſhould be always of one degree of goodneſs; and this object is obtained by diſſolving ſubſtances of a homogenous nature in nitrous acid of the ſame degree of ſtrength. Metals. therefore, whoſe nature is moſt uniform, muſt be preferred for this purpoſe. Accordingly, braſs yields nitrous air of a more uniform nature than iron, copper is ſuperior to braſs, and pure mercury is ſtill ſuperior to copper; and upon all accounts this metal ſeems the moſt proper for the purpoſe.

The properties of nitrous air are as follows:

I. The moſt remarkable property of this kind of air is, the great diminution of any quantity of reſpirable air with which it is mixed. It is found that this diminution is greater, in proportion as the reſpirable air is better adapted to the purpoſes of combuſtion and animal life. Hence, then, the method of aſcertaining the purity of atmoſpheric air, by adding to a given quantity of it, an equal bulk of nitrous air, and obſerving the ſpace occupied by both after the mixture.

EXP. If the tube AB be filled two-thirds with water, and one-third with the air to be examined, and then an equal bulk of nitrous air be added to that already in the tube, it will be found that the ſpace occupied by both after mixture, will not be equal to the ſum of the ſpaces occupied by both airs ſeparately. [Plate 13. Fig. 16.] If the equal meaſures of common and nitrous airs be diminiſhed $\frac{7}{10}$ of a meaſure, the meaſure of the *teſt* is 1.3, which number denotes the reduced bulk of the air which was originally two.

Pure vital air will admit of the addition of three times its own bulk of nitrous air, without ſenſibly increaſing the ſpace which it occupies. Thus if AB be $\frac{1}{4}$ full of vital air, 3 equal meaſures of nitrous air may be added without any viſible augmentation of its bulk.

II. Nitrous air poſſeſſes the power of preſerving animal ſubſtances from putrefaction, and reſtoring thoſe that are already putrid.

EXP. 1. Dr. Prieſtley took two mice, one juſt killed, and the other ſoft and putrid, and put them both into a jar of nitrous air ſtanding in the uſual temperature of the weather in the months of July and Auguſt; and after 25 days he found both the mice perfectly ſweet.

2. In order to compare the antiſeptic power of this kind of air with that of fixed air, Dr. Prieſtley examined a mouſe which he had encloſed in a phial full of fixed air; upon the opening of which, about a month after, he perceived that a large quantity of putrid effluvium had been generated, which ruſhed out of the phial with a ſmell inſufferably offenſive.

Dr. Prieſtley made ſome experiments with a view of rendering nitrous air advantageous for culinary purpoſes, but the reſult did not anſwer his expectations. See Prieſtley on Air, Vol. I. p. 393—7. Ann. 1790.

III. Nitrous air is not fit for the purpoſes of animal or vegetable life. Plants die very ſoon in nitrous gas. A mouſe dies the moment it is put into nitrous air; but frogs, and ſnails, and therefore probably, other animals whoſe reſpiration is not very frequent, will bear being expoſed to it a conſiderable time, though they die at length.

From the fatal effects of this air to inſects, and its great antiſeptic power, Dr. Prieſtley conjectured that it might hereafter be uſed in medicine by way of clyſters, as deſtructive to worms of all kinds, and to correct putrefaction.

IV. It will not be deemed quite foreign to our purpoſe, though not ſtrictly connected with the elaſtic gaſes of which we are treating, to mention a moſt important diſcovery of Dr. Carmichael Smyth, who for a number of years has applied the *vapour* ariſing from the decompoſition of nitre in the vitriolic acid, to the purpoſes of preventing and even deſtroying the contagion of putrid vapours.

Dr. Smyth, after examining the uſual methods employed for deſtroying contagion, ſuch as burning ſulphur, nitre, tobacco; boiling vinegar, with and without camphire, &c. and ſhewing their inefficacy for the purpoſes to which they have been applied, aſſumes as an acknowledged fact, that all the *mineral acids*, particularly when in a ſtate of vapour, with the different gaſes produced by them, have great influence over putrefaction. Dr. Smyth, in conjunction with Mr. Hume, an able chemiſt of Long-acre, in order to aſcertain the efficacy of the vapours ariſing from the mineral acids, and the ſafety with which they may be employed, made various experiments on ſmall animals, and afterwards tried the effects of theſe vapours on themſelves. From all which, he concludes, that the *vapour ariſing from the decompoſition of nitre by vitriolic acid*, is by far the moſt proper to be employed in ſituations where people muſt neceſſarily be preſent; and he entertains no doubt but that it is the *deſideratum* ſo long ſought after for deſtroying infection in places from whence the ſick cannot be removed. His own experience juſtifies the concluſion ; for, having made uſe of it for 16 or 17 years, both in hoſpitals and private practice, he has had the moſt deciſive evidence of its happy effects in preventing the ſpreading or farther communication of contagion. He adds, that "in private practice I can declare with truth, that where the nitrous vapour has been conſtantly uſed as a fumigation, I have not known an inſtance of a contagious fever having been communicated even to a nurſe or an attendant." The Doctor has no doubt of its efficacy in preventing the progreſs even of the plague. The method recommended for obtaining the nitrous vapour is extremely ſimple, and as follows.*

Exp. Put half an ounce of vitriolic acid into a crucible, or into a glaſs, or china cup, or deep ſaucer, warm this over a lamp, or in heated ſand, adding to it from time to time ſome nitre. Theſe veſſels ſhould be placed at 20 or 30 feet diſtance from each other, according to the height of the ceiling, or virulence of the contagion. See "The Effects of the Nitrous Vapour in deſtroying Contagion, &c. &c. by James Carmichael Smyth, M. D."

SECT. VI.—*Of the* Construction *of the* Atmosphere ; *and of* Azotic Air.

The foregoing conſiderations on the formation of different elaſtic aeriform fluids, throw conſiderable light on the conſtruction of the atmoſphere attached to the earth, which may readily be conceived to be a mixture of all bodies ſuſceptible of evaporation, and capable of retaining the elaſtic ſtate, under the uſual preſſure and temperature of the air ; and likewiſe of all thoſe ſubſtances which are capable of being diſſolved by the mixture of different gaſes.

If the temperature of the earth were ſuddenly and materially altered, by being tranſported, for inſtance, to a ſituation not farther from the ſun than that of the planet Mercury, the water, and all fluids with which we are acquainted, would become rarefied and changed into permanently elaſtic fluids, and form part of a new atmoſphere. In proportion, however, to the increaſe of elaſtic fluids, the preſſure of the atmoſphere would be alſo increaſed ; and as every degree of preſſure tends, in ſome meaſure, to prevent evaporation, and as even the moſt evaporable fluids can reſiſt the operation of a very high temperature, if prevented by a proportionally ſtronger compreſſion ; it muſt be admitted, that the new atmoſphere would at laſt arrive at ſuch a degree of weight, that the water which had not hitherto evaporated, would ceaſe to boil, and conſequently remain liquid.

By a contrary ſuppoſition, if the earth were ſuddenly removed into ſome very cold region, as the orbit of Saturn ; water, and probably the greater part of the fluids, would be converted into ſolid rocks, which in time might become opake ſtones of various colours. In this caſe, the air, or at leaſt ſome part of the aeriform fluids which now compoſe the maſs of our atmoſphere, would loſe its elaſticity for want of ſufficient temperature to retain it in that ſtate, and thus new liquids would be formed.

It is generally admitted, that the atmoſphere in which we live is compoſed of two permanently elaſtic fluids ; the one unfit for the purpoſes of combuſtion and animal life, and therefore called azotic air, the other poſſeſſing both theſe properties in a very eminent degree, and, on that account, denominated vital air. Of the latter we have already treated in the third ſection of this chapter. The properties of the azotic gas are, (1.) It is improper for reſpiration and combuſtion. (2.) Plants live in this air, and freely vegetate in it. (3.) This gas mixes with the other airs without combining with them. (4.) It is lighter than atmoſpheric air in the proportion of 985 to 1000. And (5.) when mixed with vital air in the proportion of 72 to 28, it conſtitutes our atmoſphere.

M. Lavoiſier has furniſhed us with methods of determining the nature and before mentioned proportions of theſe gaſes, for, exiſting merely in a ſtate of intimate mixture, they are eaſily ſeparable by any ſubſtance poſſeſſing an elective attraction for either of them. Thus we have ſeen, in the beginning of the third ſection of this chapter, that during the calcination of metals in common air, the pure part was abſorbed, and that which remained was azotic gas, ſuch as we have juſt deſcribed. We ſhall, however, in this place, briefly mention ſome experiments inſtituted by M. Lavoiſier, with the intention of analyſing atmoſpherical air.

Exp. 1. Four ounces of pure mercury were expoſed for ſeveral days, in a matraſs containing atmoſpherical air, to a degree of heat almoſt at the boiling point. On the ſecond day, ſmall red particles began to appear on the ſurface of the mercury, which, during the four of five following days, gradually increaſed in ſize and number. During the experiment, $\frac{1}{6}$th of the bulk of the air was abſorbed, and the red particles collected, were found to amount to 45 grains. The air which remained after the calcination of the mercury, and which was reduced to $\frac{5}{6}$ths of its former bulk, was no longer ſit either for reſpiration or combuſtion.

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* Note. It having been objected that nitrous vapour is highly noxious ; Mr. Keir, a chemiſt of very conſiderable reputation, obſerves, "that the fumes in Dr. Smyth's proceſs are quite different from the ordinary nitrous vapour in the diſtillation of aquifortis, or from that which exhales in the ſolution of metals by nitrous acid the latter is highly ſuffocating and noxious. The vapour made in Dr. Smyth's manner, *(if there is no metal employed in the veſſel*, &c.) is highly dephlogiſticated. and is mixed with a large quantity of pure vital air, which is extricated from the materials. Theſe fumes are not only *not* ſuffocating, but have a very pleaſant ſmell."

EXP. 2. M. Lavoiſier put the 45 grains of red matter formed during the experiment, in a ſmall glaſs retort, which when almoſt red hot, the red matter began gradually to decreaſe in bulk, and in a few minutes after, it diſappeared altogether; at the ſame time $41\frac{1}{2}$ grains of running mercury were collected in the recipient, and 7 or 8 inches of elaſtic fluid, greatly more capable of ſupporting both reſpiration and combuſtion than atmoſpherical air.

It appears, therefore, from theſe experiments, that the mercury during its calcination, that is, at a degree of heat approaching ebullition, poſſeſſes the power of decompoſing atmoſpherical air, and of combining with the pure part, which is accordingly ſeparated from the other conſtituent part, or the azotic gas: conſequently, atmoſpheric air is compoſed of two elaſtic fluids of different and oppoſite qualities. As a proof of this important truth, if the two elaſtic fluids which have been ſeparately obtained in the above experiment, be re-combined, an air is produced preciſely ſimilar, in all its properties, to that of the atmoſphere. By other experiments it has been found that the proportion of the vital air to that of the azotic is as 28 to 72.

The phyſical properties of air are elaſticity; gravity; fluidity; inviſibility; and a negation of ſmell.

Of the elaſticity and gravity of the air we have treated in a former part of this work. We ſhall now proceed with the other properties.

1. The *fluidity* of the air expoſes it to thoſe frequent and repeated motions which conſtitute the winds. The air is not, however, capable of penetrating all bodies. Tranſparent ſubſtances through which light readily paſſes, are ſecure againſt the impulſe of the air. Water, ſolutions of ſalts, &c. paſs through bodies whoſe contexture is ſuch as not to admit air.

2. Air, when confined in veſſels, is abſolutely *inviſible*. It owes this property to the ready paſſage which it affords to the rays of light which are refracted without being reflected by it.

3. Naturaliſts have conſidered the air as an *inſipid* body. Wounds, however, when uncovered, and expoſed to the air, are affected with acute pain. New-born infants expreſs, by their cries, a diſagreeable ſenſation which they ſeem to feel from the firſt contact of the external air. Atmoſpheric air impedes the growth of new bark on ſuch vegetables as have been ſtripped of their covering. Theſe facts would lead one to ſuppoſe that there is a conſiderable pungency in the air.

4. Air is perfectly *inodorous*. If the atmoſphere ſometimes affects our organs with a fetid kind of ſmell, it is to be attributed to the extraneous bodies that are mixed with it, as may be obſerved in miſts and vapours.

SECT. VII.—*Of* WATER, *and of its Compoſition and Decompoſition.*

Water is found in almoſt all natural bodies. It conſtitutes the moſt conſiderable part of animal and vegetable fluids, and even enters into the compoſition of the ſolid parts of ſuch bodies. It may be conſidered in two ſtates;—either in a ſtate of ſimple mixture;—or in a ſtate of combination. In the former caſe, it renders bodies humid;—is perceptible to the eye;—and may be eaſily diſengaged. In the latter, it exhibits no character which ſhews that it is in a ſtate of mixture. In this form it exiſts in cryſtals, ſalts, plants, animals, &c.

Water is ſuſceptible of various forms, as the ſolid, the liquid, and the vaporous.

1. Of water in a ſtate of ice. The converſion of water into ice is attended with theſe phenomena. (1) It is attended with a ſenſible production of heat which eſcapes into contiguous bodies, conſequently the ſpecific heat of ice is leſs than that of the water. (2) Ice occupies a larger ſpace than fluid water.

2. Of water in the liquid ſtate. This is the uſual ſtate in which we ſee water. It enters into combination moſt readily in this form. Water which flows on the ſurface of the earth is never pure. M. Chaptal found that rain water in ſtorms is leſs pure than that of a gentle ſhower;—that the water which falls firſt, is not ſo pure as that which falls after ſeveral days rain;—that the water which is blown from the ſea ſouthward, contains ſea-ſalt; whereas that which is produced by a northerly wind does not contain a particle. By diſtillation, water may be obtained to any degree of purity. The characters of potable water are, (1) A lively, freſh, and agreeable taſte. (2) The property of boiling readily, and alſo that of boiling peas and other pulſe. (3) The virtue of diſſolving ſoap without curdling.

3. Of water in a ſtate of gas. Many ſubſtances are naturally in a ſtate of aeriform fluidity, at the common temperature of our atmoſphere, ſuch as the fixed and inflammable airs. Others evaporate at a degree of heat very near that in which we live. Ether and alkohol are of this kind. Water requires a ſtronger heat.

EXP. In order to convert water into an aeriform fluid, M. Lavoiſier filled a glaſs veſſel with mercury, and reverſed it over a diſh filled with the ſame metal. Two ounces of water were transferred beneath this veſſel, and the mercury heated to about 250°. or 260°. The incloſed water became rarefied, and occupied the whole veſſel. The volatilization of water, therefore, ſeems nothing more than a direct combination of caloric with it.

It is now very generally admitted, that the combuſtion of vital and inflammable airs produces water. By ſome experiments of Mr. Cavendiſh, it appears that he changed common air into pure water by decompoſing it in conjunction with inflammable gas. Meſſrs. Lavoiſier and others, ſeem to have proved, by ſatisfactory experiments, that the whole maſs of water may be converted into vital and inflammable airs, and that the combuſtion of theſe two gaſes produce a volume of water proportioned to the weight of the two principles employed.

EXP. 1. If a ſmall glaſs veſſel be inverted over mercury, and a known quantity of diſtilled water and iron filings be put into the upper part of the veſſel, inflammable air will be gradually diſengaged, the iron will ruſt, and the water with which it is moiſtened will be diminiſhed, and at length diſappear, the weight of the inflammable air which is produced, and the augmentation of the iron, will be equivalent to the weight of water made uſe of.

It is therefore inferred, that the water is reduced into two principles, the one of which is inflammable air, and the other is the principle which has entered into combination with the metal. Now it is known that the calcination of metals is owing to vital air; and conſequently the two ſubſtances produced, namely, the vital and inflammable air, ariſe from the decompoſition of water.

2. When water is converted into a ſtate of vapour, in its paſſage through an ignited iron tube, the iron becomes calcined, and inflammable air is obtained. The increaſe of weight in the metal, and the weight of inflammable air obtained, form a ſum equal to that of the water employed.

3. From the experiments of the decompoſition of water, it appears that 100 parts of this fluid contained 84 parts of vital air and 16 of inflammable air. The following is the method which Dr. Higgins, in his lectures, uſed to exhibit the compoſition of water. AB is a large and very ſtrong barometrical glaſs tube ſtanding in a trough of mercury F. [Plate 13. Fig. 21.] Suppoſe the mercury to ſtand at *x*, introduce two meaſures of inflammable, and one of vital air, which will depreſs the mercury to *z*. By means of the two wires D and E, cemented carefully into the tube, and connected with an electrical machine, the gaſes air inflamed by the electrical ſpark, and being nearly conſumed, the mercury riſes almoſt to its former height, and the inſide of the tube is covered with water. The apparatus made uſe of in this experiment is very uſeful for the inflammation of any of the factitious airs. In this way Mr. Cavendiſh obtained nitrous acid by an inflammation of certain proportions of vital and azotic gaſes

4. Mr. Lavoiſier made uſe of the following apparatus in generating water. He took a large cryſtal balloon, to which were adapted four tubes, of theſe, three had ſtop-cocks; one fitted to an air-pump to exhauſt the air from the balloon; another for the purpoſe of admitting the vital air; and the third to admit the inflammable air: the other was intended to convey the electric ſpark, by which the gaſes were inflamed, and in proportion to the combuſ- the balloon was ſupplied with freſh gas. the combuſtion advanced there was a depoſition of water which gradually fell down the ſides of the veſſel; weight of which being compared with that of the airs conſumed, it was found that 85 parts by weight of vital, and 15 of inflammable air, were required to the formation of 100 parts of water.

Thus it appears that water is compoſed of vital and inflammable airs, or in the language of the French Nomenclature, of hydrogen and oxygen, which elements, when exiſting ſeparately, have ſo ſtrong an affinity for caloric as only to ſubſiſt under the form of gas, in the common temperature and preſſure of the atmoſphere.

5. Sixteen ounces of alkohol, burnt in an apparatus adapted for collecting all the water diſengaged during the combuſtion, will yield from 17 to 18 ounces of water.

Now, as no ſubſtance can furniſh more than its original quantity of matter; it follows that ſomething has united with the alkohol during its combuſtion, which muſt be the vital part of the atmoſpheric air.

If then it be admitted, that water is compoſed of two known principles, it muſt act like all other compound bodies, that is, according to the affinities of its conſtituent parts. It will, therefore in ſome inſtances, yield its inflammable air, and in others its vital air. If it be placed in contact with bodies, which have the ſtrongeſt affinity to vital air, as metals, oils, charcoal, &c. the oxygen will unite with theſe ſubſtances, and the hydrogen will eſcape. In vegetables, on the contrary, it ſeems, that the hydrogen is the principle which is abſorbed, and the oxygen is diſengaged and eſcapes.

FINIS.

Pl.VII.

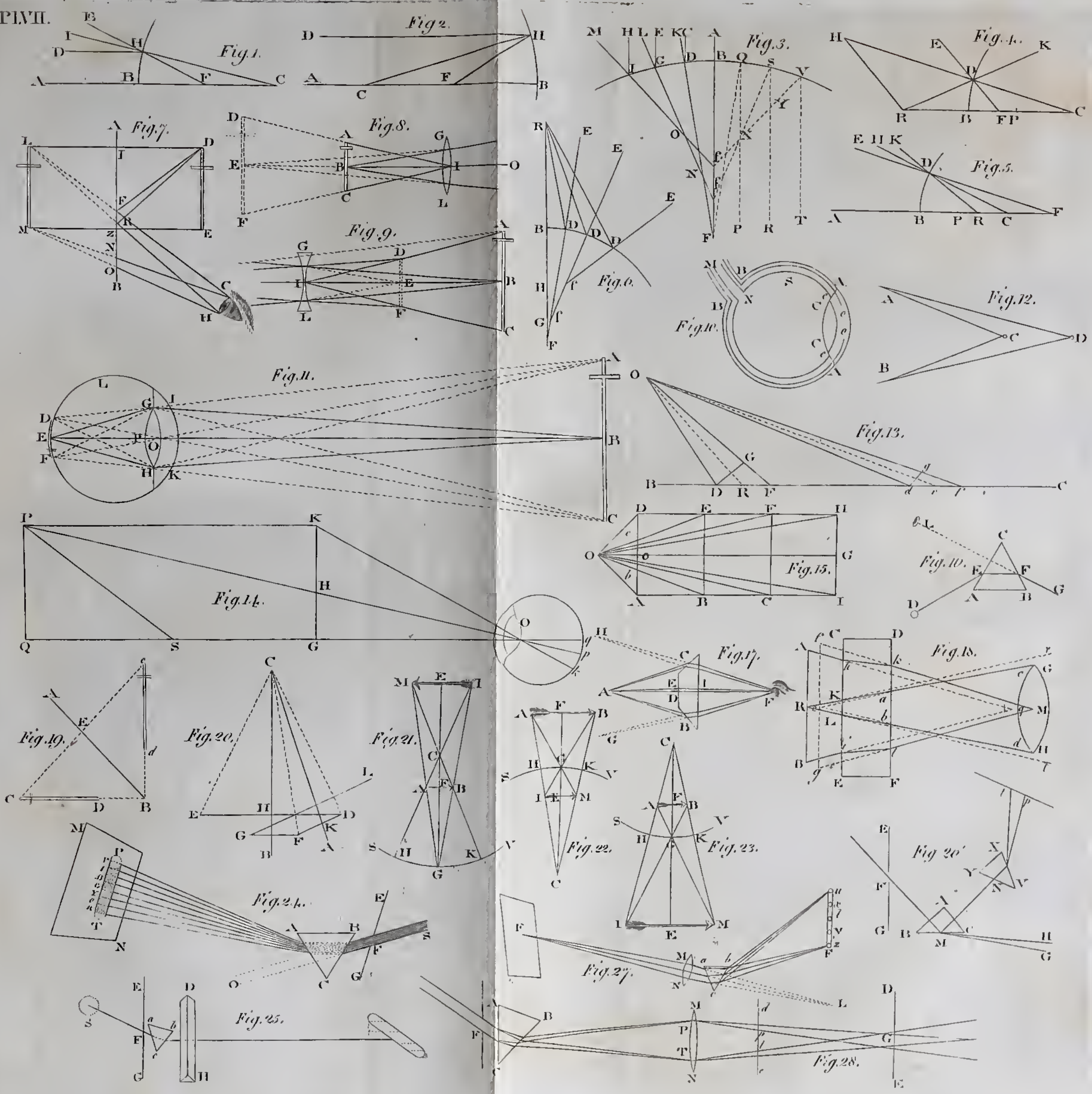
Fig.1.
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Fig.4.
Fig.5.
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Fig.11.
Fig.12.
Fig.13.
Fig.14.
Fig.15.
Fig.10.
Fig.17.
Fig.18.
Fig.19.
Fig.20.
Fig.21.
Fig.22.
Fig.23.
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Fig.25.
Fig 26'
Fig.27.
Fig.28.

Pl. VIII.

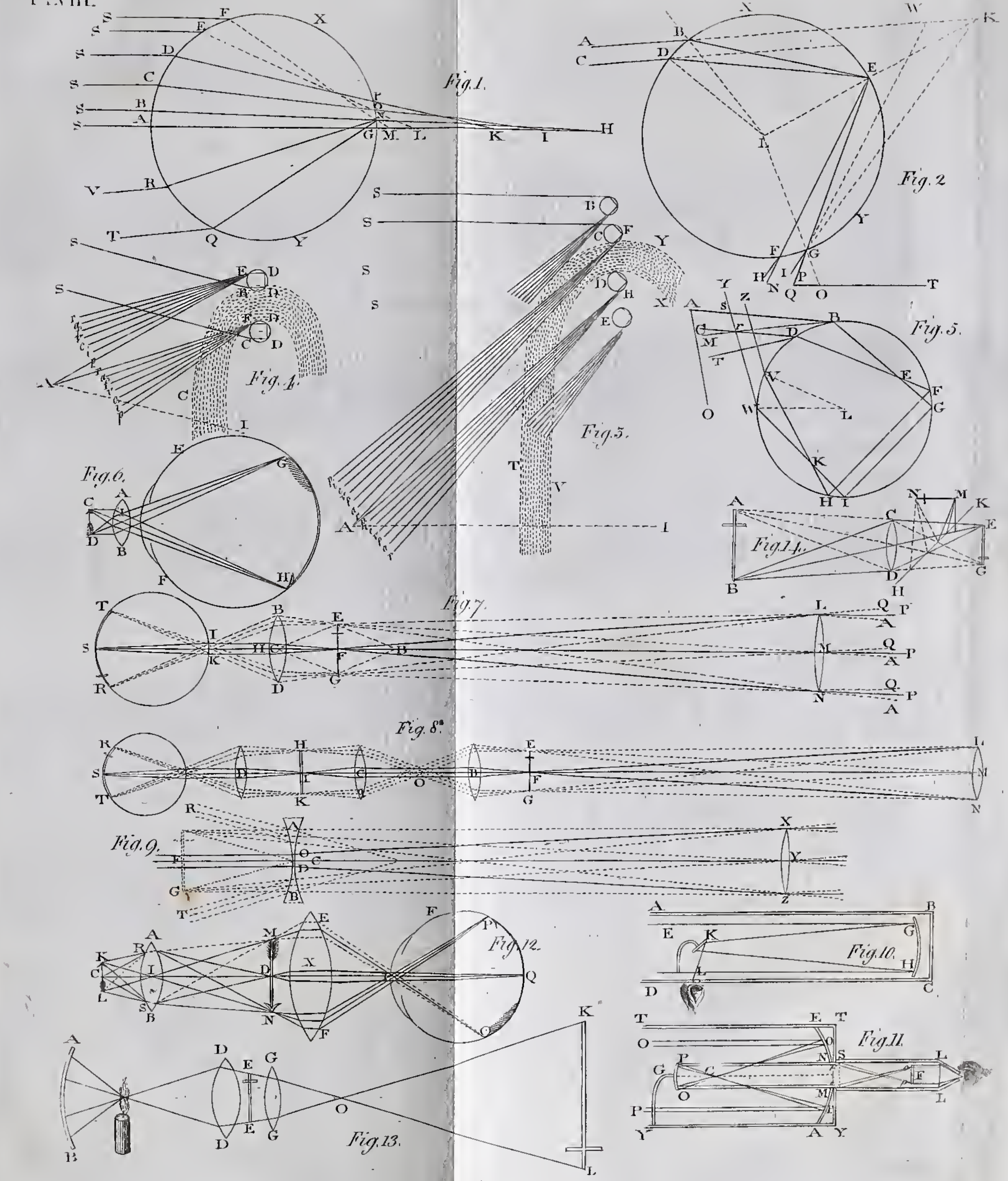

Fig. 1.
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Fig. 3.
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Fig. 12.
Fig. 13.
Fig. 14.

Pl.IX.

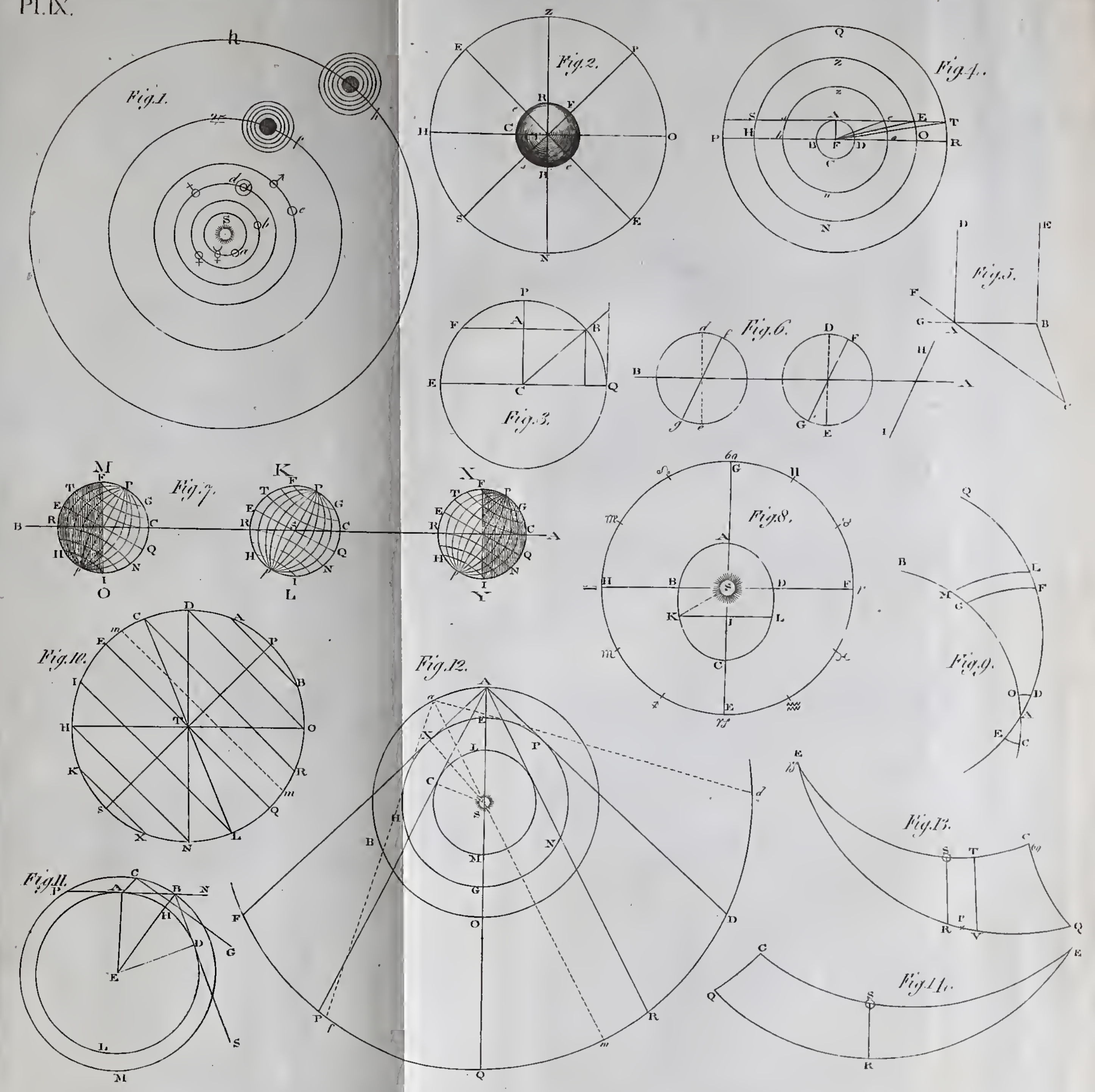
Fig.1.
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Fig.14.

Pl. X.

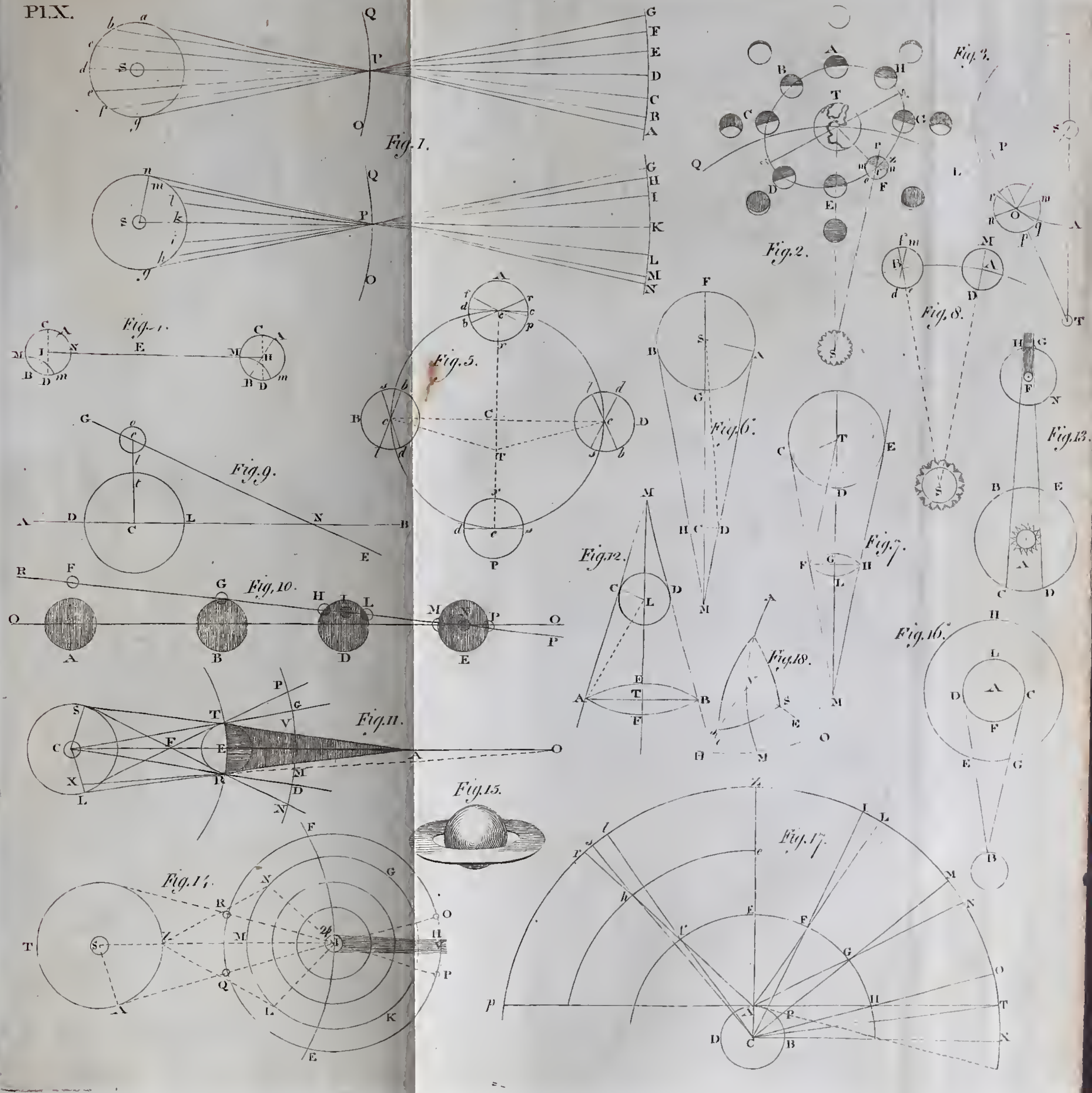
Fig. 1.
Fig. 2.
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Fig. 11.
Fig. 12.
Fig. 13.
Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
a b c d e f g S Q P O G F E D C B A
n m l k i h g S Q P O G H I K L M N
C A N M I E D m B H
G e l t A D C L N B E
R F G H I L M N P O A B D E
S T C F E R X L P G V M D N A O
T S Z R M Q A F N G L K E O H P
A B C D E F G H T Q S r z m u
A f d c b r c p F B C D l d s b e T P
F S A B G H C D M
M C L D A E T B F
C T E D G F H L M
r m u n O q p A S P L T
B A M D Fig. 8. S
H G F N B E A C D
H L D A C F E G B
Z I L M N O T X l r h e E F G H P A D C B p
A H M S E O

PL. XIII.

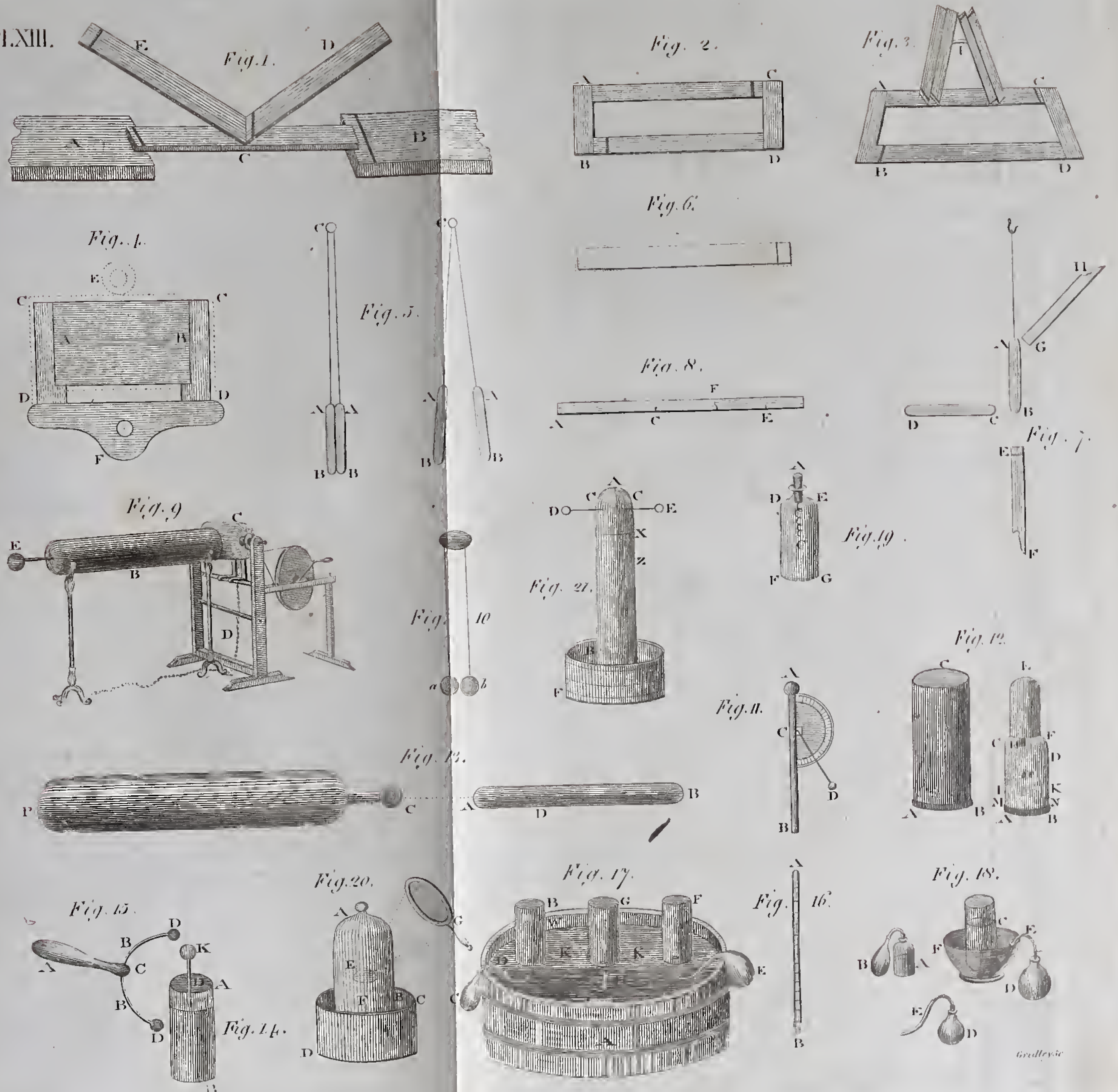
Fig. 1.
E
D
A
B
C
Fig. 2.
A
C
B
D
Fig. 3.
I
A
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Fig. 4.
E
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Fig. 5.
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A A
B B
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Fig. 6.
Fig. 8.
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Fig. 7.
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Fig. 9
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Fig. 10
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Fig. 21.
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Fig. 19.
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Fig. 12.
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Fig. 11.
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Fig. 13.
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Fig. 15.
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Fig. 14.
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Fig. 20.
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Fig. 17.
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Fig. 16.
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Fig. 18.
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PL.XIV.

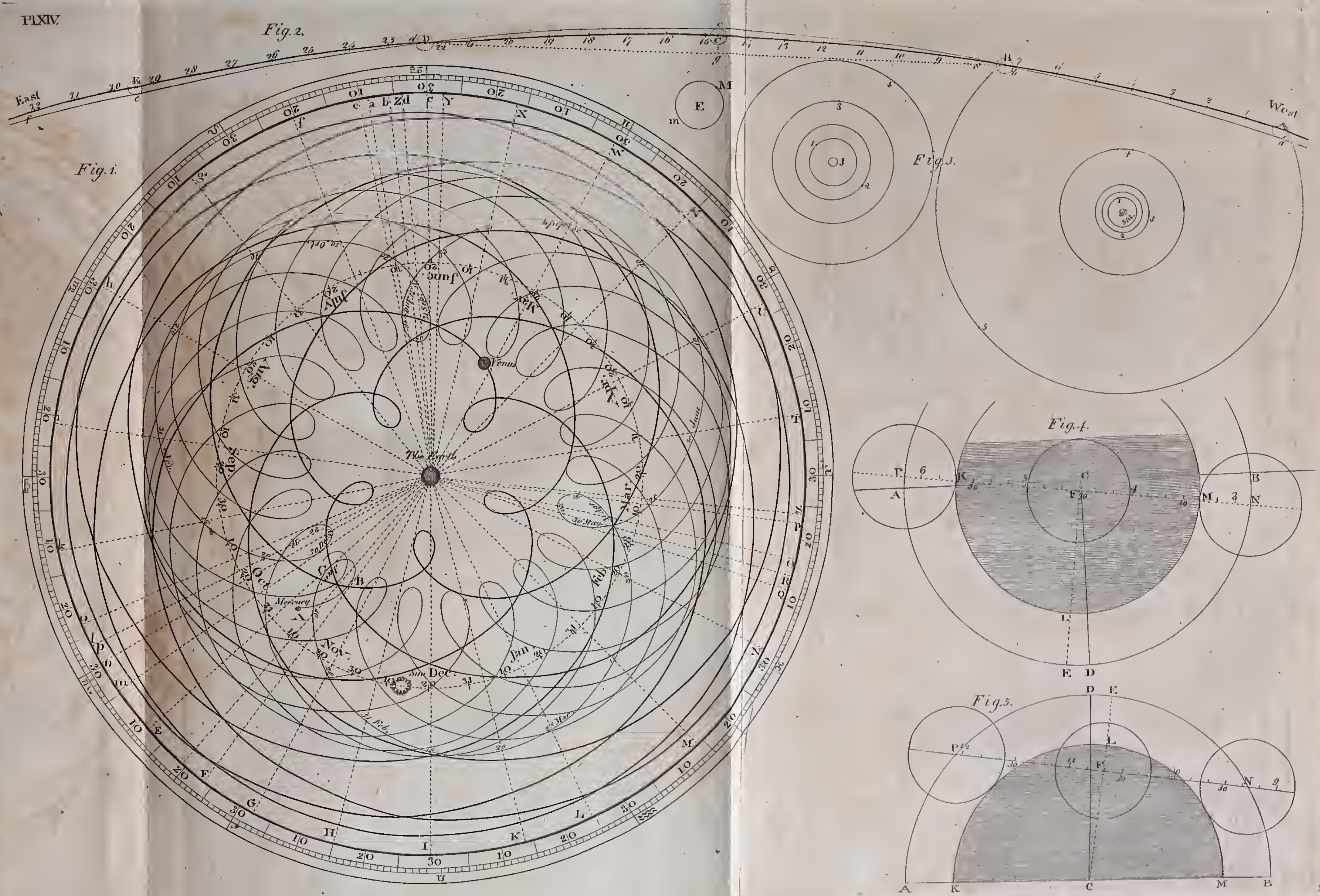

Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
East
West
The Earth
Venus
Mercury
Sun
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
Sep.
Oct.
Nov.
Dec.
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M
m
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Sat.
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LXV.

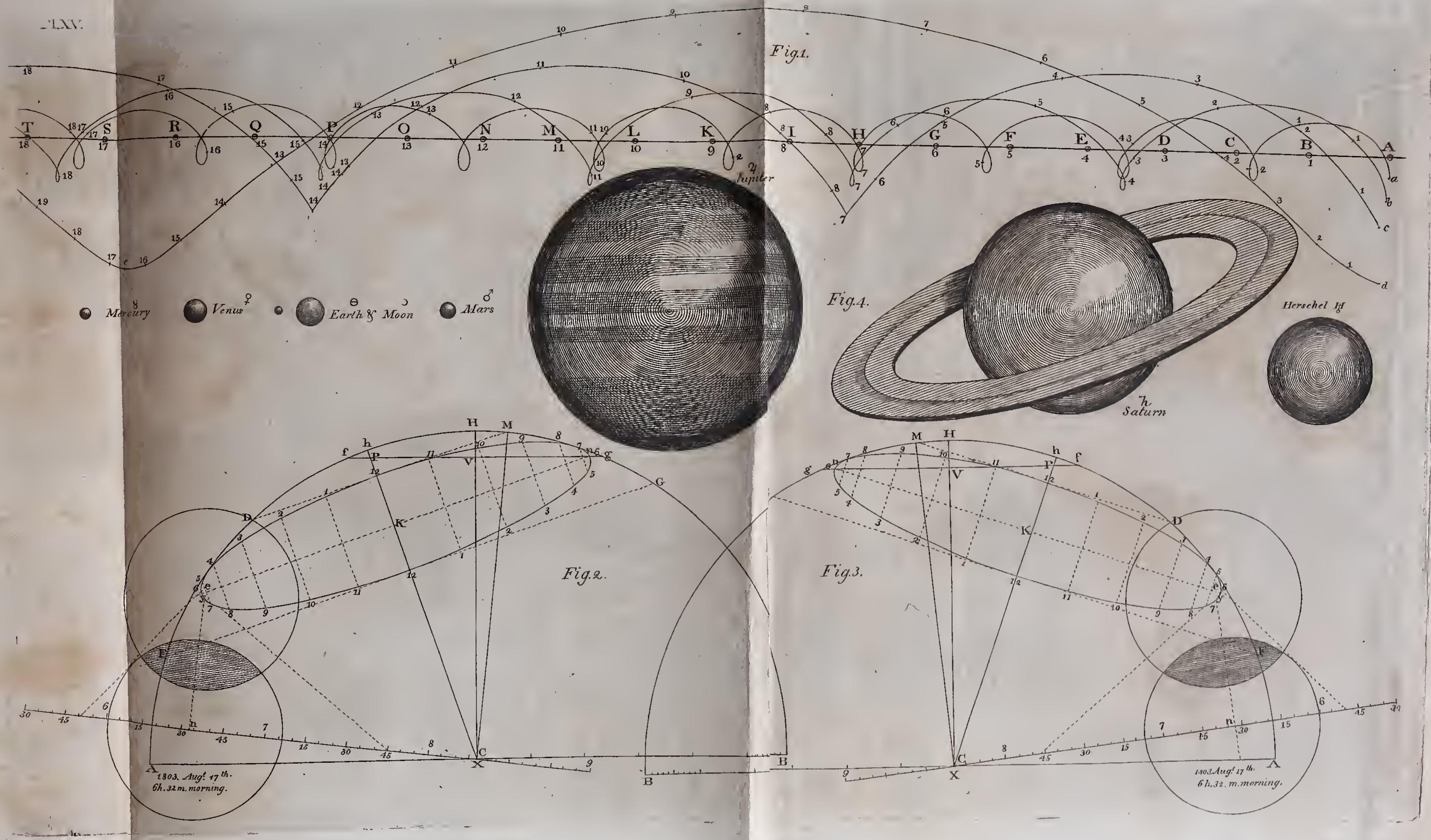
Fig.1.
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F
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H
I
K
L
M
N
O
P
Q
R
S
T
a
b
c
d
Mercury ☿
Venus ♀
Earth ⊕ & Moon ☽
Mars ♂
♃ Jupiter
Fig.4.
♄ Saturn
Herschel ♅
Fig.2.
Fig.3.
1803. Aug.t 17th. 6h. 32m. morning.
1803. Aug.t 17th. 6h. 32m. morning.